16 Sampling

Recommended Problems

P16.1

The sequence $x[n] = (-1)^n$ is obtained by sampling the continuous-time sinusoidal signal $x(t) = \cos \omega_0 t$ at 1-ms intervals, i.e.,

$$\cos(\omega_0 nT) = (-1)^n, \qquad T = 10^{-3} \text{ s}$$

Determine three *distinct* possible values of ω_0.

P16.2

Consider the system in Figure P16.2.

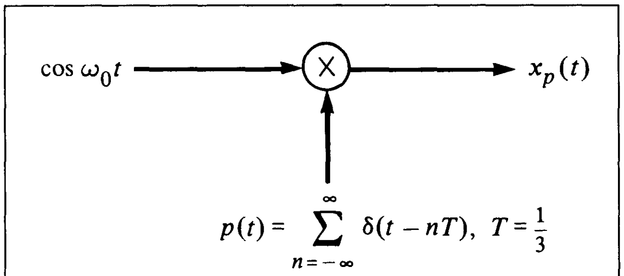

Figure P16.2

(a) Sketch $X_p(\omega)$ for $-9\pi \leq \omega \leq 9\pi$ for the following values of ω_0.

(i) $\omega_0 = \pi$

(ii) $\omega_0 = 2\pi$

(iii) $\omega_0 = 3\pi$

(iv) $\omega_0 = 5\pi$

(b) For which of the preceding values of ω_0 is $x_p(t)$ identical?

P16.3

In the system in Figure P16.3, $x(t)$ is sampled with a periodic impulse train, and a reconstructed signal $x_r(t)$ is obtained from the samples by lowpass filtering.

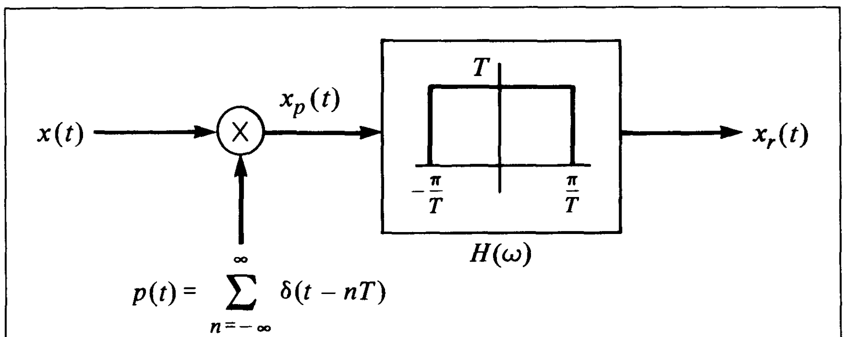

Figure P16.3

The sampling period T is 1 ms, and $x(t)$ is a sinusoidal signal of the form $x(t) = \cos(2\pi f_0 t + \theta)$. For each of the following choices of f_0 and θ, determine $x_r(t)$.

(a) $f_0 = 250$ Hz, $\theta = \pi/4$

(b) $f_0 = 750$ Hz, $\theta = \pi/2$

(c) $f_0 = 500$ Hz, $\theta = \pi/2$

P16.4

Figure P16.4 gives a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.

