6.172
Performance
Engineering
of Software
Systems

LECTURE 7

Races and Parallelism

Julian Shun

Recall: Basics of Cilk

```
int fib(int n)
{
   if (n < 2) return n;
   int x, y;
   x = cilk_spawn fib(n-1);
   y = fib(n-2);
   cilk_sync;
   return x + y;
}
```

The named *child* function may execute in parallel with the *parent* caller.

Control cannot pass this point until all spawned children have returned.

Cilk keywords grant permission for parallel execution. They do not command parallel execution.

Loop Parallelism in Cilk

Example:
In-place
matrix
transpose

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{pmatrix}$$

The iterations of a cilk_for loop execute in parallel.

```
// indices run from 0, not 1
cilk_for (int i=1; i<n; ++i) {
    for (int j=0; j<i; ++j) {
        double temp = A[i][j];
        A[i][j] = A[j][i];
        A[j][i] = temp;
    }
}
```

Determinacy Races

Race Conditions

Race conditions are the bane of concurrency. Famous race bugs include the following:

- Therac-25 radiation therapy machine — killed 3 people and seriously injured many more.
- North American Blackout of 2003 — left 50 million people without power.

Race bugs are notoriously difficult to discover by conventional testing!

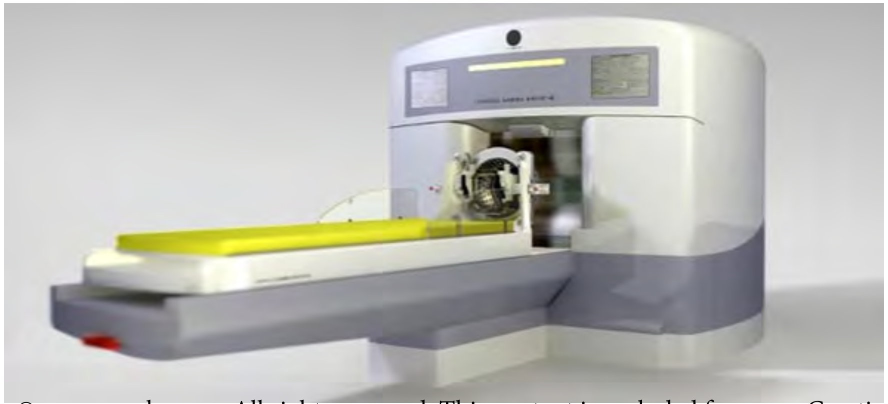


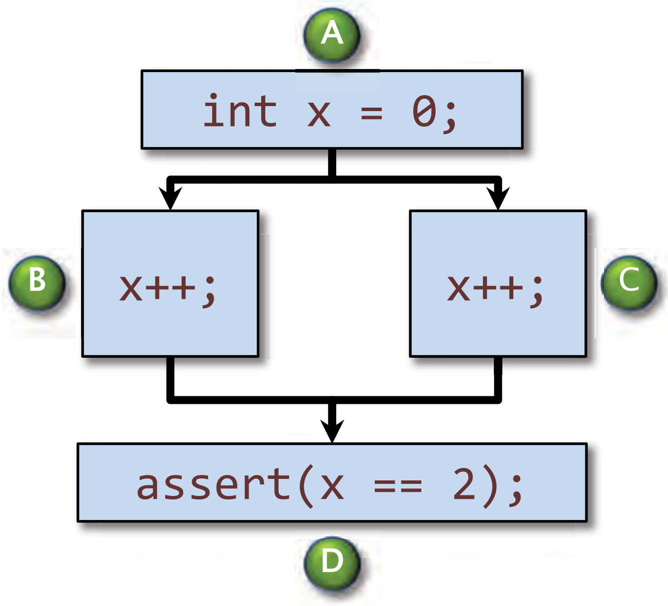
Image created by MIT OpenCourseWare from public domain image.

Determinacy Races

Definition. A determinacy race occurs when two logically parallel instructions access the same memory location and at least one of the instructions performs a write.

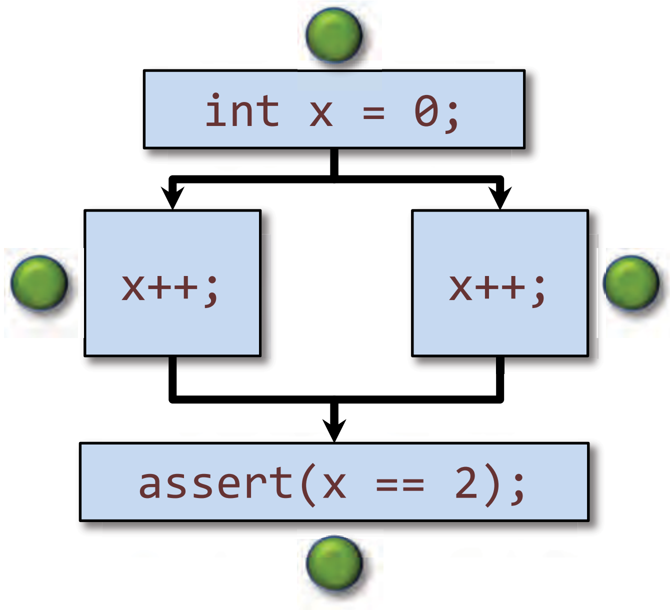
Example

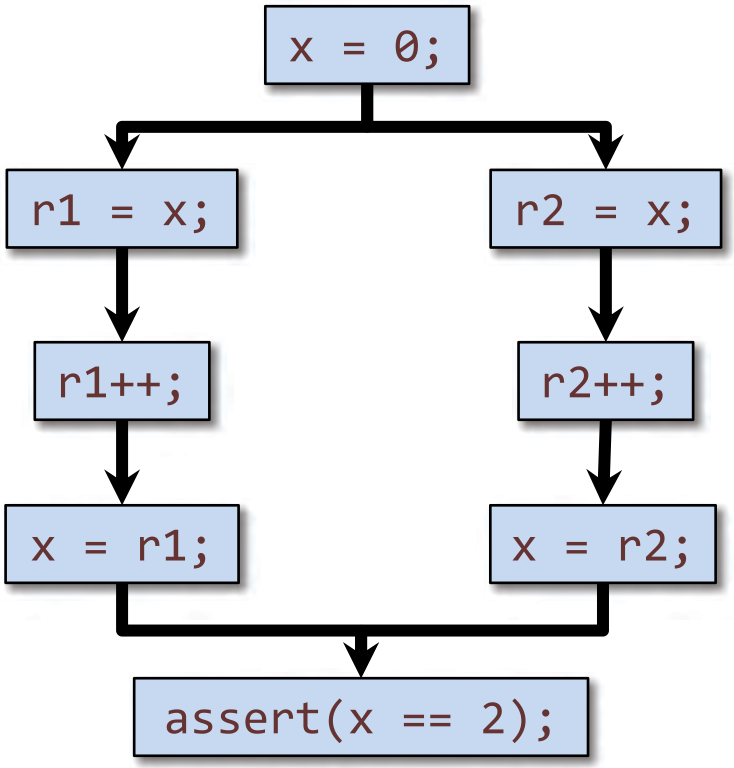
```
int x = 0;
cilk_for (int i=0, i<2, ++i) {
    x++;
}
assert(x == 2);
```



dependency graph

A Closer Look





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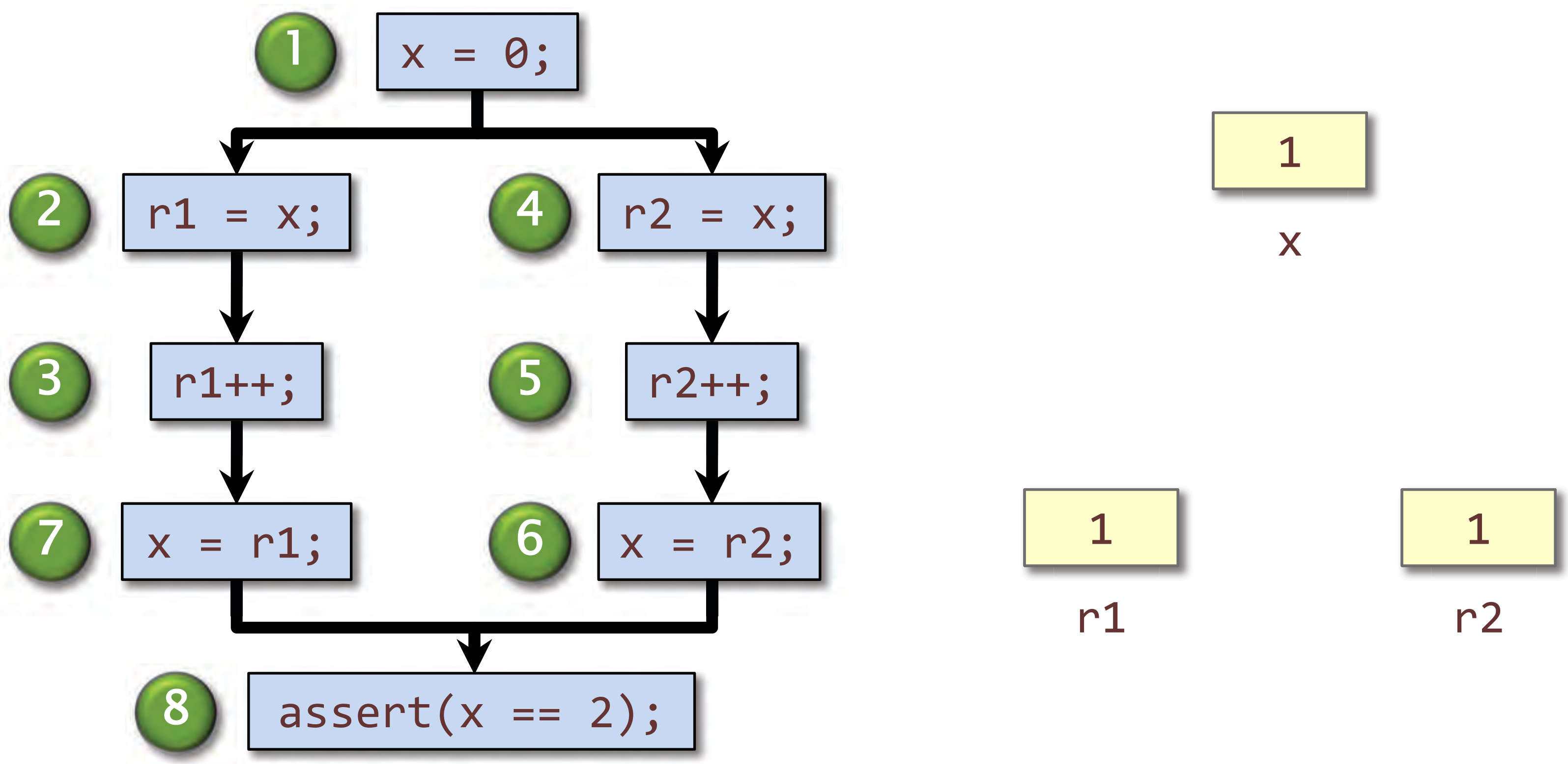

Types of Races

Suppose that instruction A and instruction B both access a location x, and suppose that A∥B (A is parallel to B).

A	B	Race Type
read	read	none
read	write	read race
write	read	read race
write	write	write race

Two sections of code are independent if they have no determinacy races between them.

Avoiding Races

- Iterations of a `cilk_for` should be independent.
- Between a `cilk_spawn` and the corresponding `cilk_sync`, the code of the spawned child should be independent of the code of the parent, including code executed by additional spawned or called children.
  - **Note:** The arguments to a spawned function are evaluated in the parent before the spawn occurs.
- Machine word size matters. Watch out for races in packed data structures:

```
struct {
  char a;
  char b;
} x;
```

Ex. Updating `x.a` and `x.b` in parallel may cause a race! Nasty, because it may depend on the compiler optimization level. (Safe on Intel x86-64.)

Cilksan Race Detector

- The Cilksan-instrumented program is produced by compiling with the `-fsanitize=cilk` command-line compiler switch.
- If an ostensibly deterministic Cilk program run on a given input could possibly behave any differently than its serial elision, Cilksan guarantees to report and localize the offending race.
- Cilksan employs a regression-test methodology, where the programmer provides test inputs.
- Cilksan identifies filenames, lines, and variables involved in races, including stack traces.
- Ensure that all program files are instrumented, or you'll miss some bugs.
- Cilksan is your best friend.

Cilksan Output

```
$ cilksan ./mm_dac
…
Race detected at address 0x65c070
  Write access to C (declared at mm/mm_dac.c:27)
             from 0x400ed0 mm_base mm/mm_dac.c:34:15
      Called from 0x401868 mm_dac mm/mm_dac.c:57:5
      Called from 0x4025d0 mm_dac mm/mm_dac.c:63:5
     Spawned from 0x401548 mm_dac mm/mm_dac.c:63:5
      Called from 0x4025d0 mm_dac mm/mm_dac.c:63:5
     Spawned from 0x401548 mm_dac mm/mm_dac.c:63:5
  Read access to  C (declared at mm/mm_dac.c:27)
             from 0x400e27 mm_base mm/mm_dac.c:34:15
      Called from 0x401868 mm_dac mm/mm_dac.c:57:5
  Common calling context
      Called from 0x401c02 main mm/mm_dac.c:85:3

0.686637

Race detector detected total of 47 races.
Race detector suppressed 147409 duplicate error messages
```

What Is Parallelism?

