MITOCW | motion

How did MIT undergraduates design a robot to lift a small model police car and place it on top of a model of MIT's great dome? Calculus! In this video, we'll use calculus to describe the motion of rigid bodies and see how these concepts are used in the field of robotics. This video is part of the Derivatives and Integrals video series. Derivatives and integrals are used to analyze the properties of a system. Derivatives describe local properties of systems, and integrals quantify their cumulative properties. Hello. My name is Dan Frey. I am a professor in the Mechanical Engineering department at MIT, and today I'll be talking with you about the motion of rigid bodies-- both translation and rotation. In order to understand this video you should be very comfortable with linear motion, including position, velocity, and acceleration. You will want to know how to turn measurements in polar coordinates into measurements in Cartesian coordinates. You should also know enough introductory calculus to apply the chain rule. After watching this video you should be able to explain what is meant by the phrase "rigid body." You should be able to describe restrictions on the motion of an object by using constraint equations. Finally, you should be able to use derivatives and integrals to connect different mathematical descriptions of rigid body motion. Our primary examples today will be robots. Let's look at how basic ideas of motion are used in the field of robotics. Here you can see some footage from a robotics competition at MIT. The competition is part of a Mechanical Engineering course, number 2.007. One typical task that robots perform is to grab something, pick it up, and move it. This robot uses a gripper, at the end of the arm, to pick up objects. One of the difficulties in programming a robot arm is that we typically have no direct measurement of where that gripper is. There's usually no convenient sort of "position meter" that could tell its location. Instead we might determine the location for the base of the arm, and we can measure the angles for different parts of the arm. We need to use the measurements that we can make in order to determine the location of the gripper. This is made easier by the concept of a "rigid body." Rigid bodies can translate and rotate, but they do not bend, stretch, or twist. In mathematical terms, the distance between any two points in the object does not change. Rigid bodies are idealizations -- to simplify our work, we imagine that we are working with objects that do not deform. This idealization works best when the object only experiences low amounts of force. Higher amounts of force can lead to objects deforming or breaking, depending on the object. The robot we saw earlier is a good example of a rigid body. You can see in this video that as our robot moves, its pieces do not bend or distort noticeably, so we can treat each piece as a rigid body. We could use the definition of a rigid body to help us determine the location of points on that robot. This robot, on the other hand, has a less sturdy frame. You can see the arm flex as the robot moves. We might not want to treat this arm as a rigid body. Let's try to solve a problem involving a rigid body. Here is a very simple robot arm. It has a "joint" on the left that can tilt up and down, and a piston that can extend its arm. In addition, this part of the piston can be considered a rigid body, so its length will be a constant. Here is a task for you: describe the acceleration of the gripper at the end of the arm. First, set the origin for your coordinate system. Then, write an expression for the x and y position of the gripper in terms of the quantities shown here. Once you have that position, use derivatives to find the velocity and the acceleration of the gripper. Pause the video here to carry out your calculations. Let's take a look at the answer. First, we need to choose an origin. Let's choose an arbitrary location as the origin of our coordinate system. Our robot's joint may be moving, so we will use a pair of functions x sub j of time and y sub j of time to describe its location. X sub j will be the horizontal distance from our origin to the joint, and y sub j will be the vertical distance. We can use derivatives of these functions to describe any relative movement that the joint has when compared with our coordinate system, such as velocity or acceleration. This slide shows just the X components of the answer, with the value x sub j indicating the location of the joint. You can see that the expressions can easily become complicated if both s and theta change at the same time. One reason that we want to know the acceleration is because some objects respond poorly to a high acceleration. Here you can see a different sort of robot arm lifting a car. Instead of a gripper, these two robots use a forklift design. Their arms must move very gently, especially as they slow down, or the car will fall off. The arms are capable of moving more quickly, but the robot's designers have programmed it to use a lower acceleration. This leads us to a discussion of constraints. This section will have a few examples, as well as several opportunities for you to practice. Be ready to pause the video. Constraints are any sort of restriction on a situation. When they can be expressed mathematically, we refer to Equations of Constraint. These describe the physical connection between two or more rigid bodies. Constraint equations are useful because they link one variable to another in a way that reduces the total number of variables in a problem. This helps to make otherwise impossible problems solvable. Constraint equations are used throughout physics and mechanical engineering. Some fields refer to a similar idea called "degree of freedom analysis." Here is a classic example of a situation with a constraint. The car on this roller coaster cannot leave the tracks. If the track is circular, we can use the equation for a circle to constrain our movement. We can use this to reduce the number of variables in our equations for the position of the roller coaster. Rather than an equation in x and y, we could have equations in just x, or just y. We could also use constraints that involve the distance along the track or another sensible measurement for the situation we are investigating. It's important to note that constraints mean giving up some freedom in our variables. In our example, we can only specify x or y, not both. Once we choose a value for x, there are only two y values that will work. Here's a situation where you can find the equation of constraint. A cart is being pulled across a flat surface, and the wheels turn without slippingâ€”effectively, the wheel is constrained to move only by rolling and not in any other way: no lifting up, no sliding, no peeling out. Can you find an equation that connects x, the distance the cart has moved, to theta, the amount that the wheels have turned? Pause the video here to discuss this in class. Here is an arm that is fairly complex -- it has many joints. Pause the video and write down the variables and constants you would use for this robot. There are three separate angles that must be recorded, as well as the extension of the arm. There are also three pieces of constant length. Now we have an opportunity to describe a constraint in a complex situation. We could choose, for example, to constrain the motion of the arm to just the horizontal direction. Because there are many variables in this situation, there are many possible ways to satisfy the constraint. Write an expression for just the vertical position of the gripper in terms of the quantities shown. Once you have done that, answer this question: how might we move the gripper in just the horizontal direction? You should come up with at least two ways that we could do this, and describe them mathematically. Your teacher will then lead the class in a discussion of your answers. Pause the video here to do this. Here is a simulation of a "hydrabot" doing exactly what you just calculated: moving one end horizontally. You can see that it matches up quite well with our hypothetical robot. Examine its motion closely--is this one of the motions you described? Are there extra constraints present here? What freedom of motion did we give up by making our choice? Let's review. Today you used derivatives to find the velocity and acceleration of an object based on its position. You also learned the definition of a rigid body: that it does not bend or stretch when force is applied. Finally, you saw that constraint equations can reduce the total number of equations in a system, thus making problems easier to solve. I hope you enjoyed seeing some applications of basic motion concepts. Good luck in your further investigation of physics and engineering!