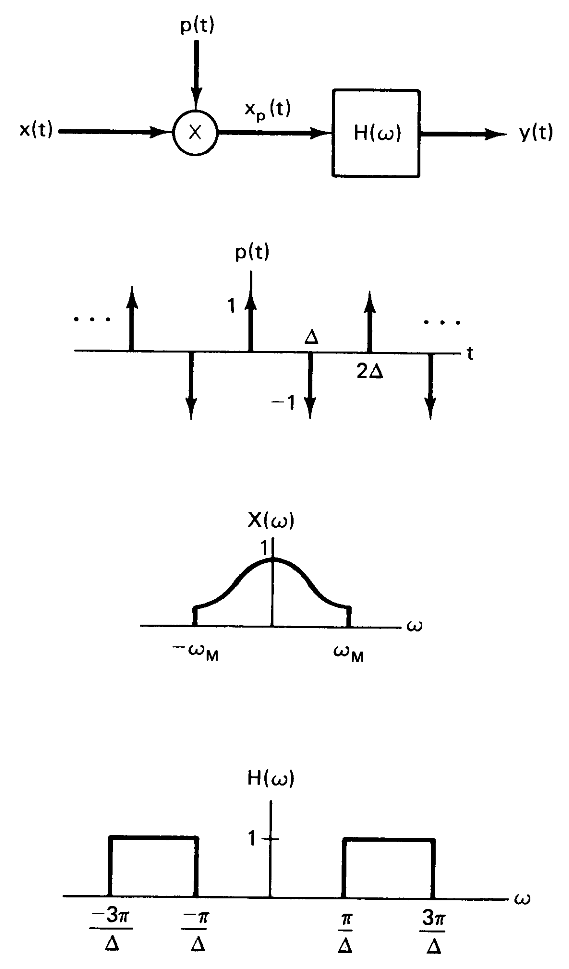

Figure P16.4

(a) For $\Delta < \pi/2\omega_M$, sketch the Fourier transform of $x_p(t)$ and $y(t)$.

(b) For $\Delta < \pi/2\omega_M$, determine a system that will recover $x(t)$ from $x_p(t)$.

(c) For $\Delta < \pi/2\omega_M$, determine a system that will recover $x(t)$ from $y(t)$.

(d) What is the *maximum* value of Δ in relation to ω_M for which $x(t)$ can be recovered from either $x_p(t)$ or $y(t)$.

P16.5

Consider the system in Figure P16.5-1.

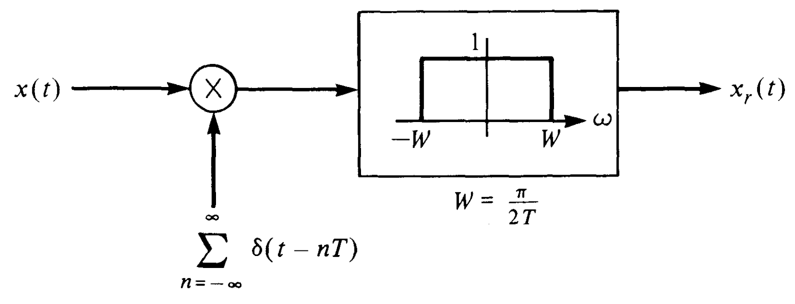

Figure P16.5-1

Figures P16.5-2 and P16.5-3 contain several Fourier transforms of $x(t)$ and $x_r(t)$. For each input spectrum $X(\omega)$ in Figure P16.5-2, identify the correct output spectrum $X_r(\omega)$ from Figure P16.5-3.