Execution Model

```
int fib (int n) {
  if (n < 2) return n;
  else {
    int x, y;
    x = cilk_spawn fib(n-1);
    y = fib(n-2);
    cilk_sync;
    return x + y;
  }
}
```

Example:
`fib(4)`

Execution Model

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Example:
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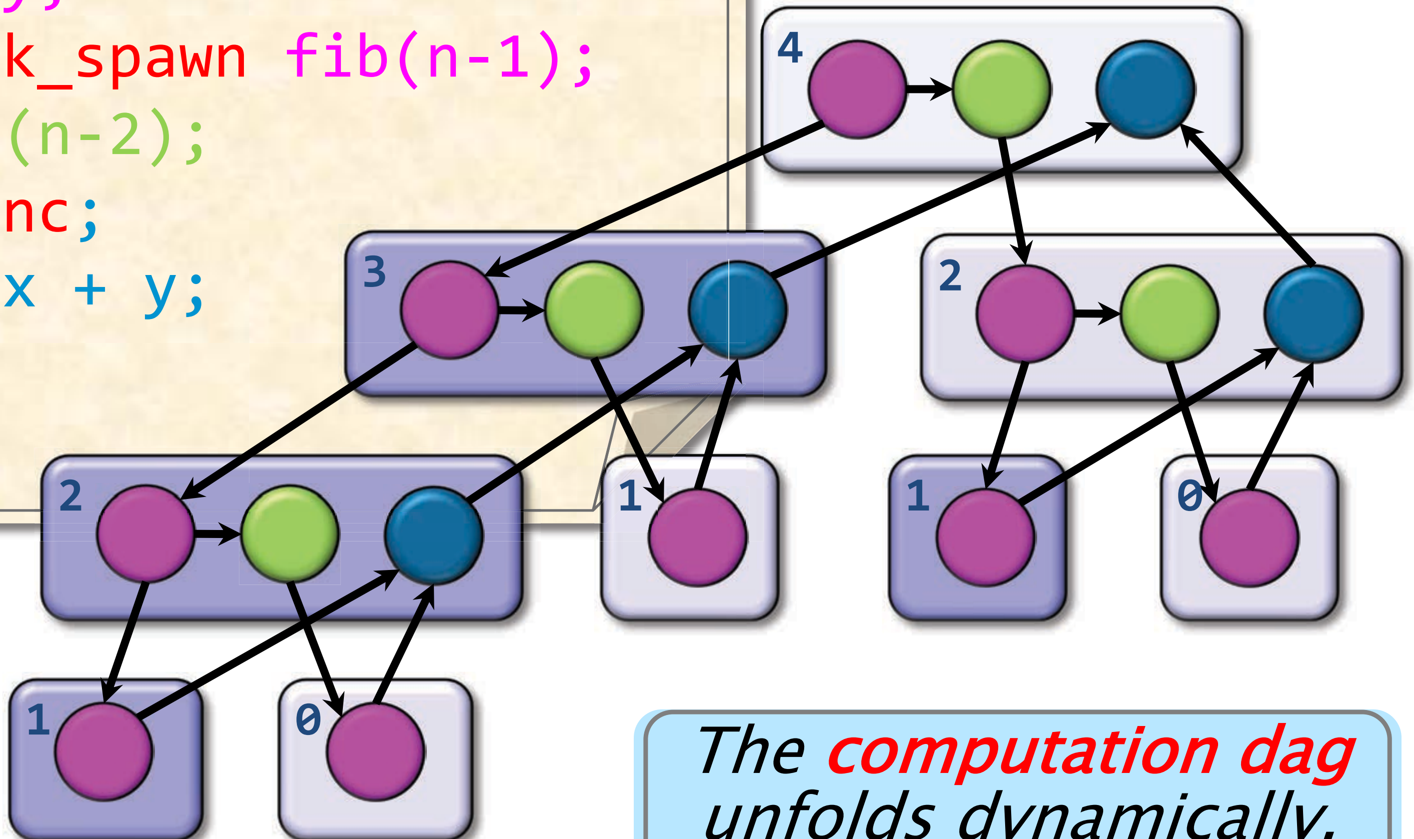
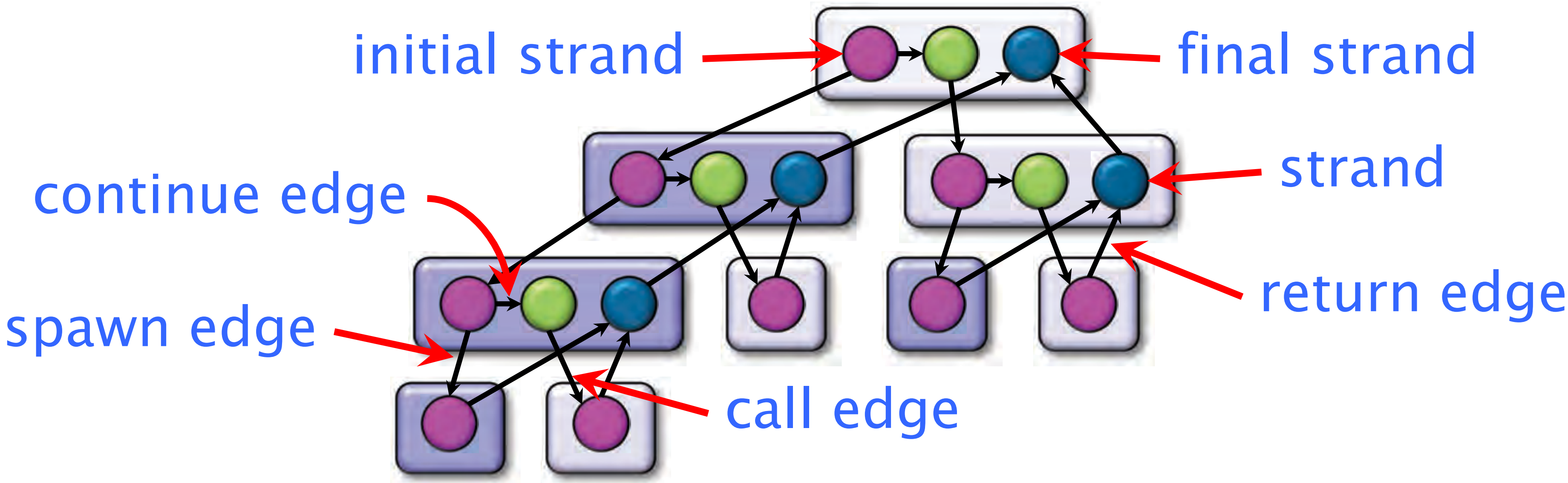


"Processor oblivious"

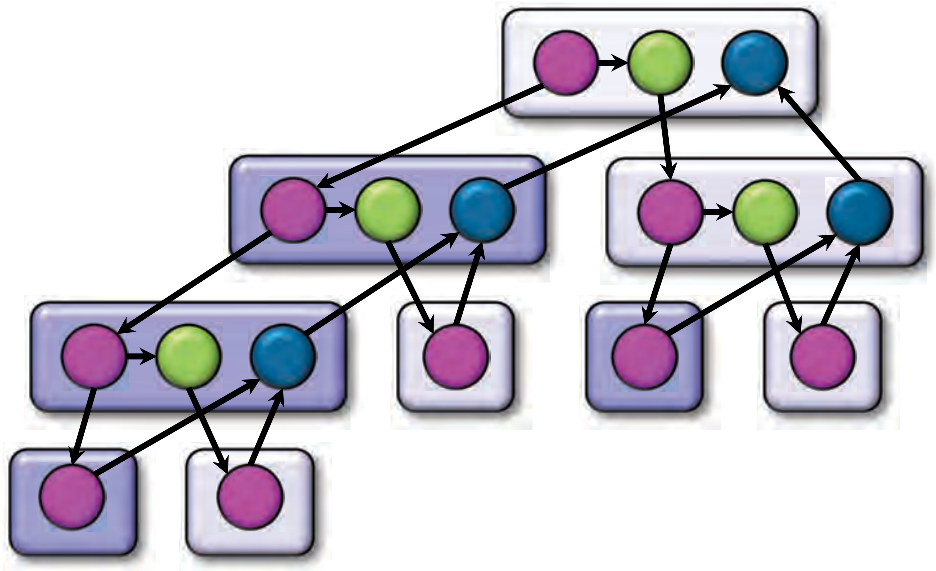
*The **computation dag** unfolds dynamically.*

Computation Dag



- A parallel instruction stream is a dag $G = (V, E)$.
- Each vertex $v \in V$ is a strand: a sequence of instructions not containing a spawn, sync, or return from a spawn.
- An edge $e \in E$ is a spawn, call, return, or continue edge.
- Loop parallelism (`cilk_for`) is converted to spawns and syncs using recursive divide-and-conquer.

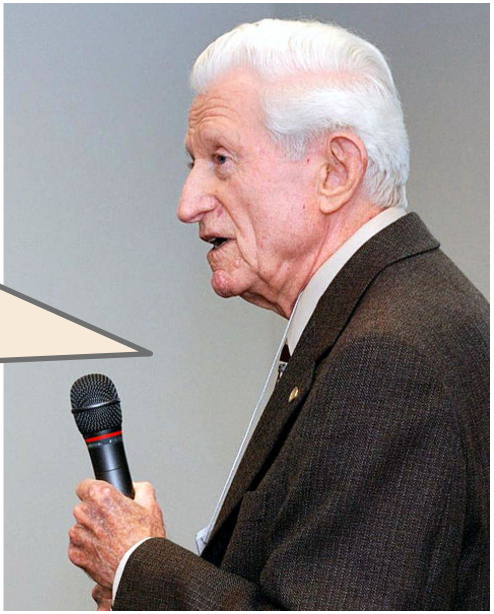
How Much Parallelism?



Assuming that each strand executes in unit time, what is the parallelism of this computation?

Amdahl's "Law"

If 50% of your application is parallel and 50% is serial, you can't get more than a factor of 2 speedup, no matter how many processors it runs on.

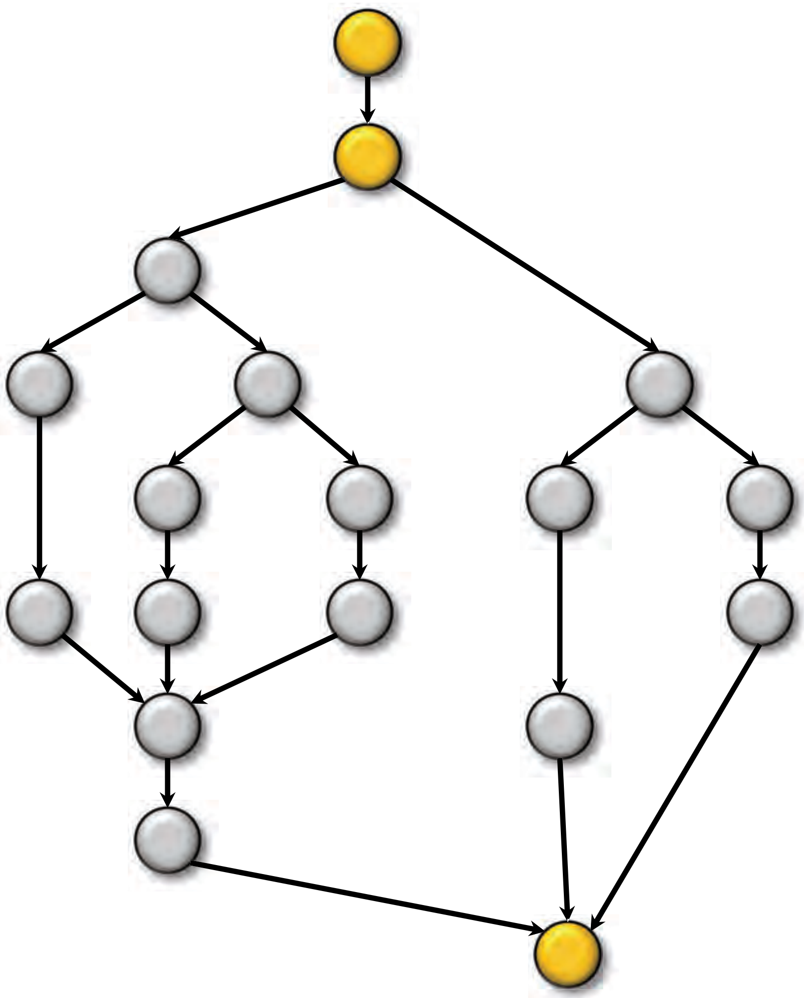


Gene M. Amdahl

In general, if a fraction α of an application must be run serially, the speedup can be at most 1/α.

Quantifying Parallelism

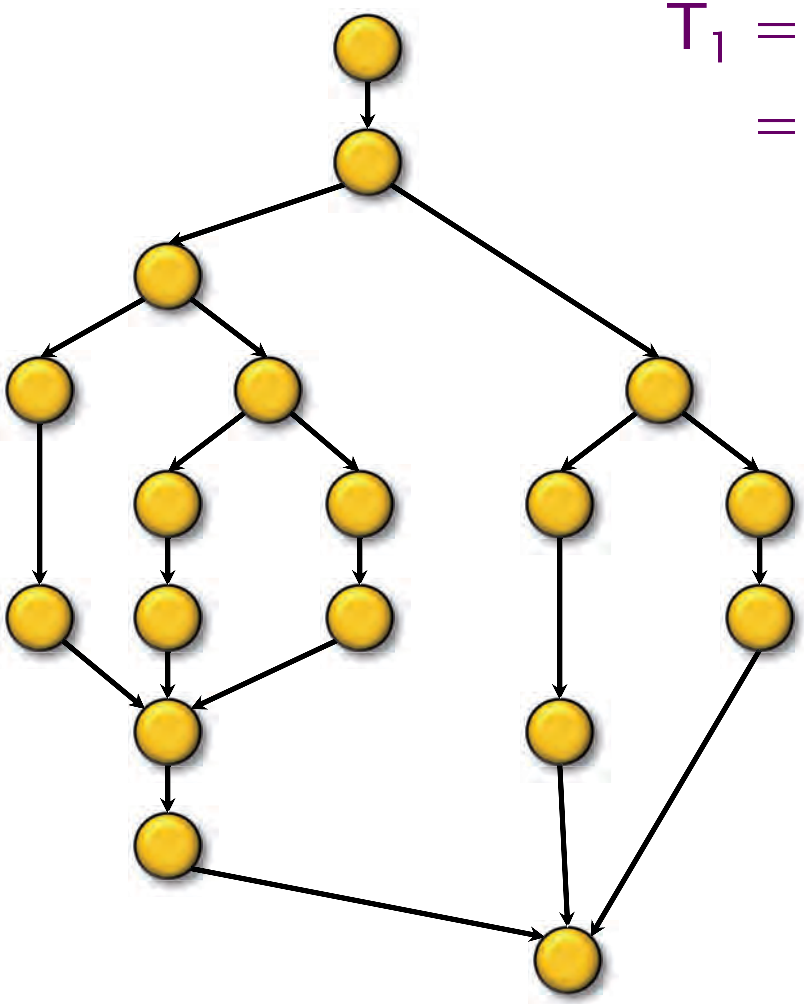
What is the parallelism of this computation?



