Art of Approximation

Problem Solving Series

Instructor's Guide

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Developed by the Teaching and Learning Laboratory at MIT
for the Singapore University of Technology and Design

Introduction

When to Use this Video

- At the beginning of a design or capstone course, as homework or during lecture; or in a Fluid Dynamics course before the introduction of drag.
- Prior knowledge: free body diagrams and dimensional analysis.

Key Information

Duration: 12:55
Narrator: Prof. Sanjoy Mahajan
Materials Needed:
• Paper
• Pencil

Learning Objectives

After watching this video students will be able to:

- Model drag to predict terminal velocities.
- Determine the fuel efficiency of a car.

Motivation

- To represent a piece of the world in our limited minds, we must discard many of the piece's features. Thus, an approximate analysis is often more useful than an exact solution.
- Without approxiation, we are paralyzed, and that is what often happens to our students. This video illustrates an approximate analysis of a complicated yet common phenomenon—drag—showing several of the most important reasoning tools for making such an analysis.

Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Predict the ratio of fall speeds for two cones, one with twice the diameter of the other.
- Test their prediction with a simple at home experiment.
- Use dimensional analysis to find exponents in an expression for drag.
- Determine the drag force ratio between two cones of equal size, but different mass.
- Check the dimensions of an expression for fuel efficiency.

Video Highlights

This table outlines a collection of activities and important ideas from the video.

Time	Feature	Comments
0:00	Cars on the highway.	Description of the problem to compute the fuel efficiency of a car.
00:46	Prerequisite Knowledge and Learning Objectives	
1:00	Chapter 1: Drag	Drag is identified as the main force needed to solve the problem, and approximation is suggested over solving Navier-Stokes (which doesn't have an analytic solution).
1:45	Chapter 2: Modeling Drag	A model for drag is determined using dimensional analysis and a simple home experiment.
2:28	Student Activity	Home experiment to determine how drag depends on the area of a cone is described.
2:54	Student Activity	Students predict the velocity ratio of two cones dropped from the same height, where one has 4 times the area of the other.
3:15	Experiemental result of cone drop	
5:48	Dimensional Analysis used to find remaining exponents in formula for drag.	
7:42	Student Activity	Check that no other combination of variables for the formula for drag gives the correct dimension.
8:05	Drag formula checked	A second home experiment using two cones with the same area but different mass is used to check the formula for drag to see if the velocity ratios predicted by the formula match the velocity ratios of the experiment.
9:33	Chapter 3: Fuel Efficiency of a Car	
11:55	To Review	

Video Summary

This video leads students through an approximation technique to compute the fuel efficiency of a car moving at highway speeds. This is accomplished by using dimensional analysis and a home experiment to identify the exponents in a formula for the drag force on a car.

Course Materials

Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.

1. Watch the Dimensional Analysis video to refresh students' memories about how dimensional analysis works.

2. In the video, an experiment is used to identify the dependence of drag on the area of an object. Repeat this experiment. Cut out two circles, one with radius 7cm and one with radius 3.5cm. Cut out one quarter of each circle and tape the edges together to make two cones, one whose area is 4 times that of the smaller cone. (Use the template from Appendix A1.)

Drop the cones so that the tips of each cone begin at the same height and determine the ratio of velocities.

3. In the video, we checked the drag formula with another experiment. Create 5 small cones using the template from Appendix A1. You can create a cone with 4 times the mass by stacking 4 small cones inside each other. Use the formula to predict the ratio of velocities.

Check that the more massive cone has twice the velocity by dropping the cones so that the cone with 4 times the mass is dropped from twice the height.

Post-Video Materials

1. Estimate how far a person can bicycle, at say 10 m/s (22 mph or 36kph), on 8 liters (roughly 2 gallons) of peanut butter. How does this distance compare with the typical car as estimated in the video?

2. Suppose that the government, hoping to reduce oil consumption, reduces highway speed limits by 15 percent. By roughly what percent does that change reduce highway fuel consumption?

3. Estimate the terminal speed of a typical raindrop (diameter of about 0.5 cm). How could you check your estimate?

4. If you run the cone experiment with very tiny cones, in a very viscous fluid such as cold glycerin, you find that the big cone (with twice the radius) falls twice as fast as the small cone. With that information, estimate the drag force F in terms of the fluid density rho, the object's speed v, the radius r, and the fluid's kinematic viscosity (dimensions of length squared per time). This is known as Stokes' drag.

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Going Further

Approximation must be used artfully in combination with science knowledge and expertise. Once a student has taken a fluid dynamics course and is familiar with Navier-Stokes and numerical methods, it would be valuable to try this problem again, but from a more rigorous standpoint and compare the two solutions to determine the validity of the approximation.

References

The following resource contain problems, examples, and methods for using approximation to solve engineering design problems.

- Mahajan, S. (2010). *Streetfighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving*, Cambridge MA: MIT Press.
- Harte, J. (1988). *Consider a Spherical Cow,* Sausalito CA: Univeristy Science Books.
- Swarz, C. (2003). *Back of the Envelope Physics,* Baltimore MD: JHU Press.

The following videos introduce dimension and dimensional anlaysis.

- Lewin, W., *8.01 Classical Mechanics*, Fall 1999. (Massachusetts Institute of Technology: MIT OpenCouseWare), http://ocw.mit.edu (Accessed November 27, 2012). License: Creative Commons BY-NC-SA
  -Lecture 1: Units and Dimensional Analysis
- Kamrin, K., *Dimensional Analysis: Problem Solving Series,* 2012. (Massachusetts Institute of Technology: Teaching and Learning Lab SUTD Concept Vignettes), http://tll.mit.edu/help/sutd-concept-vignettes-0 (Accessed May 7, 2013). License: Creative Commons BY-NC-SA

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