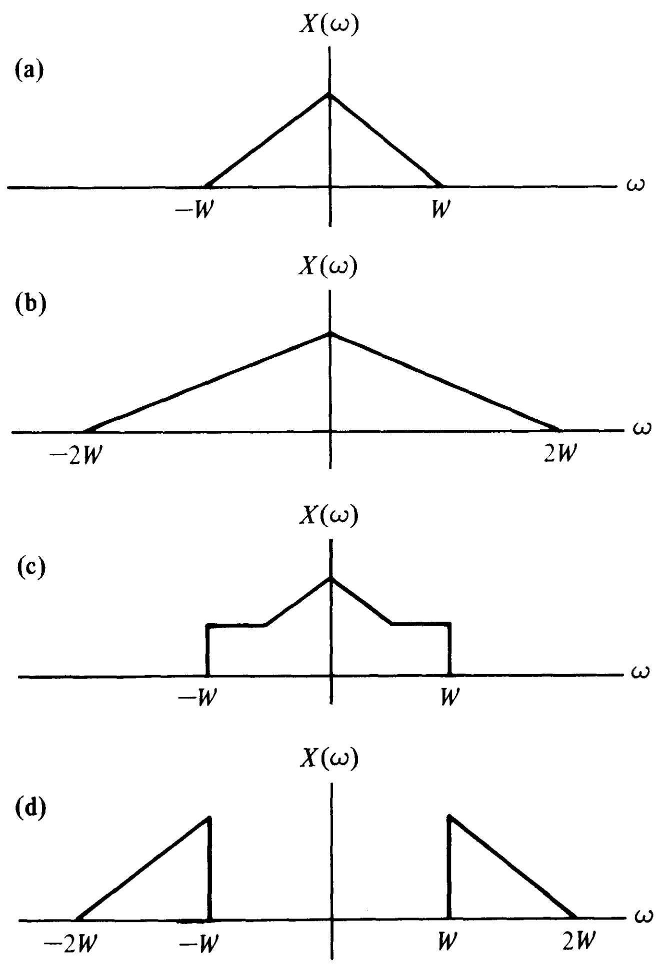

Figure P16.5-2

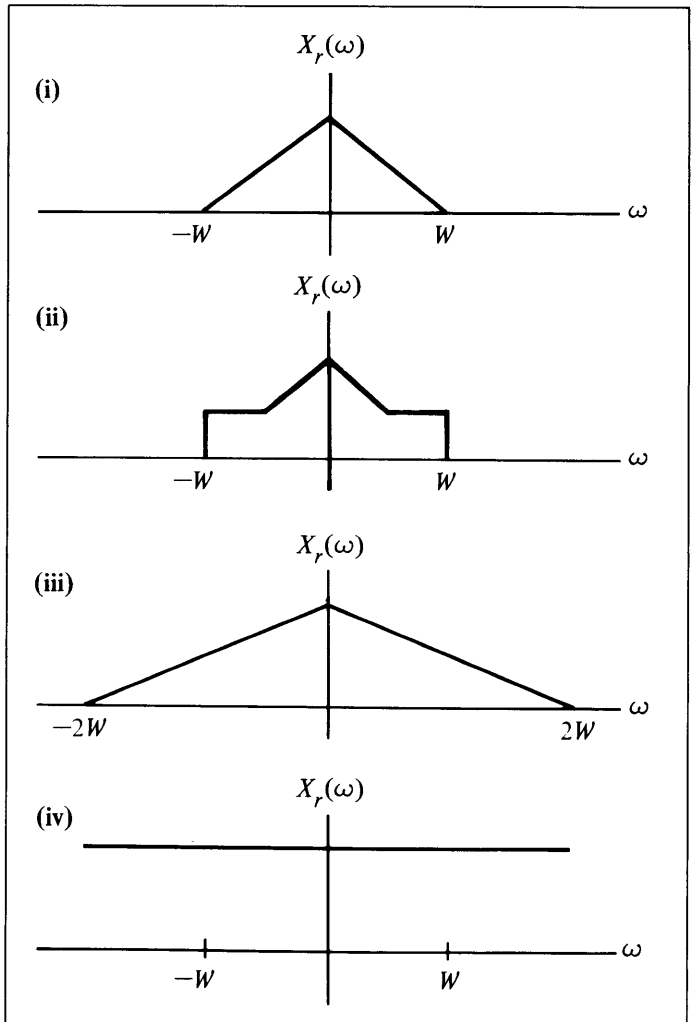

Figure P16.5-3

P16.6

Suppose we sample a sinusoidal signal and then process the resultant impulse train, as shown in Figure P16.6-1.

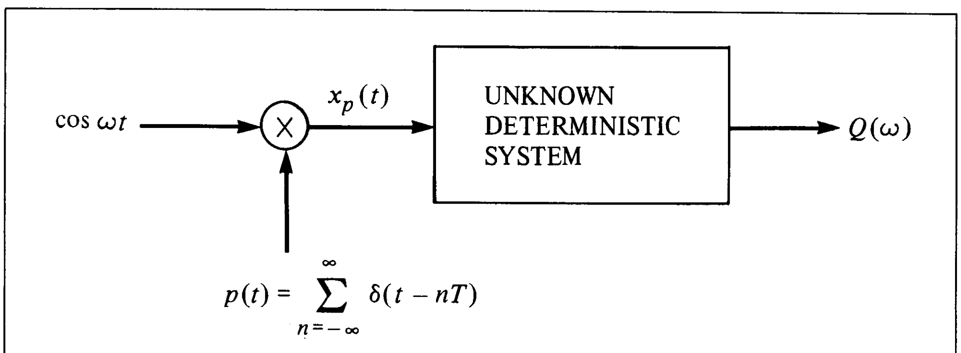

Figure P16.6-1

The result of our processing is a value Q. As ω changes, Q may change. Determine which of the plots in Figures P16.6-2 and P16.6-3 are possible candidates for the variation of Q as a function of ω.