Amdahl's Law says that since the serial fraction is 3/18 = 1/6, the speedup is upper-bounded by 6.

Performance Measures

T_P = execution time on P processors

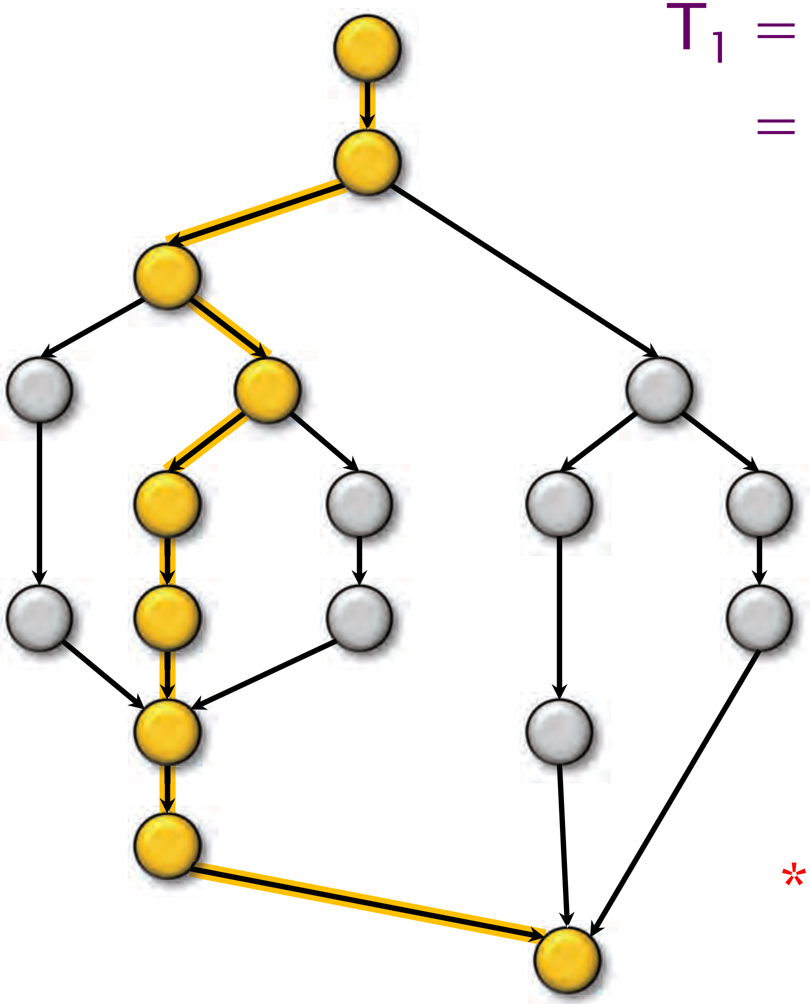


T_1 = work

= 18

Performance Measures

T_P = execution time on P processors



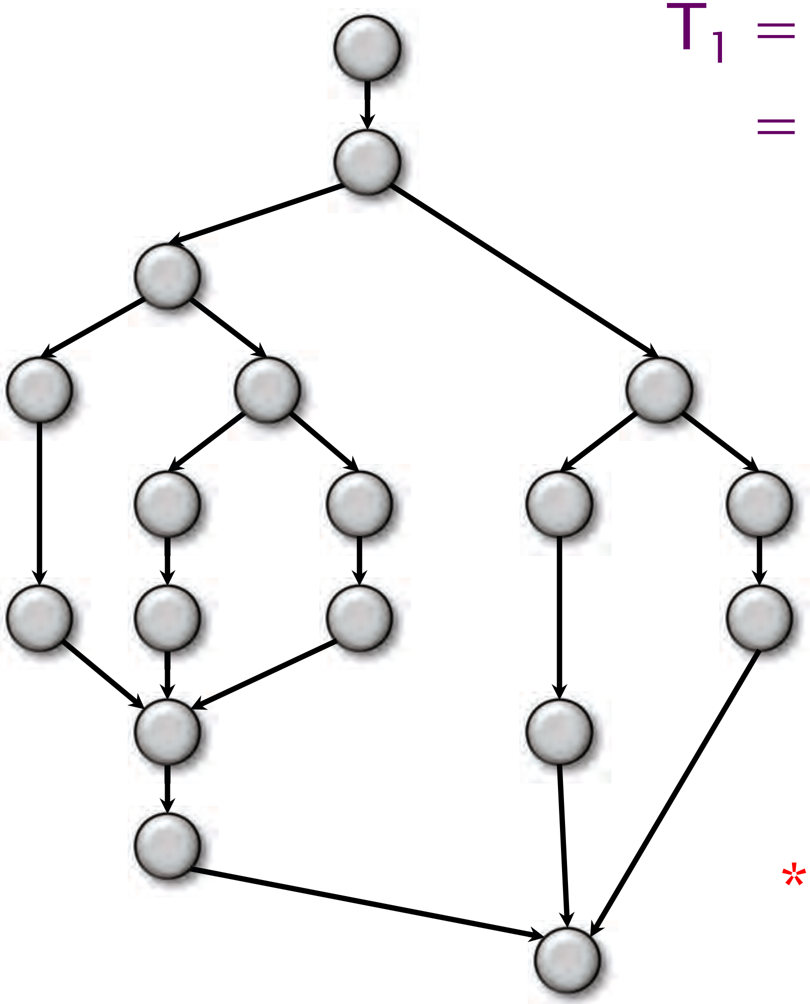
T_1 = work
= 18

T_∞ = span*
= 9

*Also called critical-path length or computational depth.

Performance Measures

T_P = execution time on P processors



T_1 = work
= 18

T_∞ = span*
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Work Law

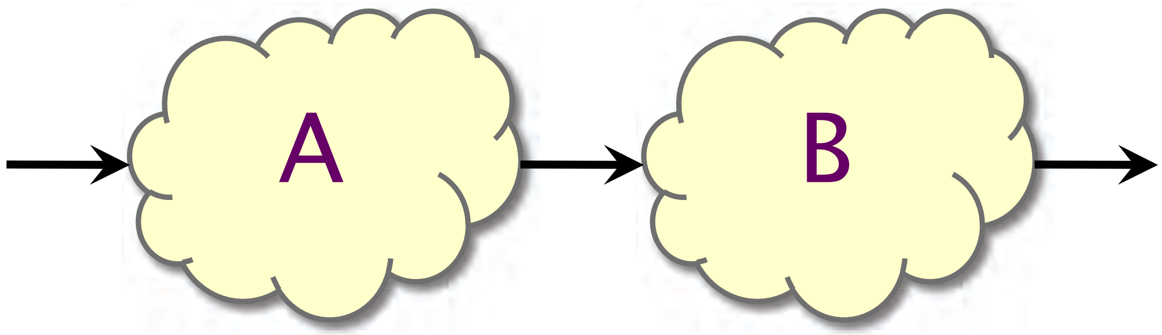
- $T_P \geq T_1/P$

Span Law

- $T_P \geq T_\infty$

*Also called critical-path length or computational depth.

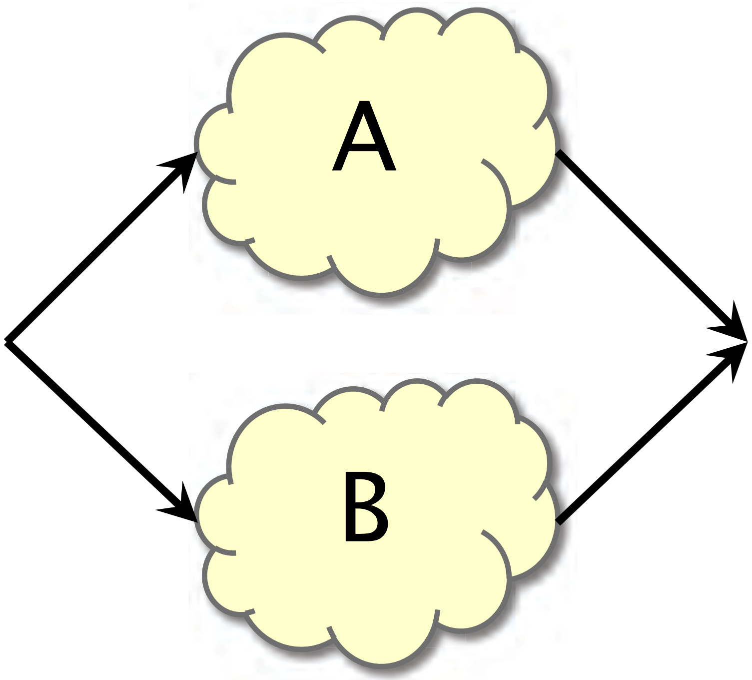
Series Composition



Work: $T_1(A \cup B) = T_1(A) + T_1(B)$

Span: $T_\infty(A \cup B) = T_\infty(A) + T_\infty(B)$

Parallel Composition



Work: $T_1(A \cup B) = T_1(A) + T_1(B)$

Span: $T_\infty(A \cup B) = \max\{T_\infty(A), T_\infty(B)\}$

Speedup

Definition. T_1/T_P = speedup on P processors.

- If $T_1/T_P < P$, we have sublinear speedup.
- If $T_1/T_P = P$, we have (perfect) linear speedup.
- If $T_1/T_P > P$, we have superlinear speedup, which is not possible in this simple performance model, because of the WORK LAW $T_P \geq T_1/P$.

Parallelism

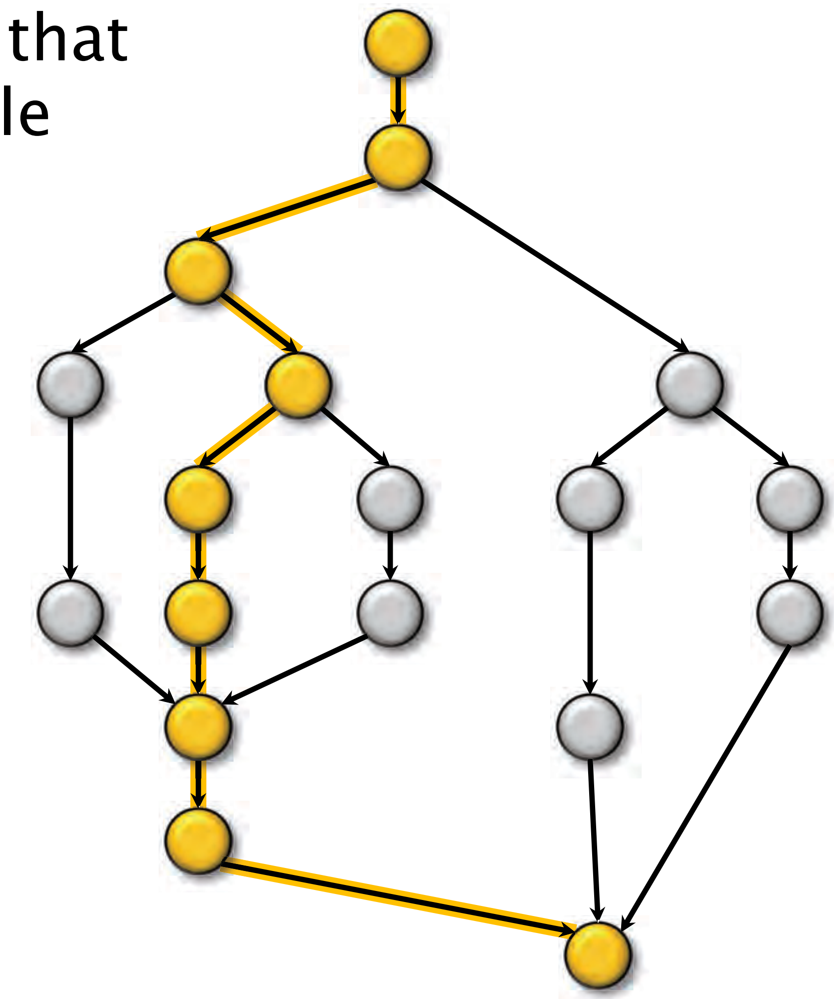
Because the SPAN LAW dictates that $T_P \geq T_\infty$, the maximum possible speedup given T_1 and T_∞ is

T_1/T_∞ = parallelism

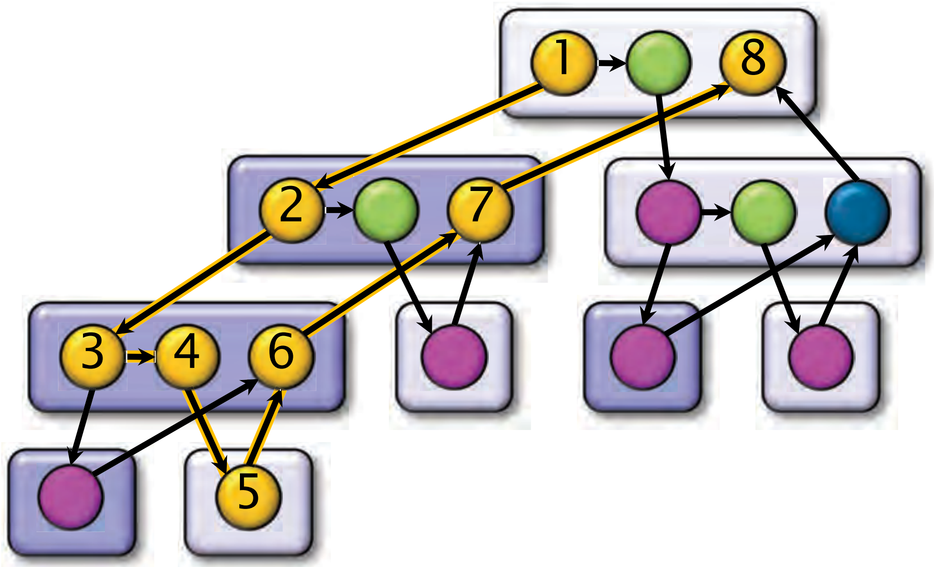
= the average amount of work per step along the span

= 18/9

= 2 .



