Solution 9.1

(a)

$$X(k) = \left[\sum_{n=0}^{N-1} x(n)\ W_N^{kn}\right] R_N(k)$$

$$= R_N(k)$$

(b)

$$X(k) = W_N^{kn_0}\ R_N(k)$$

(c)

$$X(k) = \left[\sum_{n=0}^{N-1} a^n\ W_N^{kn}\right] R_N(k)$$

$$= \left[\frac{1-a^N\ W_N^{kN}}{1-a\ W_N^{k}}\right] R_N(k)$$

$$= \left[\frac{1-a^N}{1-a\ W_N^{k}}\right] R_N(k)$$

Solution 9.2

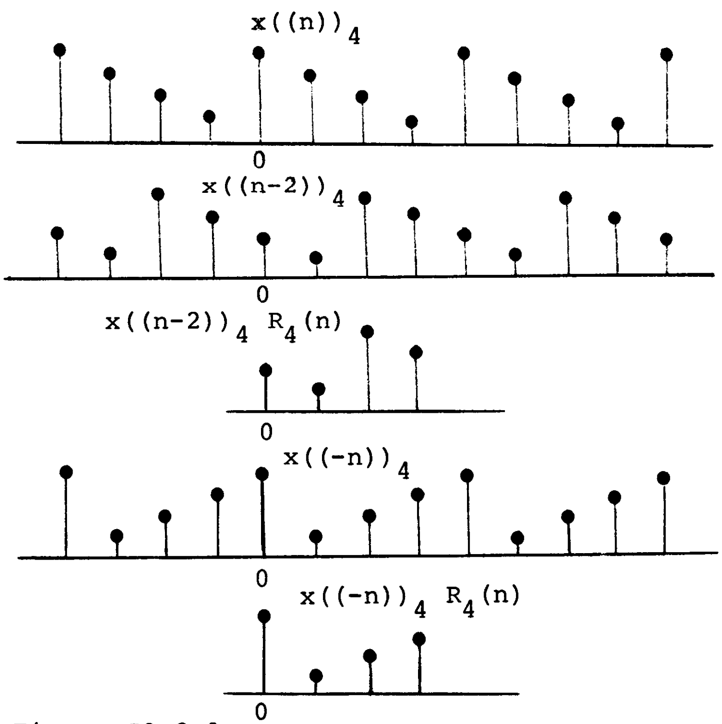

Figure S9.2-1

Solution 9.3

Since $X_I(k) = -X_I((N-k))_N R_N(k)$

$X_I(0) = -X_I((N))_N R_N(k) = -X_I(0)$

Therefore $X_I(0) = 0$

Also, for N even,

$$X_I(\tfrac{N}{2}) = -X_I((N-\tfrac{N}{2}))_N R_N(k)$$

$$= -X_I(\tfrac{N}{2})$$

Therefore $X_I(\frac{N}{2}) = 0$.

Solution 9.4

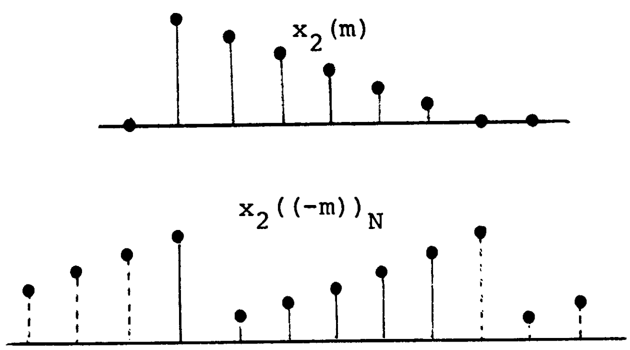

Figure S9.4-1

Solution 9.5

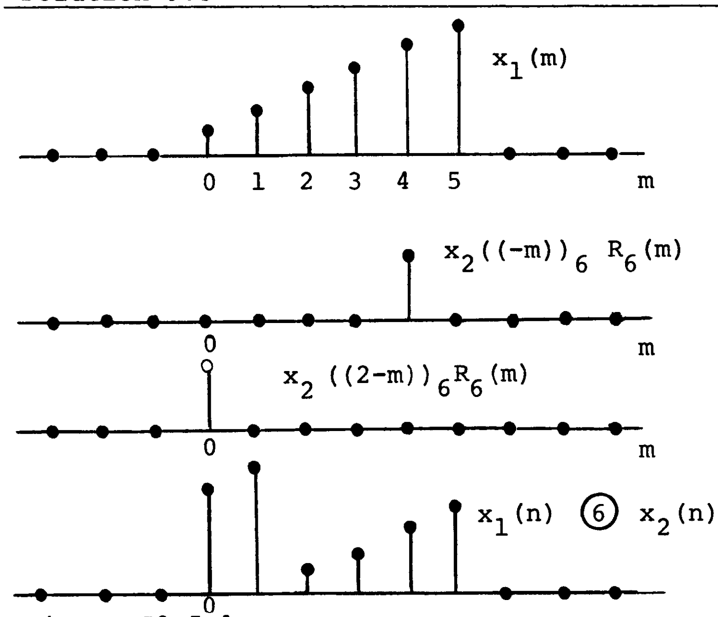

Figure S9.5-1

Note that this corresponds to $x_1(n)$ circularly shifted to the right by two points.