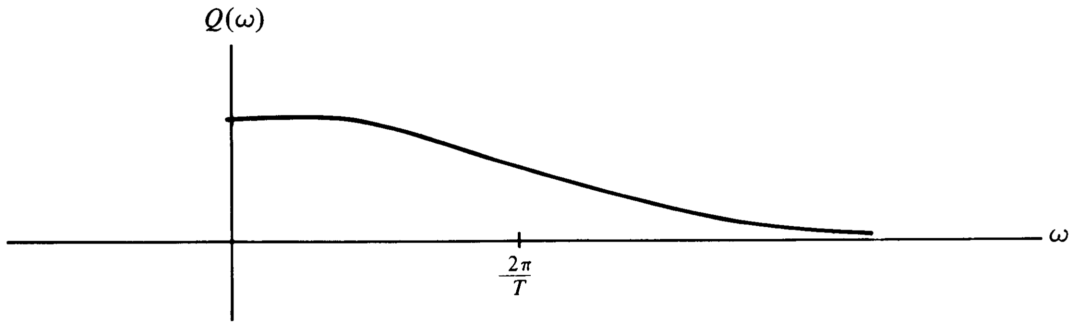

Figure P16.6-2

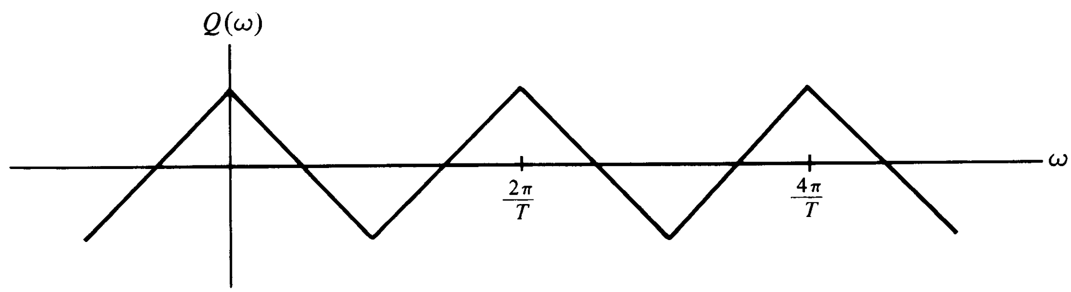

Figure P16.6-3

Optional Problems

P16.7

The sampling theorem as we have derived it states that a signal $x(t)$ must be sampled at a rate greater than its bandwidth (or, equivalently, a rate greater than twice its highest frequency). This implies that if $x(t)$ has a spectrum as indicated in Figure P16.7-1, then $x(t)$ must be sampled at a rate greater than $2\omega_2$. Since the signal has most of its energy concentrated in a narrow band, it seems reasonable to expect that a sampling rate lower than twice the highest frequency could be used. A signal whose energy is concentrated in a frequency band is often referred to as a *bandpass signal.* There are a variety of techniques for sampling such signals, and these techniques are generally referred to as *bandpass sampling.*

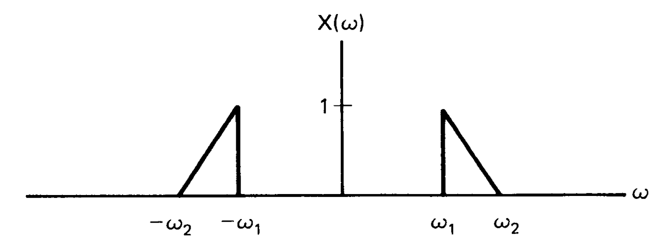

Figure P16.7-1

To examine the possibility of sampling a bandpass signal at a rate less than the total bandwidth, consider the system shown in Figure P16.7-2. Assuming that $\omega_1 > (\omega_2 - \omega_1)$, find the maximum value of T and the values of the constants A, ω_a, and ω_b such that $x_r(t) = x(t)$.