Example: `fib(4)`



Assume for simplicity that each strand in `fib(4)` takes unit time to execute.

Work: $T_1 = 17$

Span: $T_\infty = 8$

Parallelism: $T_1/T_\infty = 2.125$

Using many more than 2 processors can yield only marginal performance gains.

THE CILKSCALE SCALABILITY ANALYZER

Cilkscale Scalability Analyzer

- The Tapir/LLVM compiler provides a scalability analyzer called Cilkscale.
- Like the Cilksan race detector, Cilkscale uses compiler-instrumentation to analyze a serial execution of a program.
- Cilkscale computes work and span to derive upper bounds on parallel performance.

Quicksort Analysis

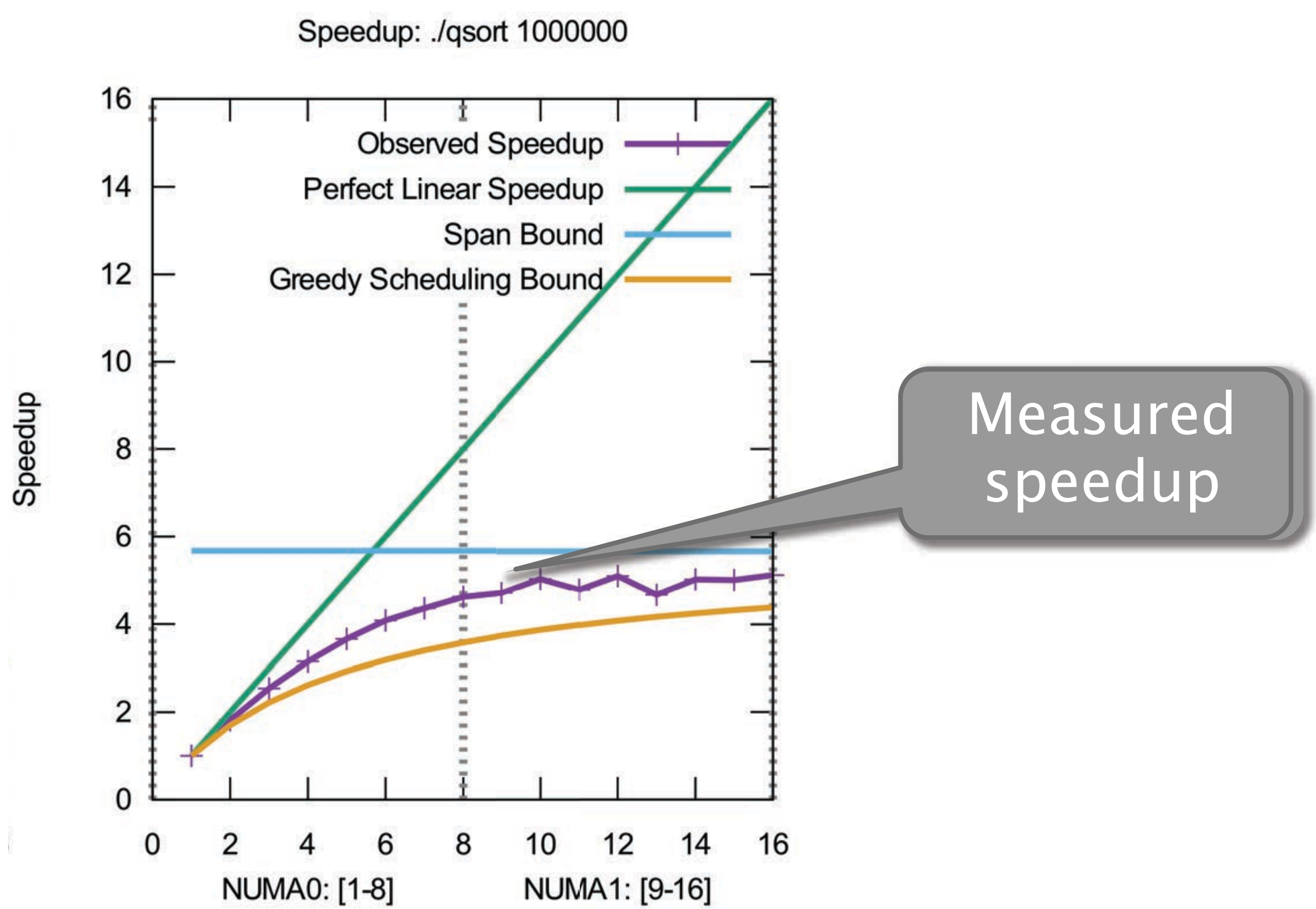
Example: Parallel quicksort

```
static void quicksort(size_t *left, size_t *right)
{
  if (left == right) return;
  size_t *p = partition(left, right); //run serially
  cilk_spawn quicksort(left, p);
  quicksort(p + 1, right);
  cilk_sync;
}
```

Analyze the sorting of 1,000,000 numbers.

★ ★ ★ *Guess the parallelism!* ★ ★ ★

Cilkscale Output



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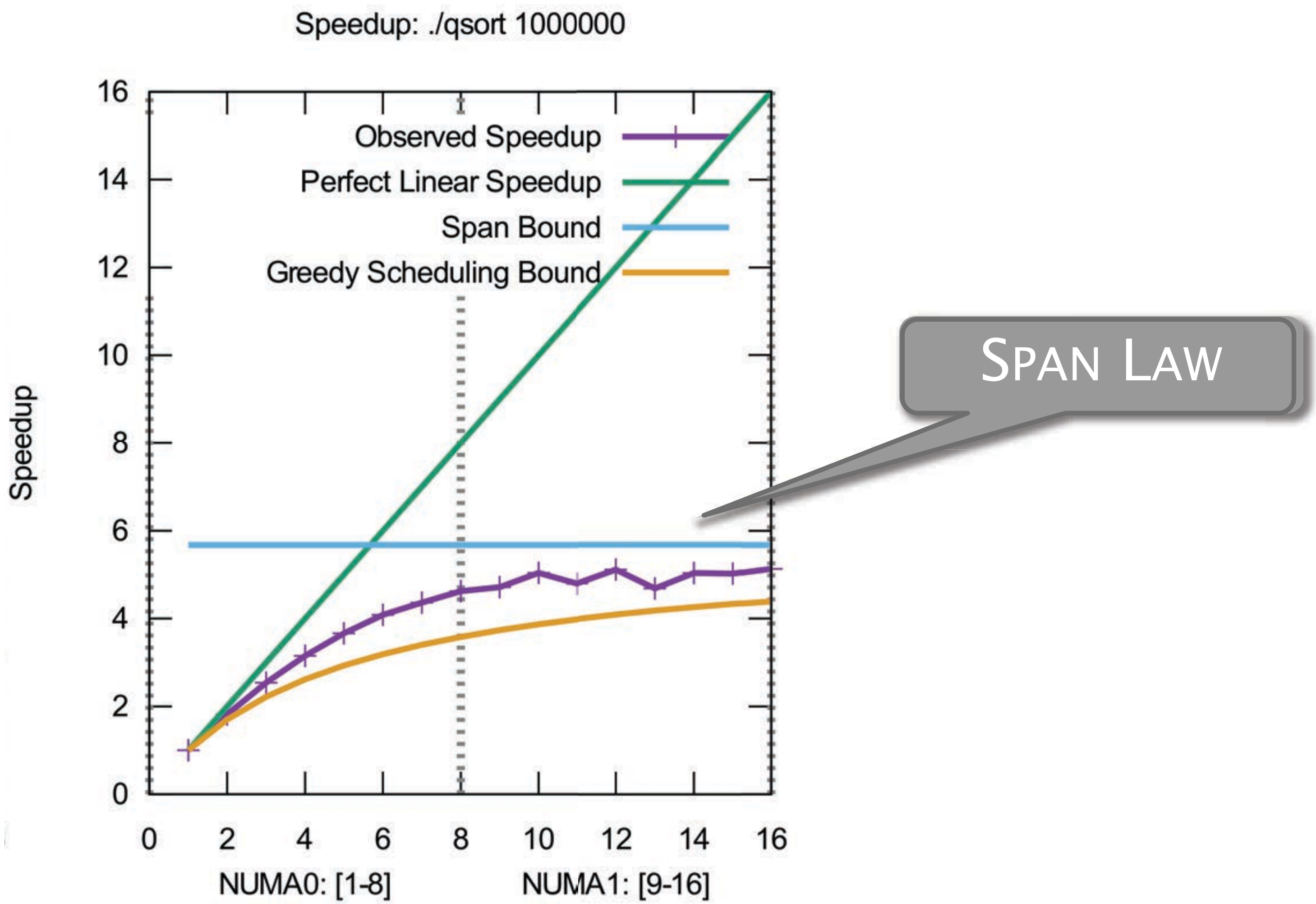

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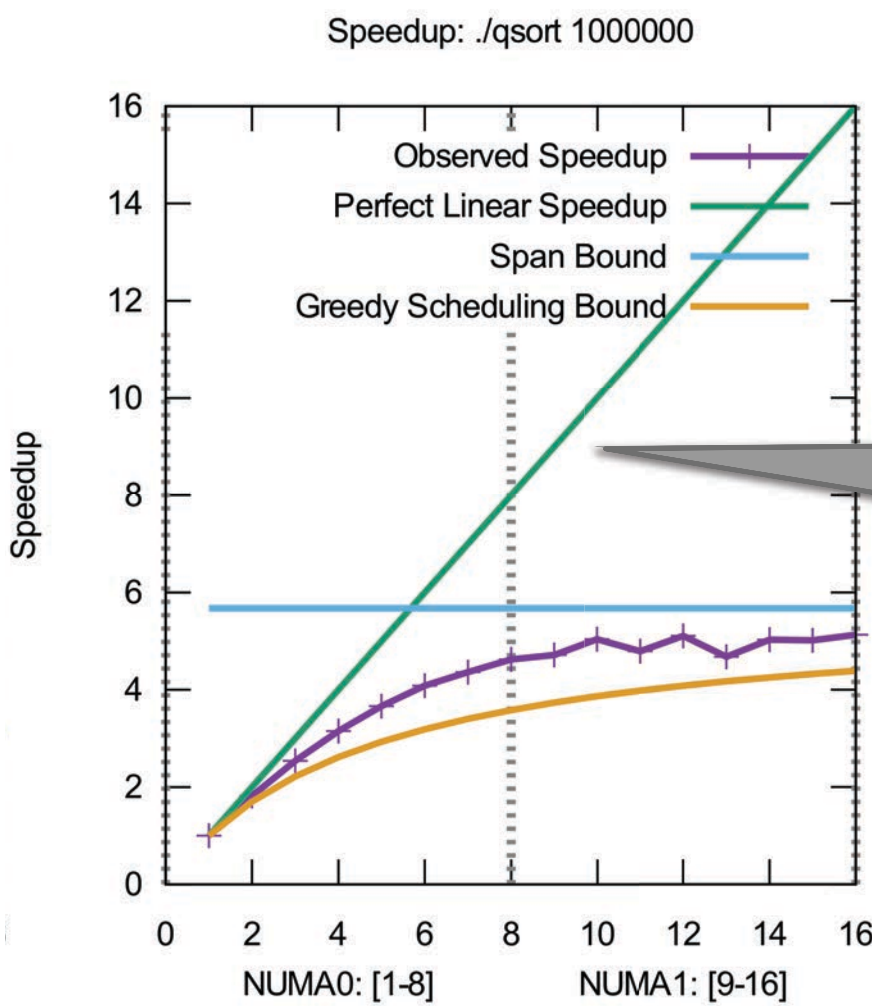
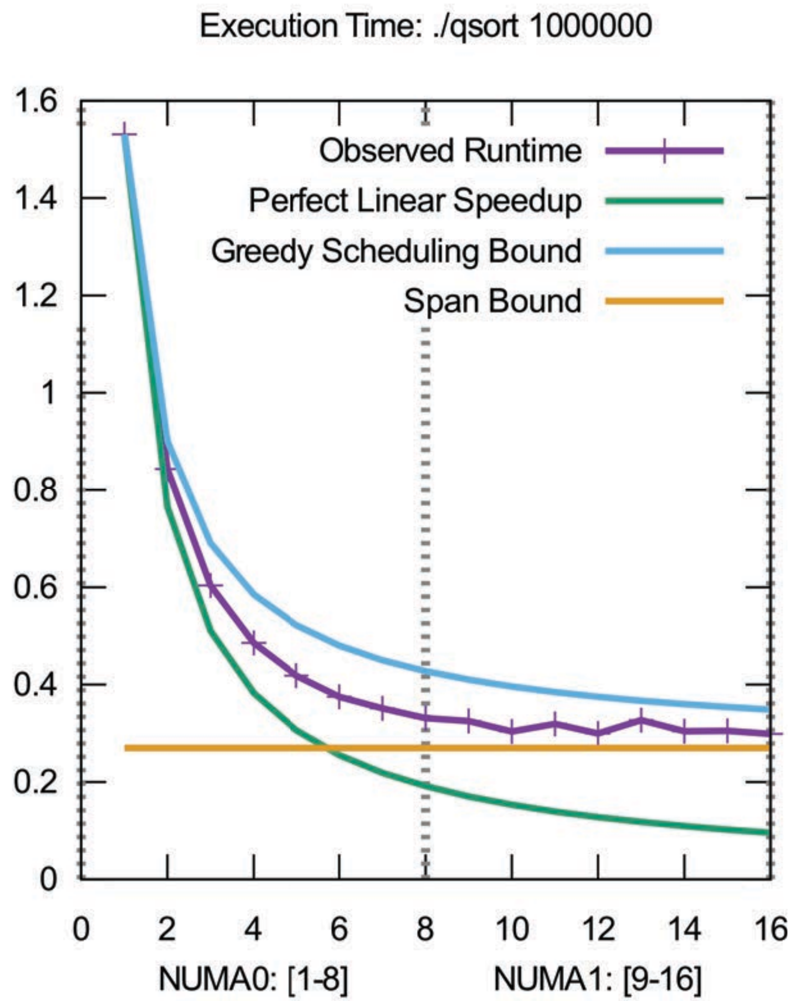