Solution 9.6

We wish to compute $X_1(k)$ given by

$$X_1(k) = \left\{\sum_{n=0}^{9} x(n)\ z_k^{-n}\right\} R_{10}(k)$$

where $z_k = 0.5\ e^{\frac{j2\pi k}{10}}\ e^{j\frac{\pi}{10}}$

so that

$$X_1(k) = \left\{\sum_{n=0}^{9} x(n) \left[\frac{1}{2}\ e^{\frac{j2\pi k}{10}}\ e^{\frac{j\pi}{10}}\right]^{-n}\right\} R_{10}(k)$$

$$= \left\{\sum_{n=0}^{9} x(n)\left[\frac{1}{2}\ e^{\frac{j\pi}{10}}\right]^{-n} e^{\frac{-j2\pi kn}{10}}\right\} R_{10}(k)$$

Thus $X_1(k)$ is the 10-point DFT of the sequence

$$x_1(n) = x(n)\left[\frac{1}{2}\ e^{\frac{j\pi}{10}}\right]^{-n}$$

Solution 9.7

In all of the following equations the DFT computed is valid only in the range $0 \le k \le N-1$ and is zero outside that range. This permits us to keep the equations somewhat cleaner by suppressing the use of the function $R_N(k)$.

$$G_1(k) = \sum_{n=0}^{N-1} x(N-1-n)\ W_N^{kn}$$

$$= \sum_{m=0}^{N-1} x(m)\ W_N^{k(N-1-m)}$$

$$= \sum_{m=0}^{N-1} x(m)\ e^{\frac{j2\pi k}{N}}\ e^{j\frac{2\pi}{N}mk}$$

$$= e^{j\frac{2\pi k}{N}}\ X(e^{-j\frac{2\pi}{N}k}) = H_7(k)$$

$$G_2(k) = \sum_{n=0}^{N-1} (-1)^n\ x(n)\ W_N^{kn} = \sum_{n=0}^{N-1} x(n)\ W_N^{\frac{Nn}{2}}\ W_N^{kn}$$

$$= \sum_{n=0}^{N-1} x(n) \; e^{-j\frac{2\pi}{N}(k+\frac{N}{2})n} = X(e^{j\frac{2\pi}{N}(k+\frac{N}{2})})$$

$$= H_8(k)$$

$$G_3(k) = \sum_{n=0}^{N-1} x(n) \; W_{2N}^{nk} + \sum_{n=N}^{2N-1} x(n-N) \; W_{2N}^{nk}$$

$$= \sum_{n=0}^{N-1} x(n) \left[W_{2N}^{nk} + W_{2N}^{(n+N)k} \right]$$

$$= \sum_{n=0}^{N-1} x(n) \; W_{2N}^{nk} \left[1 + W_{2N}^{Nk} \right]$$

$$= \left[1 + (-1)^k \right] X(e^{j\frac{2\pi}{2N}k}) = H_3(k)$$

$$G_4(k) = \sum_{n=0}^{\frac{N}{2}-1} \left(x(n) + x(n+\tfrac{N}{2}) \right) W_{\frac{N}{2}}^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n) \; W_{\frac{N}{2}}^{nk} + \sum_{n=\frac{N}{2}}^{N-1} x(n) \; W_{\frac{N}{2}}^{(n-\frac{N}{2})k}$$

$$= \sum_{n=0}^{N-1} x(n) \; W_{\frac{N}{2}}^{nk} = X(e^{j\frac{4\pi}{N}}) = H_6(k)$$

$$G_5(k) = \sum_{n=0}^{2N-1} x(n) \; W_{2N}^{nk} = X(e^{j\frac{\pi k}{N}}) = H_2(k)$$

$$G_6(k) = \sum_{n=0}^{N-1} x(n) \; W_{2N}^{2nk} = X(e^{j\frac{2\pi k}{N}}) = H_1(k)$$

$$G_7(k) = \sum_{n=0}^{\frac{N}{2}-1} x(2n) \; W_{\frac{N}{2}}^{nk}$$

$$= \sum_{n=0}^{N-1} x(n) \left[\frac{1+(-1)^n}{2} \right] W_{\frac{N}{2}}^{nk/2}$$

$$= \frac{1}{2} \sum_{n=0}^{N-1} x(n) \left[W_N^{nk} + W_N^{n(k+N/2)} \right]$$

$$= \frac{1}{2}\left[X\left(e^{j\frac{2\pi k}{N}}\right) + X\left(e^{j\frac{2\pi}{N}(k+N/2)}\right)\right] = H_5(k)$$

All of the above properties can alternatively be obtained from the basic DFT properties of sections 8.7 and 8.8, or the z-transform properties of section 4.4. Many of the properties used in this problem have important practical applications. $g_5(n)$, for example, corresponds to augmenting a finite length sequence with zeros so that a computation of the DFT for this augmented sequence provides finer spectral sampling of the Fourier transform.