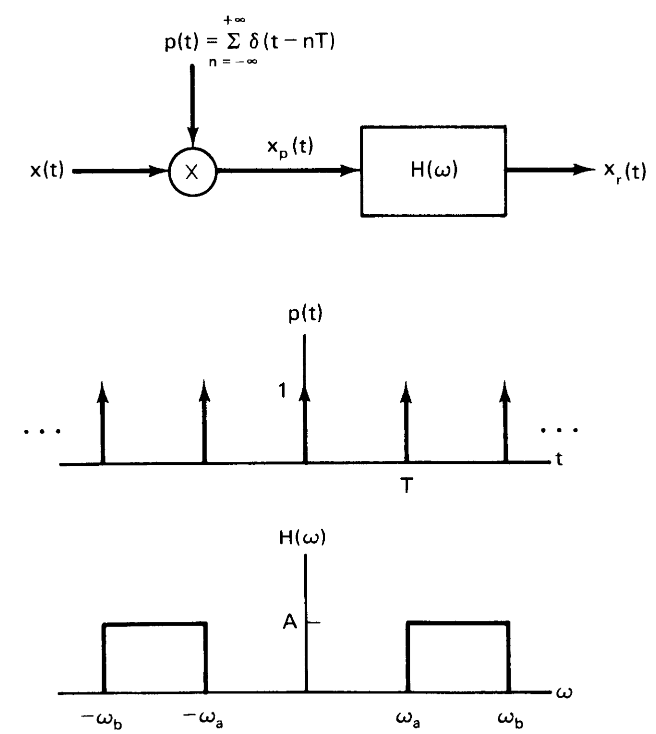

Figure P16.7-2

P16.8

In Problem P16.7 we considered one procedure for bandpass sampling and reconstruction. Another procedure when $x(t)$ is real consists of using complex modulation followed by sampling. The sampling system is shown in Figure P16.8-1.

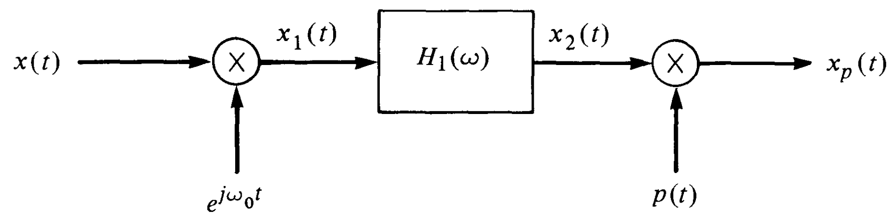

Figure P16.8-1

With $x(t)$ real and with $X(\omega)$ nonzero only for $\omega_1 < |\omega| < \omega_2$, the modulating frequency ω_0 is chosen as $\omega_0 = \frac{1}{2}(\omega_1 + \omega_2)$, and the lowpass filter $H_1(\omega)$ has cutoff frequency $\frac{1}{2}(\omega_2 - \omega_1)$.

(a) For $X(\omega)$ as shown in Figure P16.8-2, sketch $X_p(\omega)$.

(b) Determine the maximum sampling period T such that $x(t)$ is recoverable from $x_p(t)$.

(c) Determine a system to recover $x(t)$ from $x_p(t)$.

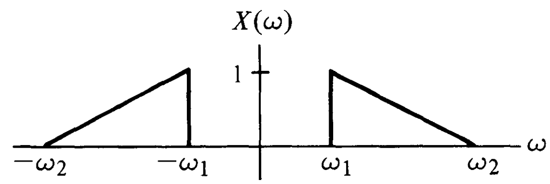

Figure P16.8-2

P16.9

Given the system in Figure P16.9-1 and the Fourier transforms in Figure P16.9-2, determine A and find the maximum value of T in terms of W such that $y(t) = x(t)$ if $s(t)$ is the impulse train

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

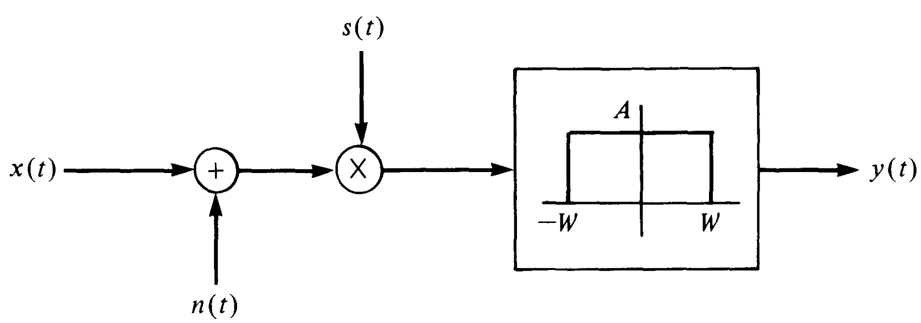

Figure P16.9-1

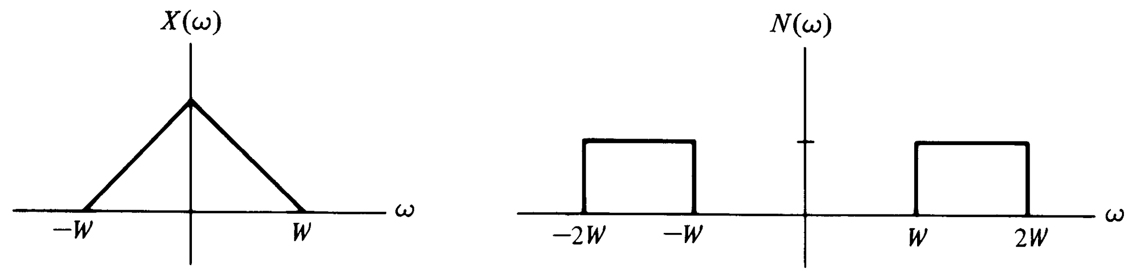

Figure P16.9-2

P16.10

Consider the system in Figure P16.10-1.