Work Law (linear speedup)

Cilkscale Output



Theoretical Analysis

Example: Parallel quicksort

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  quicksort(p + 1, right);
  cilk_sync;
}
```

Expected work = $\Theta(n \lg n)$
Expected span = $\Theta(n)$

⇒ Parallelism = $\Theta(\lg n)$

Interesting Practical* Algorithms

Algorithm	Work	Span	Parallelism
Merge sort	$\Theta(n \lg n)$	$\Theta(\lg^3 n)$	$\Theta(n/\lg^2 n)$
Matrix multiplication	$\Theta(n^3)$	$\Theta(\lg n)$	$\Theta(n^3/\lg n)$
Strassen	$\Theta(n^{\lg 7})$	$\Theta(\lg^2 n)$	$\Theta(n^{\lg 7}/\lg^2 n)$
LU-decomposition	$\Theta(n^3)$	$\Theta(n \lg n)$	$\Theta(n^2/\lg n)$
Tableau construction	$\Theta(n^2)$	$\Theta(n^{\lg 3})$	$\Theta(n^{2-\lg 3})$
FFT	$\Theta(n \lg n)$	$\Theta(\lg^2 n)$	$\Theta(n/\lg n)$
Breadth-first search	$\Theta(E)$	$\Theta(\Delta \lg V)$	$\Theta(E/\Delta \lg V)$

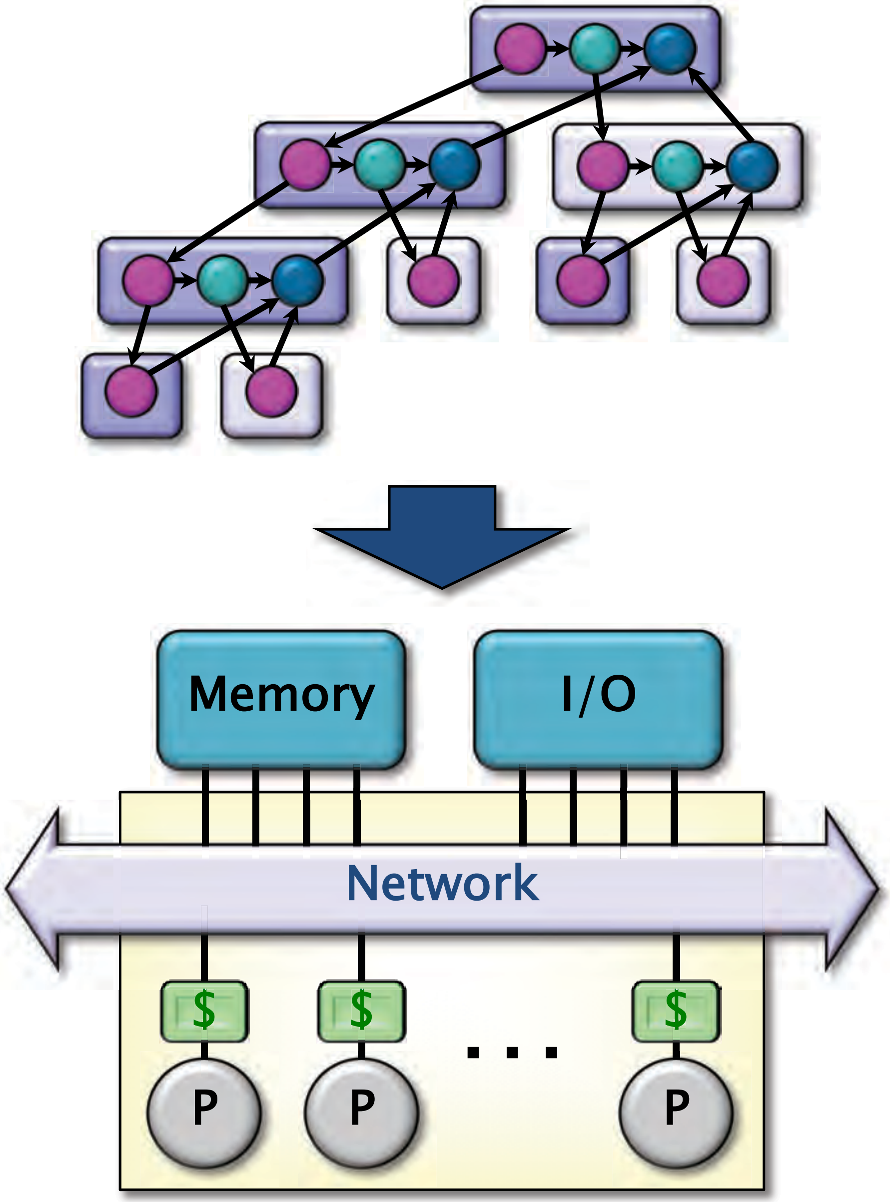
---

*Cilk on 1 processor competitive with the best C.

SCHEDULING THEORY

Scheduling

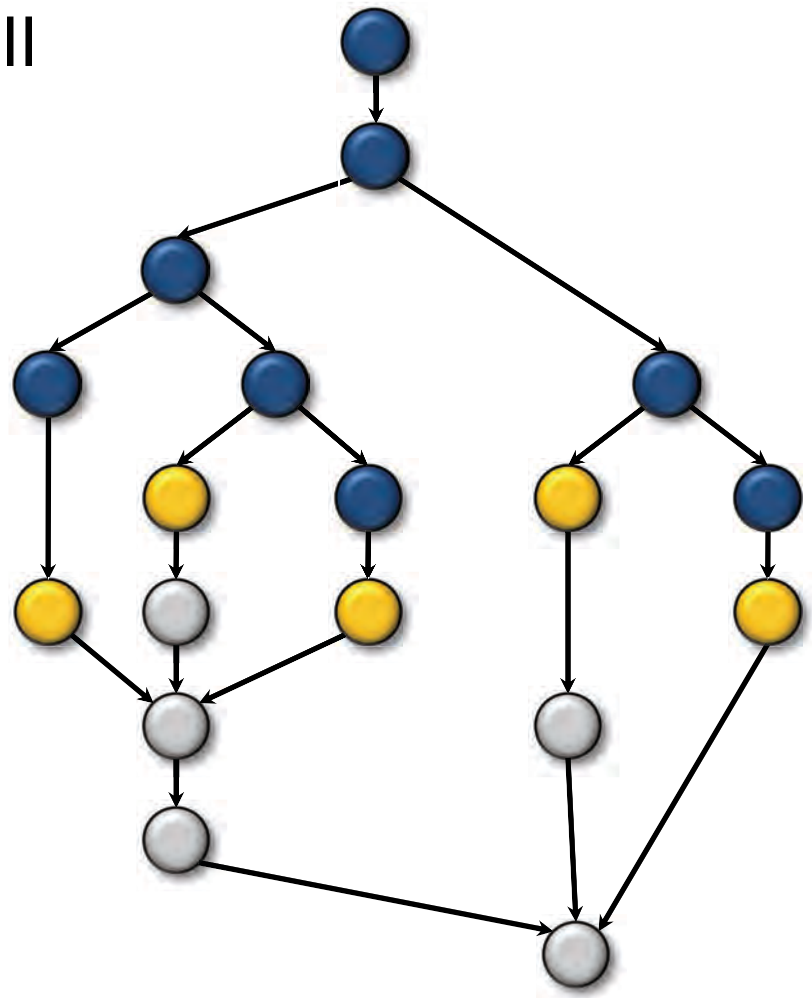
- Cilk allows the programmer to express potential parallelism in an application.
- The Cilk scheduler maps strands onto processors dynamically at runtime.
- Since the theory of distributed schedulers is complicated, we'll explore the ideas with a centralized scheduler.



Greedy Scheduling

IDEA: Do as much as possible on every step.

Definition. A strand is ready if all its predecessors have executed.



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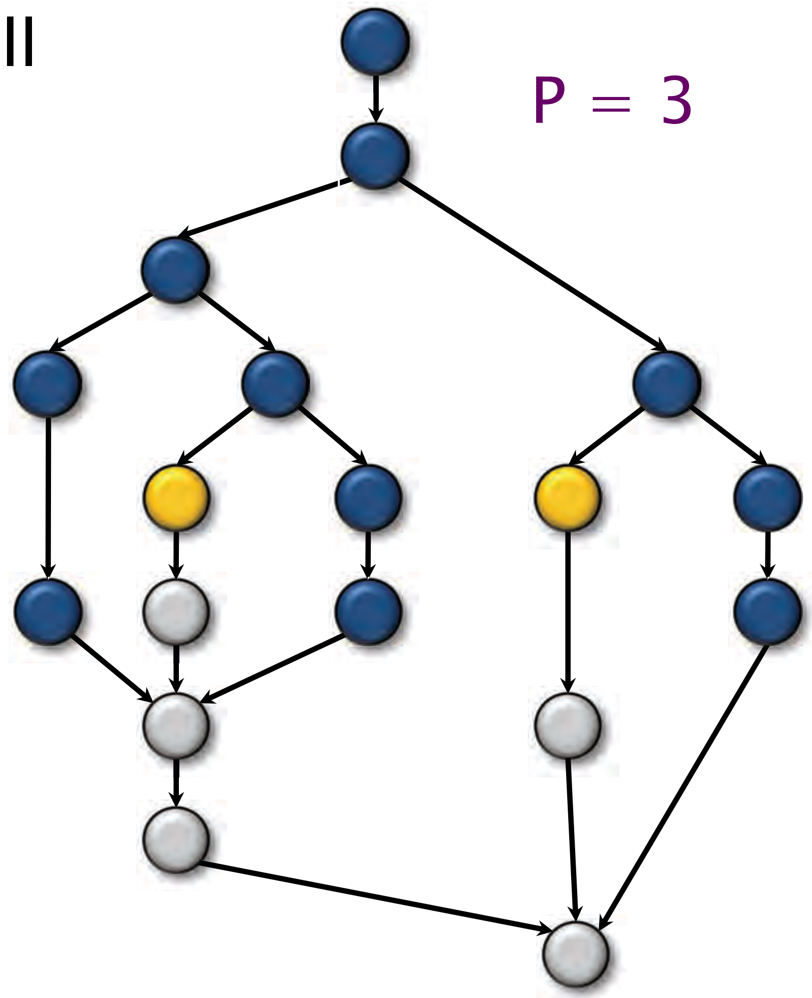

Complete step
