S O L U T I O N S

Feedback Compensation 11

Note: All references to Figures and Equations whose numbers are *not* preceded by an "S" refer to the textbook.

Solution 11.5 (P5.14)

(a) If the major loop crosses over at $\omega = 10^3$ rad/sec, then it is very likely that we can choose b and τ such that the minor-loop transmission crosses over well above this frequency. For $\omega \gg 1$, the minor-loop transmission is given approximately by

$$\text{L.T.} \simeq -10^{10}\frac{b}{\tau s + 1} \tag{S11.1}$$

At $\omega = 10^3$ rad/sec the minor-loop transmission magnitude is approximately $10^7 \dfrac{b}{\tau}$, which will be large when $10^7 b \gg \tau$. We proceed under this assumption, and check its validity later in the solution. Also note that the phase shift of the negative of the minor-loop transmission never exceeds $-90°$. Thus, stability of the minor loop is guaranteed for all positive values of b and τ.

Assuming the minor-loop transmission magnitude is large, and following the development in Section 5.3, the major-loop transmission is given approximately as

$$a(s) = 3 \times 10^{-3}\left(\frac{\tau s + 1}{bs^2}\right) \tag{S11.2}$$

To achieve 55° of phase margin, the zero must supply 55° of positive phase shift at the crossover frequency of 10^3 rad/sec. Thus, we require

$$\tan^{-1} 10^3\tau = 55°$$

or

$$\tau = 10^{-3}\tan 55° = 1.43 \times 10^{-3} \tag{S11.3}$$

That is, the zero should be located at $\omega = 700$ rad/sec. At crossover, with this value of τ, the major-loop transmission magnitude is given by

$$\begin{aligned} |a(s)| &= \frac{3 \times 10^{-3}}{b \times 10^6}\sqrt{(1.43 \times 10^{-3})^2(10^3)^2 + 1} \\ &= \frac{1}{b} \times 5.2 \times 10^{-9} \end{aligned} \tag{S11.4}$$

Thus, to set the magnitude equal to unity, we must have $b = 5.2 \times 10^{-9}$. Now to check the original assumption. At $\omega = 10^3$ rad/sec, the minor-loop transmission magnitude is about 30, which is sufficiently greater than 1 to satisfy the conditions of our analysis.

Now, we sketch the open-loop Bode plot for the amplifier. An approximate analysis follows. For frequencies well below the zero location, the feedback path of the minor loop is approximately bs^2, and the open loop is approximately given by

$$\frac{V_o(s)}{V_i(s)} \simeq 3 \times 10^{-3} \times \frac{-10^{10}}{(s+1)^2 + 10^{10}bs^2}$$
$$= 3 \times 10^7 \frac{1}{53.3s^2 + 2s + 1}, \qquad |s| \ll 700 \qquad \textbf{(S11.5)}$$

which has a complex pair of poles at $s = -1.8 \times 10^{-2} \pm j0.14$, which is lightly damped, but stable. From earlier results, we know that there is an open-loop zero at $s = -700$ rad/sec. That is, the pole $\frac{1}{\tau s + 1}$ in the minor-loop feedback path is an open-loop zero of the amplifier. Finally, for frequencies well above the zero location, the minor-loop feedback path is approximately $\frac{bs}{\tau}$, and the open-loop transfer function is approximately given by

$$\frac{V_o(s)}{V_i(s)} \simeq 3 \times 10^{-3} \frac{-10^{10}}{s^2 + \frac{10^{10}bs}{\tau}}$$
$$= \frac{-8.2 \times 10^2}{s\left(\frac{s}{3.7 \times 10^4} + 1\right)}, \quad |s| \gg 700 \qquad \textbf{(S11.6)}$$

This has a pole at the origin, which represents the net effect of the two poles and the zero as seen at frequencies much greater than 700 rad/sec. The higher frequency pole at $s = -3.7 \times 10^4$ rad/sec is due to the minor-loop transmission crossover. Thus, the open-loop transfer function has a complex pole pair at $s = -1.8 \times 10^{-2} \pm j0.14$, a zero at $s = -700$, and a pole at $s = -3.7 \times 10^4$. An exact numerical solution of the full third-order open-loop transfer function confirms these approximate results. Given the above singularity locations, we can sketch the Bode plot as shown in Figure S11.1.

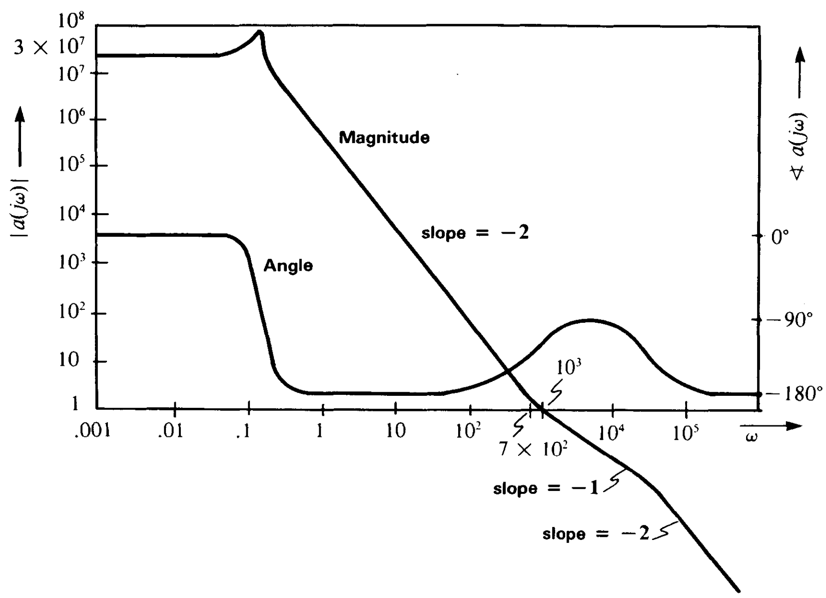

Figure S11.1 Open-loop magnitude and phase versus frequency.

(b) It is not possible to match the resonant peak at $\omega = 0.135$ rad/sec with any $|a_c(j\omega)| \leq 1$, however, we are only asked to match the magnitude characteristics asymptotically. This is possible by placing two poles at $s = -0.135$, two zeros at $s = -1$ to cancel the poles at $s = -1$, one zero at $s = -700$, and one pole at $s = -3.7 \times 10^4$. This will give a transfer function of

$$a_c(s) = \frac{(s + 1)^2 \left(\dfrac{s}{700} + 1\right)}{\left(\dfrac{s}{0.135} + 1\right)^2 \left(\dfrac{s}{3.7 \times 10^4} + 1\right)} \qquad \textbf{(S11.7)}$$

This has $|a_c(j\omega)| \leq 1$ for all ω, although it is not physically realizable, because at high frequencies $|a_c(j\omega)| \simeq 0.96$, implying infinite frequency response, which is of course impossible for any real circuit.

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