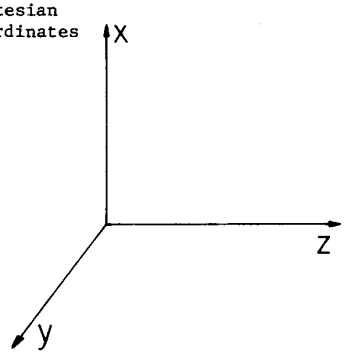
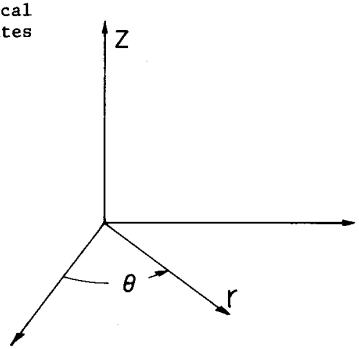
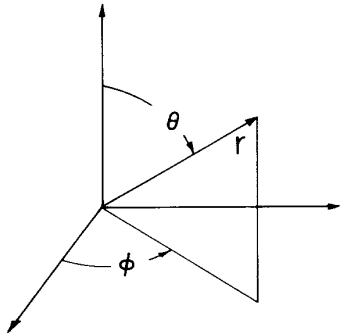
Appendix A

Differential Operators in Cartesian, Cylindrical and Spherical Coordinates

APPENDIX A. Differential Operators in Cartesian, Cylindrical and Spherical Coordinates

Operator	Cartesian coordinates 	Cylindrical coordinates 	Spherical coordinates 
$(\nabla \cdot \vec{A})$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$
$\nabla\Phi$	$\frac{\partial \Phi}{\partial x}\vec{i}_x + \frac{\partial \Phi}{\partial y}\vec{i}_y + \frac{\partial \Phi}{\partial z}\vec{i}_z$	$\frac{\partial \Phi}{\partial r}\vec{i}_r + \frac{1}{r}\frac{\partial \Phi}{\partial \theta}\vec{i}_\theta + \frac{\partial \Phi}{\partial z}\vec{i}_z$	$\frac{\partial \Phi}{\partial r}\vec{i}_r + \frac{1}{r}\frac{\partial \Phi}{\partial \theta}\vec{i}_\theta + \frac{1}{r\sin\theta}\frac{\partial \Phi}{\partial \phi}\vec{i}_\phi$
$(\nabla^2\Phi)$	$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}$	$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Phi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial \theta^2} + \frac{\partial^2\Phi}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial \Phi}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial \Phi}{\partial \theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial \phi^2}$
$(\nabla \times \vec{A})$	$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{i}_x$ $+\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{i}_y$ $+\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{i}_z$	$\left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{i}_r$ $+\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{i}_\theta$ $+\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)\vec{i}_z$	$\left(\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial \phi}\right)\vec{i}_r$ $+\left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)\right)\vec{i}_\theta$ $+\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)\vec{i}_\phi$

$\nabla^2\vec{A}$	$\left[\frac{\partial^2 A_x}{\partial x^2}+\frac{\partial^2 A_x}{\partial y^2}+\frac{\partial^2 A_x}{\partial z^2}\right]\vec{i}_x$ $+\left[\frac{\partial^2 A_y}{\partial x^2}+\frac{\partial^2 A_y}{\partial y^2}+\frac{\partial^2 A_y}{\partial z^2}\right]\vec{i}_y$ $+\left[\frac{\partial^2 A_z}{\partial x^2}+\frac{\partial^2 A_z}{\partial y^2}+\frac{\partial^2 A_z}{\partial z^2}\right]\vec{i}_z$	$\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_r)\right)+\frac{1}{r^2}\frac{\partial^2 A_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial A_\theta}{\partial\theta}+\frac{\partial^2 A_r}{\partial z^2}\right]\vec{i}_r$ $+\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 A_\theta}{\partial\theta^2}+\frac{2}{r^2}\frac{\partial A_r}{\partial\theta}+\frac{\partial^2 A_\theta}{\partial z^2}\right]\vec{i}_\theta$ $+\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 A_z}{\partial\theta^2}+\frac{\partial^2 A_z}{\partial z^2}\right]\vec{i}_z$	$\left[\nabla^2 A_r-\frac{2A_r}{r^2}-\frac{2}{r^2}\frac{\partial A_\theta}{\partial\theta}-\frac{2A_\theta\cot\theta}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\phi}\right]\vec{i}_r$ $+\left[\nabla^2 A_\theta+\frac{2}{r^2}\frac{\partial A_r}{\partial\theta}-\frac{A_\theta}{r^2\sin^2\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial\phi}\right]\vec{i}_\theta$ $+\left[\nabla^2 A_\phi-\frac{A_\phi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial\phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial\phi}\right]\vec{i}_\phi$
$\vec{C}\cdot\nabla\vec{A}$	$\left(C_x\frac{\partial A_x}{\partial x}+C_y\frac{\partial A_x}{\partial y}+C_z\frac{\partial A_x}{\partial z}\right)\vec{i}_x$ $+\left(C_x\frac{\partial A_y}{\partial x}+C_y\frac{\partial A_y}{\partial y}+C_z\frac{\partial A_y}{\partial z}\right)\vec{i}_y$ $+\left(C_x\frac{\partial A_z}{\partial x}+C_y\frac{\partial A_z}{\partial y}+C_z\frac{\partial A_z}{\partial z}\right)\vec{i}_z$	$\left(C_r\frac{\partial A_r}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_r}{\partial\theta}+C_z\frac{\partial A_r}{\partial z}-\frac{C_\theta A_\theta}{r}\right)\vec{i}_r$ $+\left(C_r\frac{\partial A_\theta}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_\theta}{\partial\theta}+C_z\frac{\partial A_\theta}{\partial z}+\frac{C_\theta A_r}{r}\right)\vec{i}_\theta$ $+\left(C_r\frac{\partial A_z}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_z}{\partial\theta}+C_z\frac{\partial A_z}{\partial z}\right)\vec{i}_z$	$\left(C_r\frac{\partial A_r}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_r}{\partial\theta}+\frac{C_\phi}{r\sin\theta}\frac{\partial A_r}{\partial\phi}-\frac{C_\theta A_\theta}{r}-\frac{C_\phi A_\phi}{r}\right)\vec{i}_r$ $+\left(C_r\frac{\partial A_\theta}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_\theta}{\partial\theta}+\frac{C_\phi}{r\sin\theta}\frac{\partial A_\theta}{\partial\phi}+\frac{C_\theta A_r}{r}-\frac{C_\phi A_\phi\cot\theta}{r}\right)\vec{i}_\theta$ $+\left(C_r\frac{\partial A_\phi}{\partial r}+\frac{C_\theta}{r}\frac{\partial A_\phi}{\partial\theta}+\frac{C_\phi}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}+\frac{C_\phi A_r}{r}+\frac{C_\phi A_\theta\cot\theta}{r}\right)\vec{i}_\phi$
$(\vec{\vec{T}}:\nabla\vec{A})$	$T_{xx}\left(\frac{\partial A_x}{\partial x}\right)+T_{yy}\left(\frac{\partial A_y}{\partial y}\right)+T_{zz}\left(\frac{\partial A_z}{\partial z}\right)$ $+T_{xy}\left(\frac{\partial A_x}{\partial y}+\frac{\partial A_y}{\partial x}\right)$ $+T_{yz}\left(\frac{\partial A_y}{\partial z}+\frac{\partial A_z}{\partial y}\right)+T_{zx}\left(\frac{\partial A_z}{\partial x}+\frac{\partial A_x}{\partial z}\right)$	$T_{rr}\left(\frac{\partial A_r}{\partial r}\right)+T_{\theta\theta}\left(\frac{1}{r}\frac{\partial