- $\geq P$ strands ready.
- Run any P.

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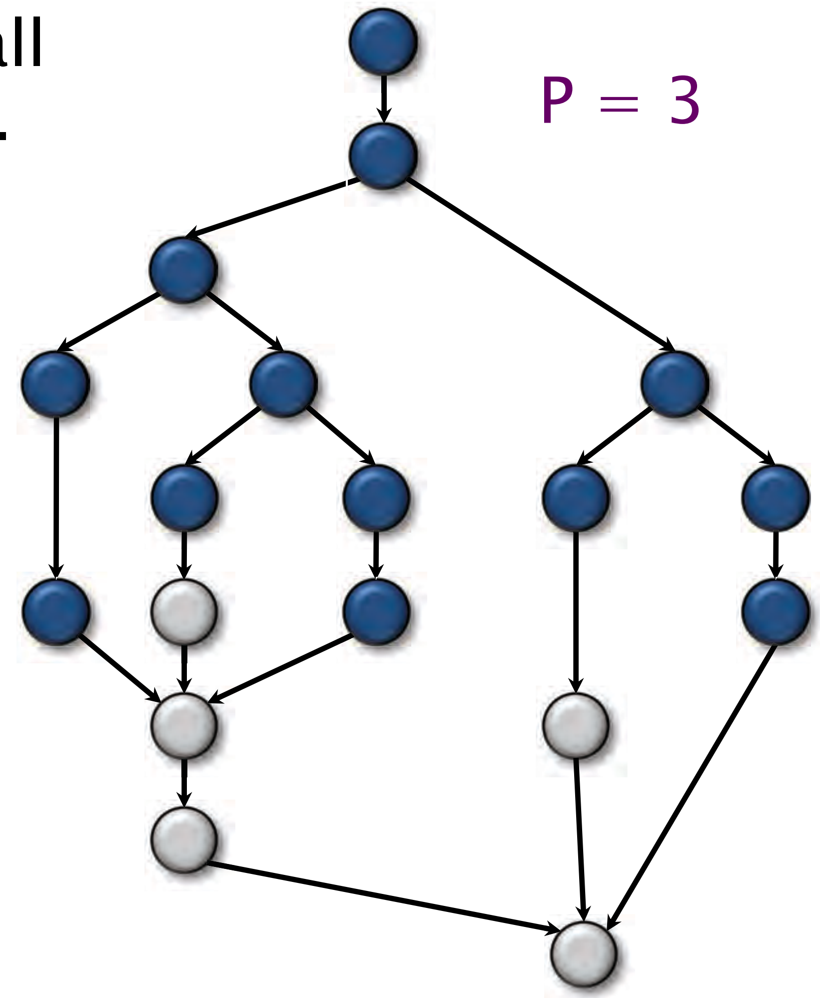


Complete step

- $\geq$ P strands ready.
- Run any P.

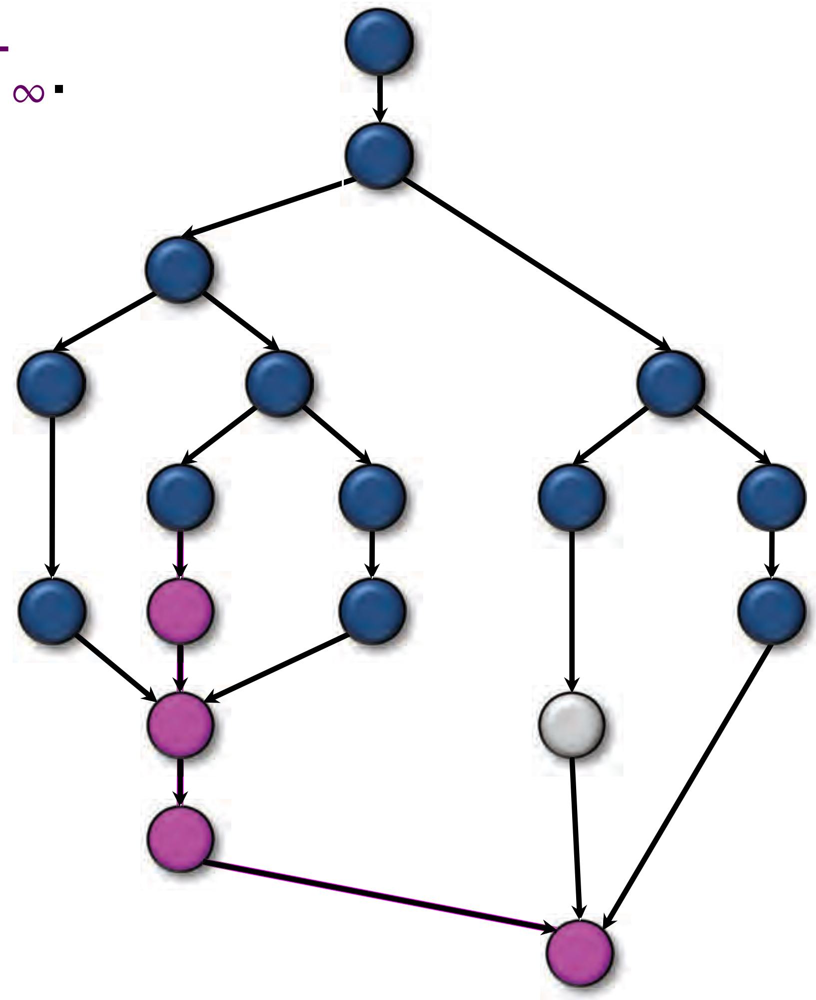
Incomplete step

- $<$ P strands ready.
- Run all of them.

Analysis of Greedy

Theorem [G68, B75, EZL89]. Any greedy scheduler achieves

$$T_P \le T_1/P + T_\infty .$$



Proof.

- # complete steps $\le T_1/P$, since each complete step performs P work.
- # incomplete steps $\le T_\infty$, since each incomplete step reduces the span of the unexecuted dag by 1. ■

Optimality of Greedy

Corollary. Any greedy scheduler achieves within a factor of 2 of optimal.

Proof. Let T_P^* be the execution time produced by the optimal scheduler. Since $T_P^* \geq \max\{T_1/P, T_\infty\}$ by the WORK and SPAN LAWS, we have

$$\begin{aligned} T_P &\leq T_1/P + T_\infty \\ &\leq 2\cdot\max\{T_1/P, T_\infty\} \\ &\leq 2T_P^* . \quad \blacksquare \end{aligned}$$

Linear Speedup

Corollary. Any greedy scheduler achieves near-perfect linear speedup whenever $T_1/T_\infty \gg P$.

Proof. Since $T_1/T_\infty \gg P$ is equivalent to $T_\infty \ll T_1/P$, the Greedy Scheduling Theorem gives us

$$\begin{aligned} T_P &\le T_1/P + T_\infty \\ &\approx T_1/P \;. \end{aligned}$$

Thus, the speedup is $T_1/T_P \approx P$. ■

Definition. The quantity $T_1/(PT_\infty)$ is called the parallel slackness.

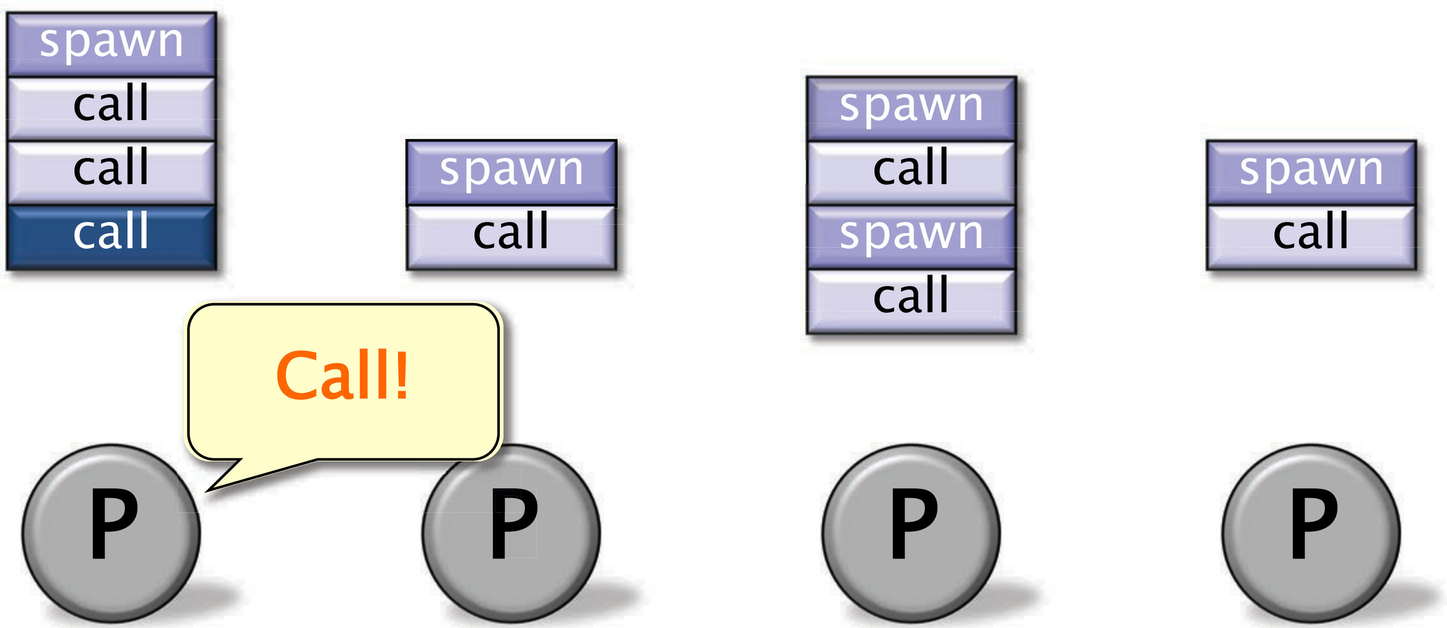
Cilk Performance

- Cilk's work-stealing scheduler achieves
  - $T_P = T_1/P + O(T_\infty)$ expected time (provably);
