What's the fuel consumption of a car? Say, on the highway moving at highway speeds? We can look it up, but can we understand those numbers? Can we predict them from our knowledge of science and engineering? That's what we're gonna do in this video. This video is part of the problem solving video series. Problem-solving skills, in combination with an understanding of the natural and human-made world, are critical to the design and optimization of systems and processes. Hi, my name is Sanjoy Mahajan, and I'm a professor of Applied Science and Engineering at Olin College. And I excel at streetfighting mathematics. Before watching this video, you should be familiar with free body diagrams and dimensional analysis. After watching this video, you will be able to: Model drag to predict terminal velocities; and Determine the fuel efficiency of a car. Fuel is consumed in fighting drag, on the highway at least. What's the force of drag on a car? That's fluid mechanics. We could try the hard wayâ€"solving the Navier-Stokes, the equations of fluid mechanics. Now after 10 years of learning mathematics, you'll discover this is a really hard problem. You can only solve it analytically in certain special cases like a sphere moving at really slow speeds and a car is not one of those special cases; certainly not a car moving at any reasonable speed. So we need another way. By applying approximations artfully, we're gonna find the drag force and the fuel consumption of a car with a simple experiment and some scientific and engineering reasoning. First, we need to model drag. To do this, we're gonna figure out how the drag force depends on the quantities that control it. Drag force F depends on the density of the air, to some power which we don't know yet, times the speed of the car moving through the air, to some power we don't know yet. It depends on how big the car is, which we'll represent by the area; traditionally, the cross- sectional area, to some power. And it also can depend on the viscosity here, the kinematic viscosity of the air to some power. And what those powers are, we don't yet know. To figure them out, let's do an experiment. For the experiment, we make two cones. First, by making a large circle and cutting out a quarter of it. The large circle has a radius of seven centimeters. And the small three-quarter circle has half the radius. Cut them out and then tape this edge to that edge and this edge to that edge in order to make the 2 cones. Next we're gonna race them, the big cone vs. the small cone. But first, make a prediction. The question is: what is the ratio of fall speeds of the big cone and the small cone approximately? Is the big cone roughly twice as fast, is the small cone twice as fast, or do they fall at roughly the same speed? Pause the video and make your best guess. You may want to try the experiment yourself. The experimental result is that they both fall at roughly the same speed. Now what does that mean for the exponents in the drag force? To decide, use a free body diagram. Here is a free body diagram of a cone as it falls. What are the forces on the cone? Pause and fill in the diagram. There is drag and gravity. Because the cones are falling at their terminal speed, in other words at the constant velocity that they reach pretty quickly, the drag must equal the force due to gravity. So the drag force can easily be measured just by sticking the cones on a scale. We don't actually have to stick the cones on a scale because we know what their relative weights are and that's what we're gonna use to figure out one of the exponents. Knowing that the drag equals the force due to gravity, let's compare the drag forces, which are the factors on the right side, of the small and large cones. The large cone was made out of a paper circle with twice the radius so it has four times the area, four times the mass, and four times the force due to gravity. Therefore, four times the drag force. Now let's look at the causes one by one. This density here is the density of the air and both cones feel the same air so there's no difference here, so that's times one no matter the exponent. Now, what about the speed? The speeds were the same; that was the result of our experiment, so no matter what the exponent here, the speed contributes to the effect by a factor of one; in other words, nothing. Now the area. This is traditionally the cross-sectional area. The big cone has four times the cross-sectional area of the small cone so this area factor is times four and raised to the unknown exponent. And here, the kinematic viscosity, that's the kinematic viscosity of air and both cones feel the same kinematic viscosity. Therefore, that contributes a factor of one just like the density does. So now we know what the question mark here has to be. The question mark here must be one so that this four to the question mark is equal to this four. And so does this one. Now that we have found one exponent, we can find the remaining three using dimensional analysis. We make sure that the exponents chosen on the right side produce dimensions of force, which are the dimensions on the left side. A force has dimensions of mass, length per time squared, that's mass times acceleration. Density has dimensions of mass per length cubed, raised to this unknown exponent. Velocity has dimensions of length per time, raised to this unknown exponent. Area has dimensions of length squared, raised to the power 1, we've