This will be a short tutorial on infinite series, their definition and their basic properties.

What is an infinite series?

We're given a sequence of numbers ai, indexed by i, where i ranges from 1 to infinity.

So it's an infinite sequence.

And we want to add the terms of that sequence together.

We denote the resulting sum of that infinity of terms using this notation.

But what does that mean exactly?

What is the formal definition of an infinite series?

Well, the infinite series is defined as the limit, as n goes to infinity, of the finite series in which we add only the first n terms in the series.

However, this definition makes sense only as long as the limit exists.

This brings up the question, when does this limit exist?

The nicest case arises when all the terms are non-negative.

If all the terms are non-negative, here's what's happening.

We consider the partial sum of the first n terms.

And then we increase n.

This means that we add more terms.

So the partial sum keeps becoming bigger and bigger.

The sequence of partial sums is a monotonic sequence.

Now monotonic sequences always converge either to a finite number or to infinity.

In either case, this limit will exist.

And therefore, the series is well defined.

The situation is more complicated if the terms ai can have different signs.

In that case, it's possible that the limit does not exist.

And so the series is not well defined.

The more interesting and complicated case is the following.

It's possible that this limit exists.

However, if we rearrange the terms in the sequence, we might get a different limit.

When can we avoid those complicated situations?

We can avoid them if it turns out that the sum of the absolute value of the numbers sums to a finite number.

Now this series that we have here is an infinite series in which we add non-negative numbers.

And by the fact that we mentioned earlier, this infinite series is always well defined.

And it's going to be either finite or infinite.

If it turns out to be finite, then the original series is guaranteed to be well defined, to have a finite limit when we define it that way, and furthermore, that finite limit is the same even if we rearrange the different terms, if we rearrange the sequence with which we sum the different terms.