  - $T_P \approx T_1/P + T_\infty$ time (empirically).
- Near-perfect linear speedup as long as $P \ll T_1/T_\infty$.
- Instrumentation in Cilkscale allows you to measure T_1 and T_∞ .

The Cilk Runtime System

Cilk Runtime System

Each worker (processor) maintains a work deque of ready strands, and it manipulates the bottom of the deque like a stack [MKH90, BL94, FLR98].



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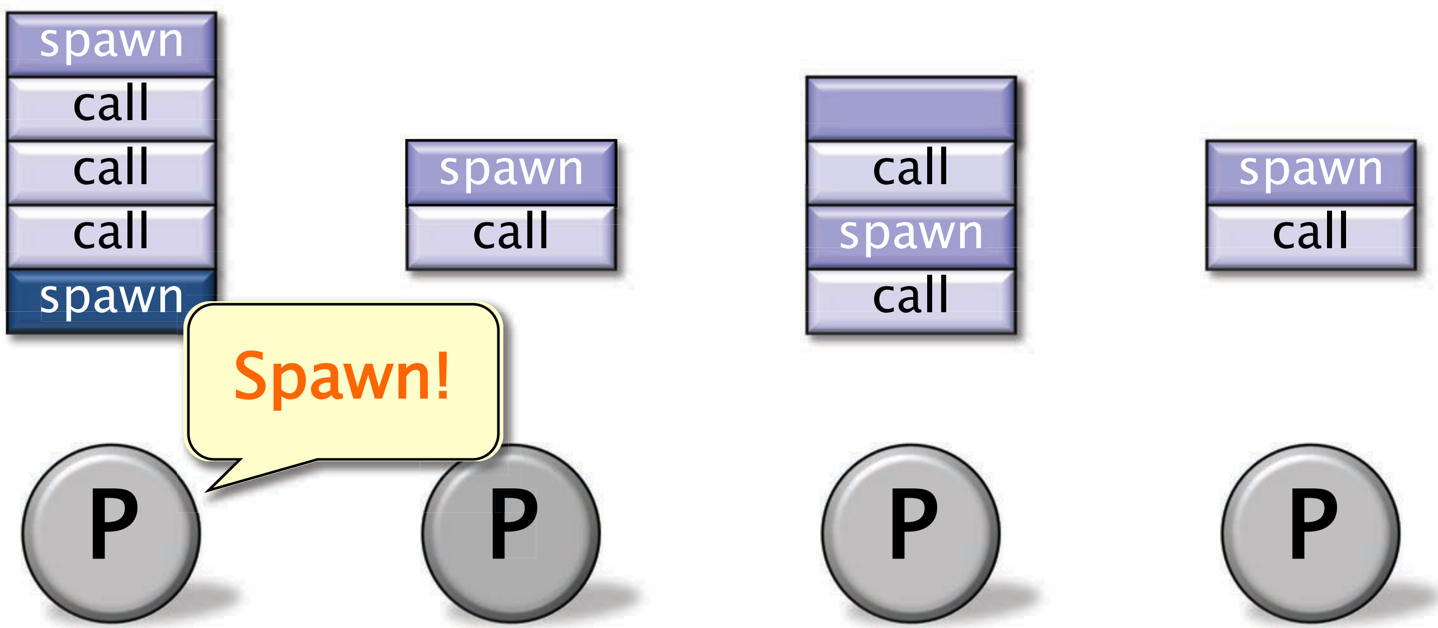

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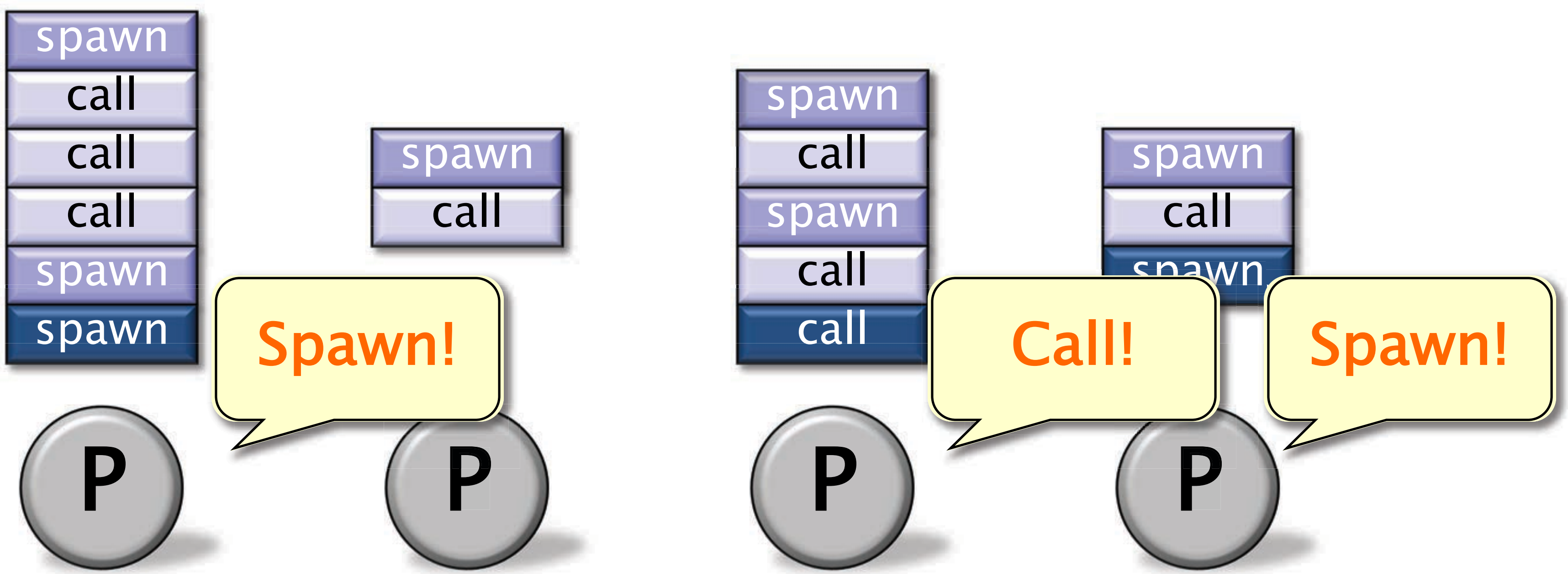

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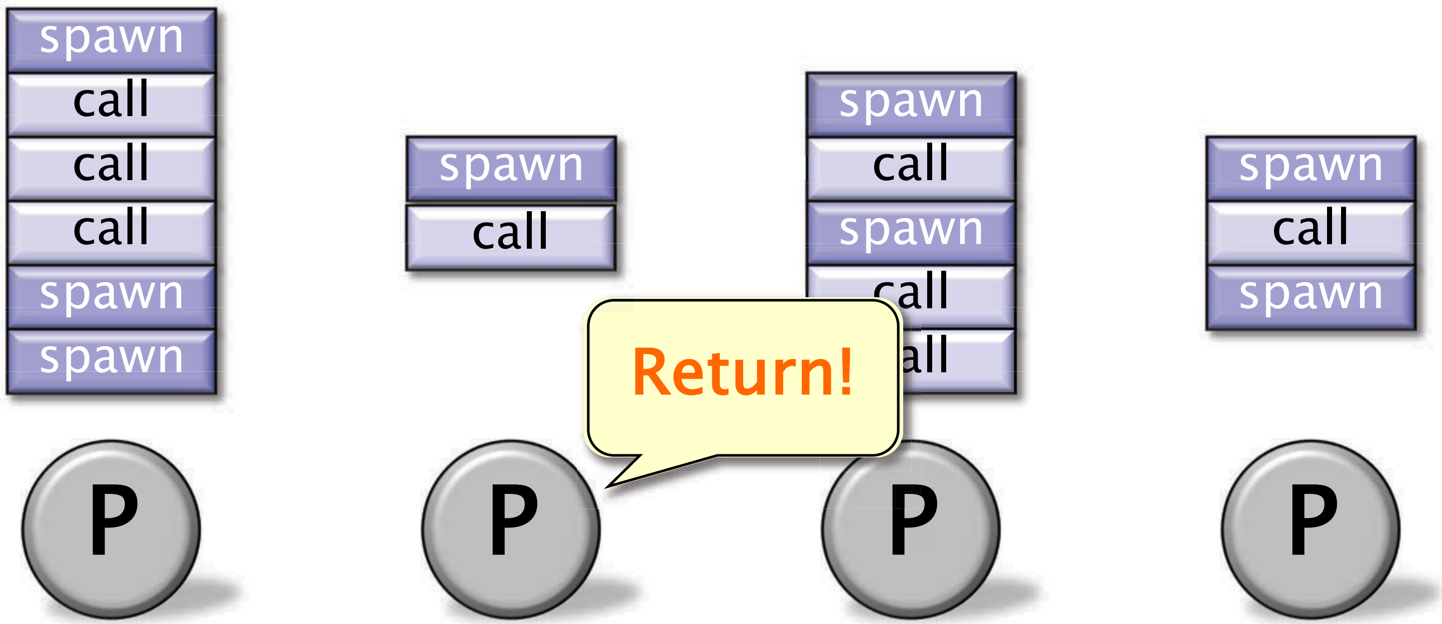

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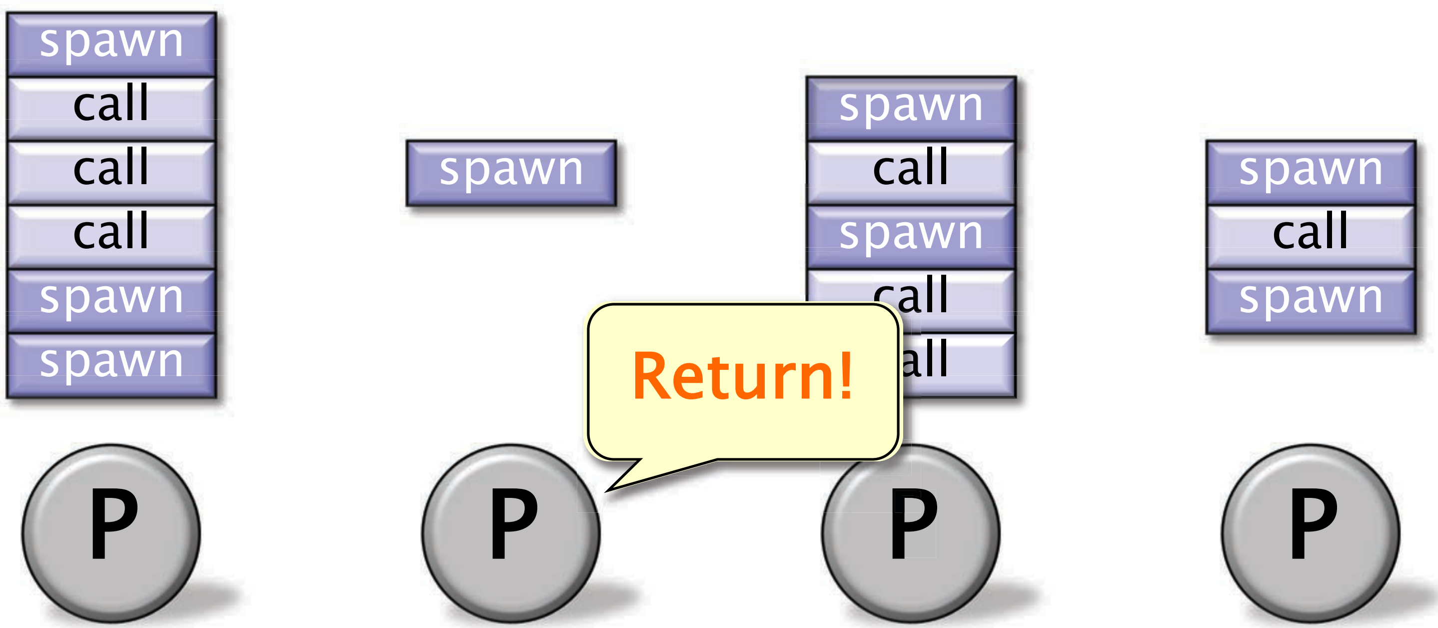

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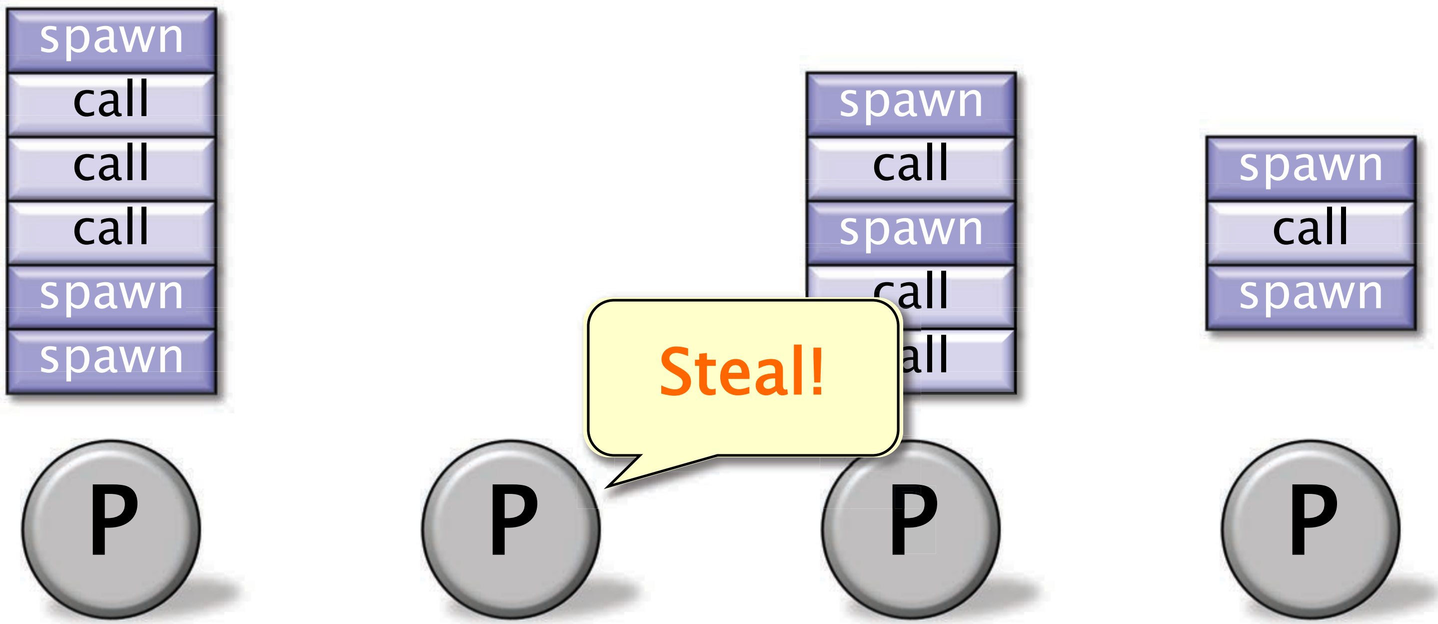

When a worker runs out of work, it steals from the top of a random victim's deque.

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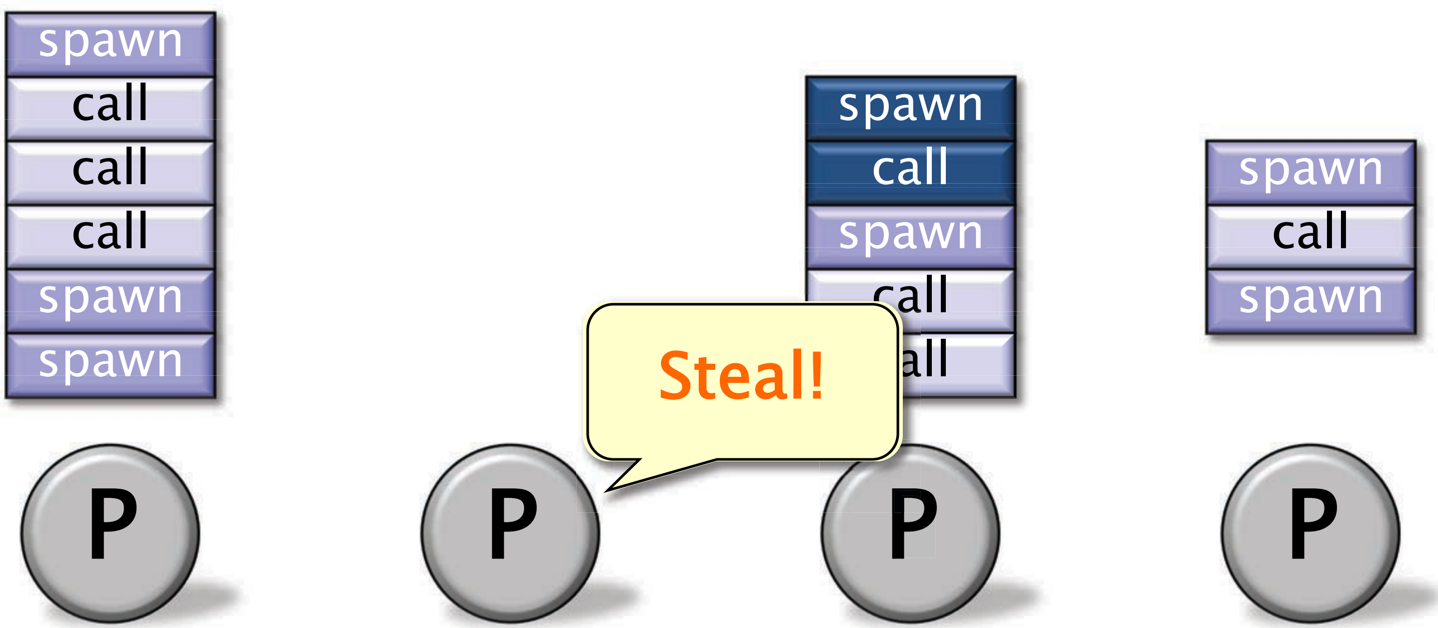

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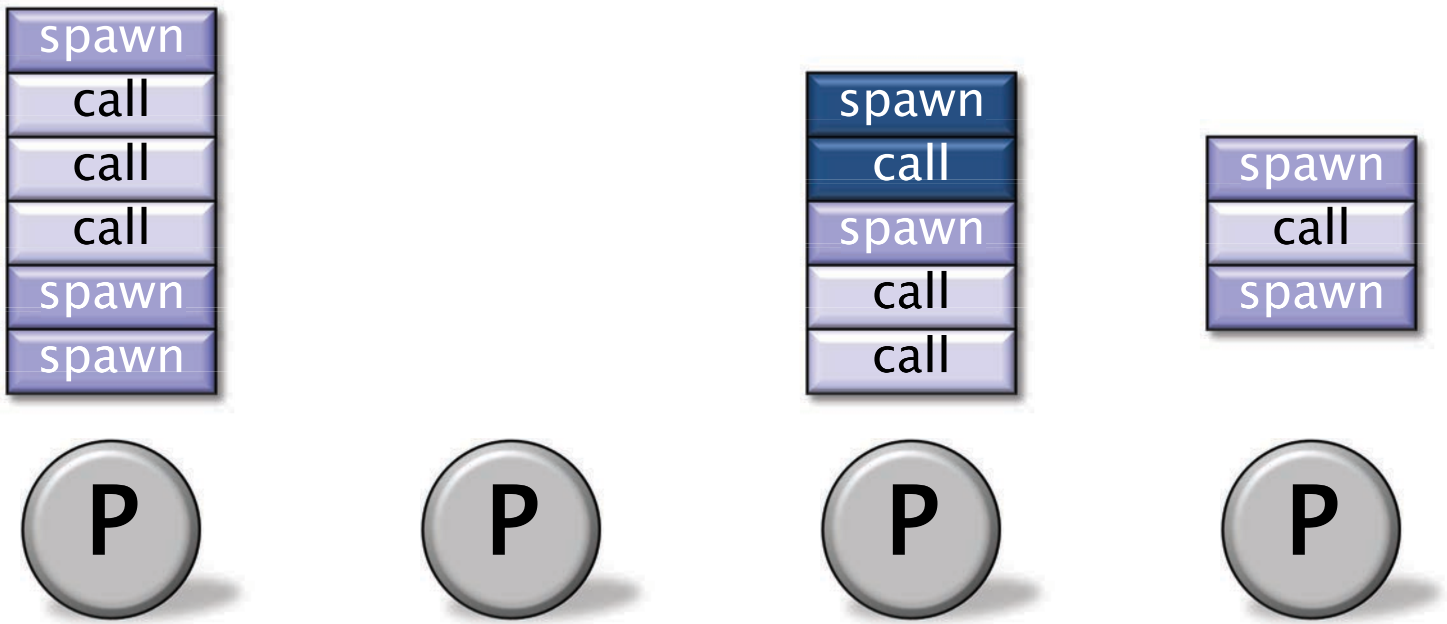

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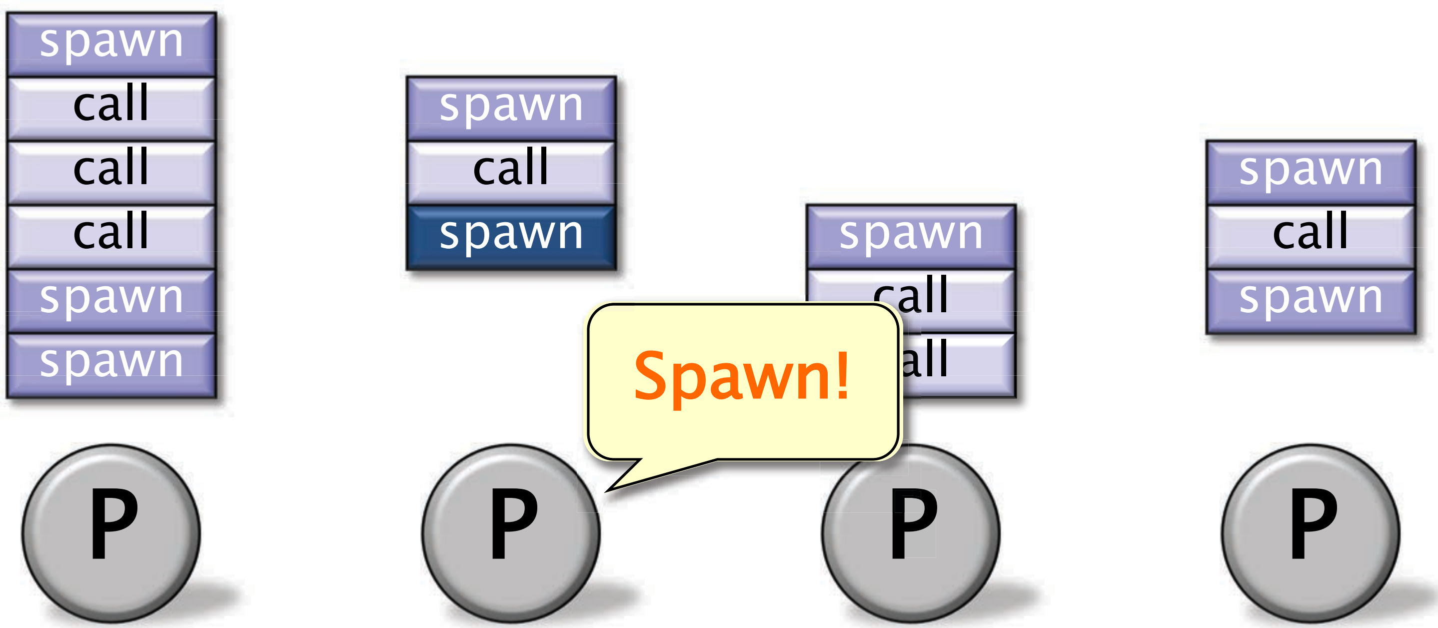

When a worker runs out of work, it steals from the top of a random victim's deque.

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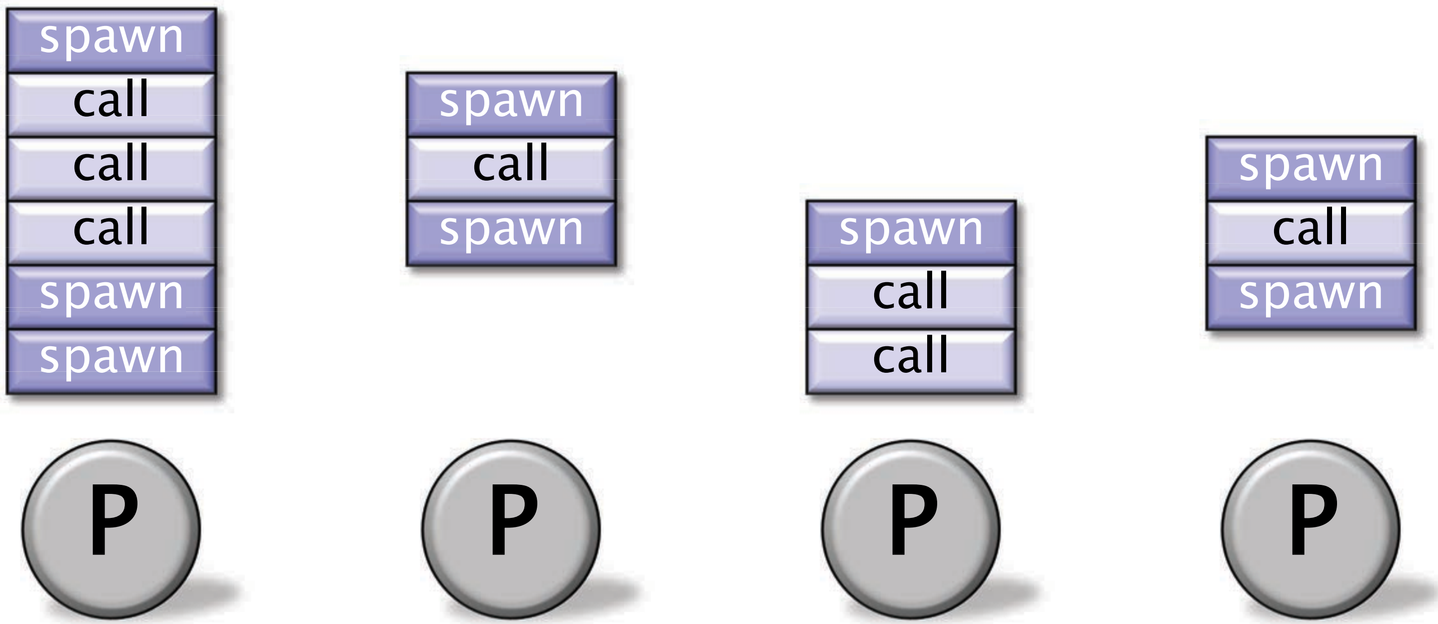

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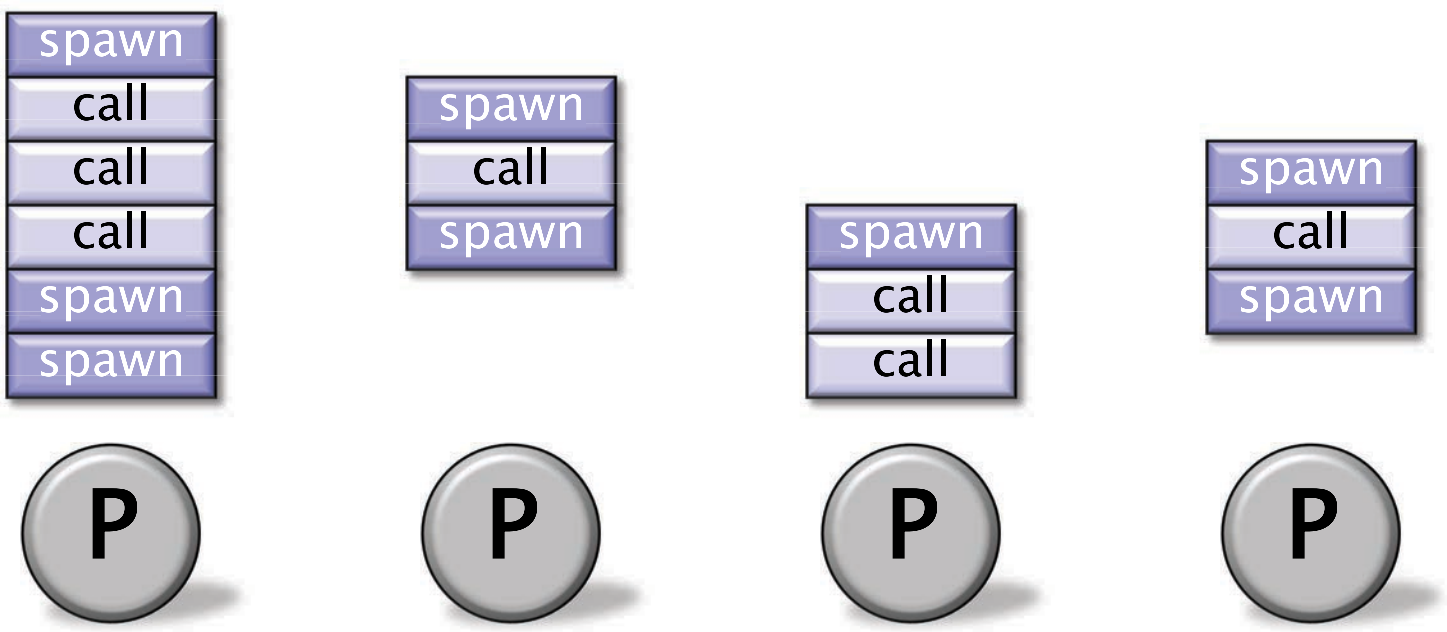


