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This is a simple example where we want to just apply the formula for conditional probabilities and see what we get.

The example involves a four-sided die, if you can imagine such an object, which we roll twice, and we record the first roll, and the second roll.

So there are 16 possible outcomes.

We assume to keep things simple, that each one of those 16 possible outcomes, each one of them has the same probability, so each outcome has the probability $1/16$.

Let us consider now a particular event B on which we're going to condition.

This is the event under which the smaller of the two die rolls is equal to 2, which means that one of the dice must have resulted in two, and the other die has resulted in something which is 2 or larger.

So this can happen in multiple ways.

And here are the different ways that it can happen.

So at 2, 2, or 2, 3, or 2, 4; then a 3, 2 and a 4, 2.

All of these are outcomes in which one of the dice has a value equal to 2, and the other die is at least as large.

So we condition on this event.

This results in a conditional model where each one of those five outcomes are equally likely since they used to be equally likely in the original model.

Now let's look at this quantity.

The maximum of the two die rolls-- that is, the largest of the results.

And let us try to calculate the following quantity-- the conditional probability that the maximum is equal to 1 given that the minimum is equal to 2.

So this is the conditional probability of this particular outcome.

Well, this particular outcome cannot happen.

If I tell you that the smaller number is 2, then the larger number cannot be equal to 1, so this outcome is impossible, and therefore this conditional probability is equal to 0.

Let's do something a little more interesting.

Let us now look at the conditional probability that the maximum is equal to 3 given the information that event B has occurred.

It's best to draw a picture and see what that event corresponds to.

M is equal to 3-- the maximum is equal to 3-- if one of the dice resulted in a 3, and the other die resulted in something that's 3 or less.

So this event here corresponds to the blue region in this diagram.

Now let us try to calculate the conditional probability by just following the definition.

The conditional probability of one event given another is the probability that both of them-- both of the two events-- occur, divided by the probability of the conditioning event.

That is, out of the total probability in the conditioning event, we ask, what fraction of that probability is assigned to outcomes in which the event of interest is also happening?

So what is this event?

The maximum is equal to 3, which is the blue event.

And simultaneously, the red event is happening.

These two events intersect only in two places.

This is the intersection of the two events.

And the probability of that intersection is 2 out of 16, since there's 16 outcomes and that event happens only with two particular outcomes.

So this gives us $2/16$ in the numerator.

How about the denominator?

Event B consists of a total of five possible outcomes.

Each one has probability $1/16$, so this is $5/16$, so the final answer is $2/5$.

We could have gotten that same answer in a simple and perhaps more intuitive way.

In the original model, all outcomes were equally likely.

Therefore, in the conditional model, the five outcomes that belong to B should also be equally likely.

Out of those five, there's two of them that make the event of interest to occur.

So given that we live in B, there's two ways out of five that the event of interest will materialize.

So the event of interest has conditional probability [equal to] $2/5$.