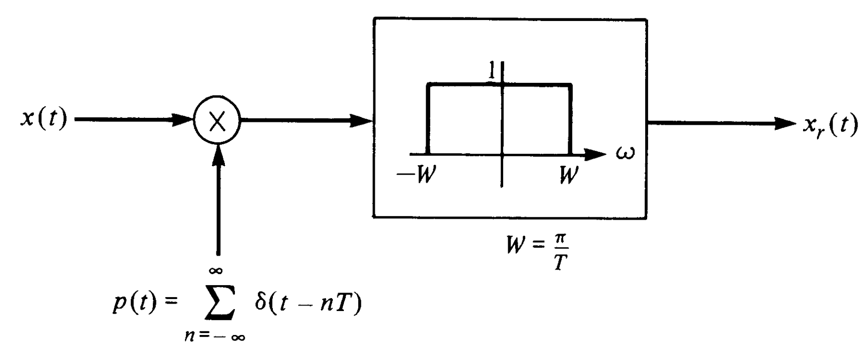

Figure P16.10-1

Given the Fourier transform of $x_r(t)$ in Figure P16.10-2, sketch the Fourier transform of two different signals $x(t)$ that could have generated $x_r(t)$.

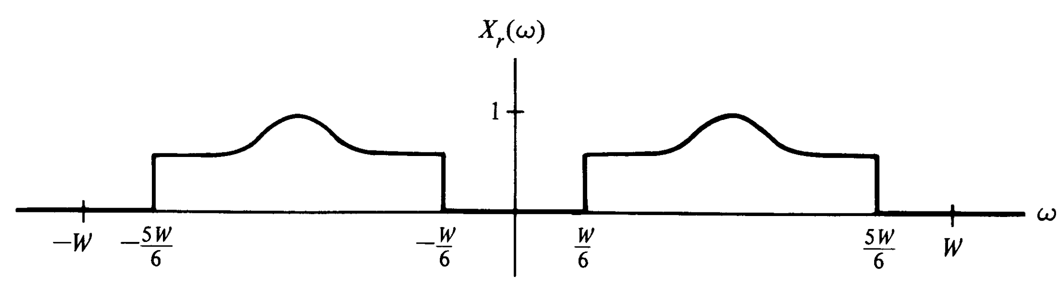

Figure P16.10-2

P16.11

Consider the system in Figure P16.11.

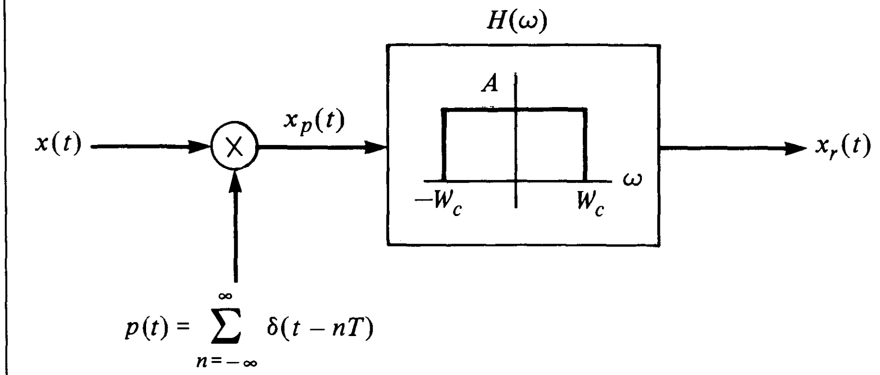

Figure P16.11

(a) If $X(\omega) = 0$ for $|\omega| > W$, find the maximum value of T, W_c, and A such that $x_r(t) = x(t)$.

(b) Let $X_1(\omega) = 0$ for $|\omega| > 2W$ and $X_2(\omega) = 0$ for $|\omega| > W$. Repeat part (a) for the following.

(i) $x(t) = x_1(t) * x_2(t)$

(ii) $x(t) = x_1(t) + x_2(t)$

(iii) $x(t) = x_1(t)x_2(t)$

(iv) $x(t) = x_1(10t)$