already figured that out. And kinematic viscosity; now, that's the regular viscosity divided by the density. It has dimensions of length squared per time raised to this unknown exponent Now let's solve for these unknown exponents one at a time. Looking first at the mass, there is mass here and mass here on the left side, but there is no other mass. So the only way to get the mass to work out is to make this exponent one. Now, what do we have left? We have to make sure the lengths and times work out and we have two unknowns, this exponent and this exponent. Let's see how many more lengths we need, given what we already have. We have mass worked out, we have two lengths here over three lengths so we have mass per length. So we have length to the minus one so far. And what about time? Well, we have no times yet except in these question marks so we need to get two more times on the bottom two more lengths on the top. So we need to multiply by length squared over time squared, and the only way to do that is to make this exponent a two... and this exponent here a zero. You can pause the video and go ahead and try other combinations of exponents. You'll find that none of them will get both length and time correct. So now we have all the exponents, we can actually enter that into our force formula. Drag force is proportional to density to the first power, speed squared, area to the first power we got from our home experiment and viscosity here to the zeroth power. Now let's test our formula with a second demonstration and see if it predicts this new situation. We're gonna race four small cones stacked on top of each other. One, two, three, four. So we stack them to make one small cone that is four times the mass of the other small cone. Next we're going to race them. But first, what's the drag force ratio between the small cone and 4 small cones stacked together. Pause the video and use the formula to make a prediction. The four cones weigh four times as much as one cone so that's times four on the left side. What about the right side? Again, both contestants feel the same air density. The velocity ratio, that's what we're trying to figure out, and let's see what all the other pieces are. The area; well here in this case, the stack of four has the same cross- sectional area as the one cone so that's just one. And the viscosity doesn't matter. So, this factor of four on the left side has to be produced by the v squared; in other words, the v goes up by a factor of two so that when it's squared, you get a factor of four. That means the correct answer should be two to one. Now, let's test our prediction with a demonstration to check whether nature behaves as we predict. It looks like our prediction was correct! Now we feel pretty confident that we can use this formula to try to figure out the fuel consumption of a car. Let's clean up our formula here just a bit and erase the viscosity since it comes in with a zero exponent so it doesn't matter. And this is our drag formula, which we're gonna use to estimate the fuel consumption of a car on the highway. Fuel efficiency is measured as the distance that that the car travels on one gallon or liter of gasoline. So it depends on how much energy you can get out of a gallon of gasoline divided by the drag force, which is the formula we've just found. So let's put in some numbers. What's the energy from a gallon of gasoline? A gallon is roughly 4 liters. And gasoline is roughly like water so, there are 1,000g/liter. How much energy do you get out of each gram of gasoline? Gasoline is like fat and from the side of a butter packet, you can read that 100 kilocalories, 10 to the 2 kilocalories come from 11 grams of butter. But now we want to convert kilocalories to some reasonable metric unit. So that's four joules/calorie or four kilojoules per kilocalorie. And that all divided by the drag force so let's put in those terms one by one. First the density here, one kilogram per meter cubed is the density of air roughly. Now v for highway speeds. Say 100 kilometers an hour, 65 miles an hour. That's roughly 30 meters per second. Don't forget to square it. And then the area of a car. Well, it's about... two meters wide by 1.5 meters high. Now we have to work all the numbers, but before you do that, pause the video to check that the dimensions all cancel appropriately. You should have found that the result has units of meters, which is good! But how many? A quick numerical approximation will tell you that you get 50 kilometers, which is 30 miles roughly. The fuel efficiency of a car on the highway is about 30 miles per gallon; or 50 kilometers per gallon. In typical European Union units, that's eight liters per hundred kilometers.To Review With our approximation methods, we've come to pretty reasonable numbers for the fuel efficiency of a car, without having to solve any of the complicated equations of fluid mechanics. Instead, we used one simple home experiment, dropping the cones, and the powerful principle of dimensional analysis. From that, we were able to find all the exponents here, in the relationship between drag and density, speed, and cross-sectional area. So the moral of this is that there is an art to approximations, and reasoning tools such as dimensional analysis, they give you an understanding of how systems behave without having to solve every single last bit of the detailed mathematics. This understanding allows us to redesign the world.