Theorem [BL94]: With sufficient parallelism, workers steal infrequently ⇒ linear speed-up.

Work-Stealing Bounds

Theorem [BL94]. The Cilk work-stealing scheduler achieves expected running time

$$T_P \approx T_1/P + O(T_\infty)$$

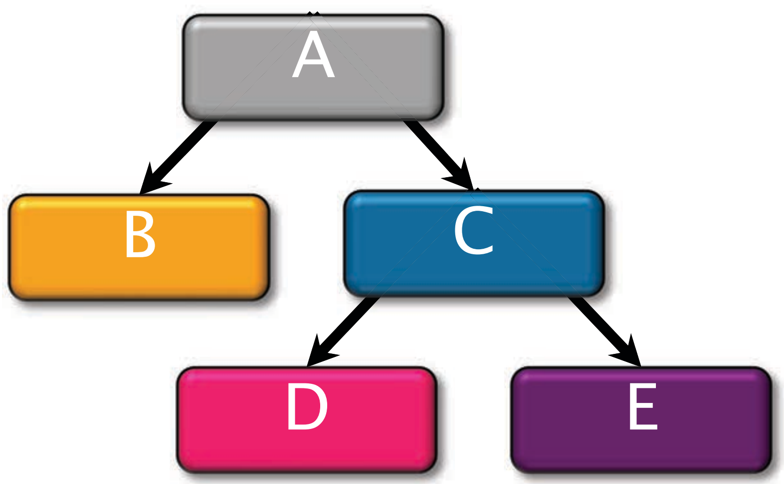
on P processors.

Pseudoproof. A processor is either working or stealing. The total time all processors spend working is T_1. Each steal has a $1/P$ chance of reducing the span by 1. Thus, the expected cost of all steals is $O(PT_\infty)$. Since there are P processors, the expected time is

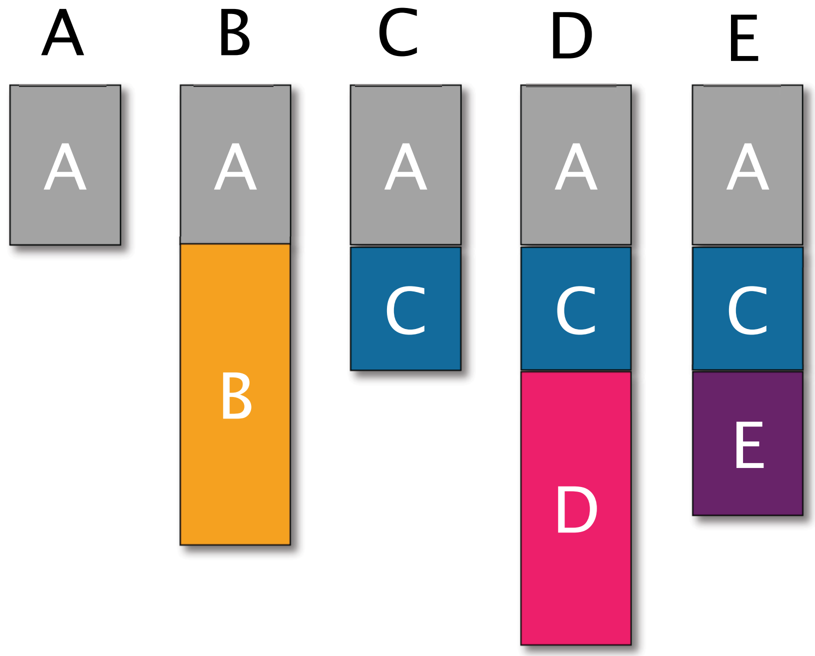
$$(T_1 + O(PT_\infty))/P = T_1/P + O(T_\infty) . \blacksquare$$

Cactus Stack

Cilk supports C's rule for pointers: A pointer to stack space can be passed from parent to child, but not from child to parent.



Views of stack

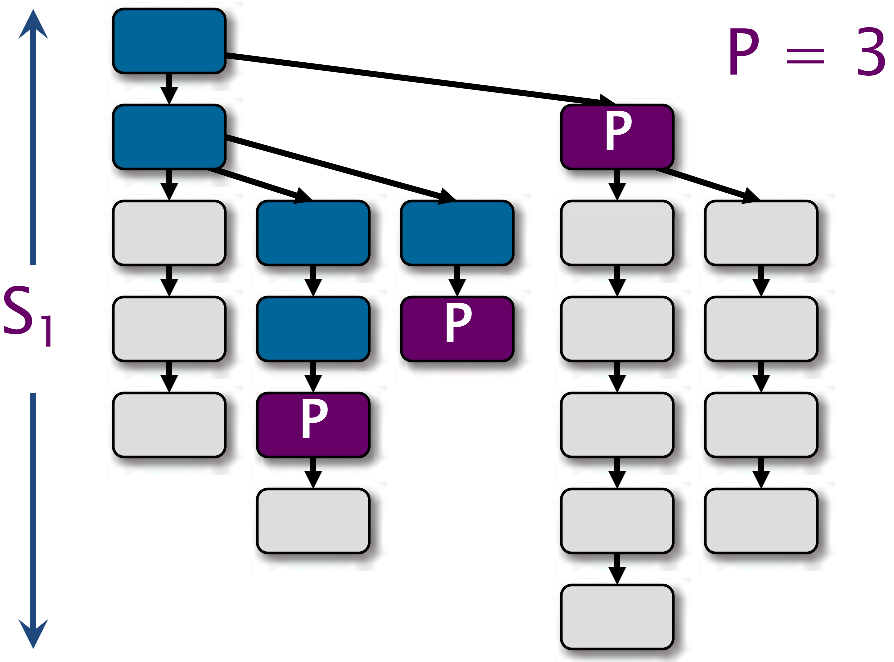


Cilk's cactus stack supports multiple views in parallel.

Bound on Stack Space

Theorem. Let S_1 be the stack space required by a serial execution of a Cilk program. Then the stack space required by a P–processor execution is at most $S_P \leq PS_1$.

Proof (by induction). The work-stealing algorithm maintains the **busy-leaves property**: Every extant leaf activation frame has a worker executing it. ■



Summary

- Determinacy races are often bugs, and they can be detected using Cilksan
- Cilkscale can analyze the work, span, and parallelism of a computation
- A greedy scheduler is within a factor of 2 of the optimal scheduler
- Cilk uses a work-stealing scheduler with strong theoretical bounds on its running time