Contents

Preface ii

1 Introduction 1

2 Algebraic Preliminaries 2

2.1 Groups . . . 2

2.1.1 Criterion for Subgroups . . . 3

2.1.2 Cyclic Groups . . . 4

2.1.3 Cosets . . . 4

2.1.4 Conjugate Elements and Classes . . . 4

2.2 The geometry of the three-dimensional rotation group. The Rodrigues-Hamilton theorem . . . 5

2.3 The n-dimensional vector space $V(n)$ . . . 8

2.4 How to multiply vectors? Heuristic considerations . . . 11

2.5 A Short Survey of Linear Groups . . . 12

2.6 The unimodular group $\mathcal{SL}(n, R)$ and the invariance of volume . . . 14

2.7 On "alias" and "alibi". The Object Group . . . 16

3 The Lorentz Group and the Pauli Algebra 18

3.1 Introduction . . . 18

3.2 The corpuscular aspects of light . . . 18

3.3 On circular and hyperbolic rotations . . . 21

3.4 The Pauli Algebra . . . 28

3.4.1 Introduction . . . 28

3.4.2 Basic Definitions and Procedures . . . 29

3.4.3 The restricted Lorentz group . . . 35

3.4.4 Similarity classes and canonical forms of active transformations . . . 40

4 Pauli Algebra and Electrodynamics **47**

4.1 Lorentz transformation and Lorentz force 47

4.2 The Free Maxwell Field 52

5 Spinor Calculus **55**

5.1 From triads and Euler angles to spinors. A heuristic introduction. 55

5.2 Rigid Body Rotation 67

5.3 Polarized light 72

5.4 Relativistic triads and spinors. A preliminary discussion. 81

5.5 Review of SU(2) and preview of quantization. 83

A Supplementary material on the Pauli algebra **88**

A.1 Useful formulas 88

A.2 Lorentz invariance and bilateral multiplication 89

A.3 Typical Examples 90

A.4 On the use of Involutions 91

A.5 On Parameterization and Integration 91

B Problems **93**

B.1 Assignments 1–7 93

List of Figures

2.1 Composition of the Rotations of the Sphere 7

2.2 Translational congruence and equal area. 15

3.1 Area in (x_0, x_3)-plane. 24

5.1 Euler Angles 66

5.2 Progressive and Retrograde Precession. 71

5.3 Representation of Polarization in the Poincaré Sphere. 86

5.4 Representation of Polarization in the Poincaré Sphere (contd). 87

B.1 Mirror Planes. 93

B.2 Problem 8 coordinate frame and angles. 95

List of Tables

3.1 Summary of the rotational transformations. (The signs of the angles correspond to the passive interpretation.) 23

3.2 Canonical Forms for the Simlarity classes of $\mathcal{A}_2$. 46

B.1 Table for Problem 8 94