A_\theta}{\partial\theta}+\frac{A_r}{r}\right)+T_{zz}\left(\frac{\partial A_z}{\partial z}\right)$ $+T_{r\theta}\left(r\frac{\partial}{\partial r}\left(\frac{A_\theta}{r}\right)+\frac{1}{r}\frac{\partial A_r}{\partial\theta}\right)+T_{\theta z}\left(\frac{1}{r}\frac{\partial A_z}{\partial\theta}+\frac{\partial A_\theta}{\partial z}\right)$ $+T_{rz}\left(\frac{\partial A_z}{\partial r}+\frac{\partial A_r}{\partial z}\right)$	$T_{rr}\left(\frac{\partial A_r}{\partial r}\right)+T_{\theta\theta}\left(\frac{1}{r}\frac{\partial A_\theta}{\partial\theta}+\frac{A_r}{r}\right)+T_{\phi\phi}\left(\frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}+\frac{A_r}{r}+\frac{A_\theta\cot\theta}{r}\right)$ $+T_{r\theta}\left(\frac{\partial A_\theta}{\partial r}+\frac{1}{r}\frac{\partial A_r}{\partial\theta}-\frac{A_\theta}{r}\right)+T_{r\phi}\left(\frac{\partial A_\phi}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi}-\frac{A_\phi}{r}\right)$ $+T_{\theta\phi}\left(\frac{1}{r}\frac{\partial A_\phi}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial\phi}-\frac{\cot\theta}{r}A_\phi\right)$
$\nabla\cdot\vec{\vec{T}}$	$\left(\frac{\partial T_{xx}}{\partial x}+\frac{\partial T_{xy}}{\partial y}+\frac{\partial T_{xz}}{\partial z}\right)\vec{i}_x$ $+\left(\frac{\partial T_{yx}}{\partial x}+\frac{\partial T_{yy}}{\partial y}+\frac{\partial T_{yz}}{\partial z}\right)\vec{i}_y$ $+\left(\frac{\partial T_{zx}}{\partial x}+\frac{\partial T_{zy}}{\partial y}+\frac{\partial T_{zz}}{\partial z}\right)\vec{i}_z$	$\left(\frac{1}{r}\frac{\partial}{\partial r}(rT_{rr})+\frac{1}{r}\frac{\partial}{\partial\theta}T_{r\theta}-\frac{1}{r}T_{\theta\theta}+\frac{\partial T_{rz}}{\partial z}\right)\vec{i}_r$ $+\left(\frac{1}{r}\frac{\partial T_{\theta\theta}}{\partial\theta}+\frac{\partial T_{r\theta}}{\partial r}+\frac{2}{r}T_{r\theta}+\frac{\partial T_{\theta z}}{\partial z}\right)\vec{i}_\theta$ $+\left(\frac{1}{r}\frac{\partial}{\partial r}(rT_{zr})+\frac{1}{r}\frac{\partial T_{z\theta}}{\partial\theta}+\frac{\partial T_{zz}}{\partial z}\right)\vec{i}_z$	$\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2T_{rr})+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(T_{r\theta}\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial T_{r\phi}}{\partial\phi}-\frac{T_{\theta\theta}+T_{\phi\phi}}{r}\right)\vec{i}_r$ $+\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2T_{r\theta})+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(T_{\theta\theta}\sin\theta)+\frac{1}{r\sin\theta}\frac{\partial T_{\theta\phi}}{\partial\phi}+\frac{T_{r\theta}}{r}-\frac{\cot\theta}{r}T_{\phi\phi}\right)\vec{i}_\theta$ $+\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2T_{r\phi})+\frac{1}{r}\frac{\partial T_{\theta\phi}}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial T_{\phi\phi}}{\partial\phi}+\frac{T_{r\phi}}{r}+\frac{2\cot\theta}{r}T_{\theta\phi}\right)\vec{i}_\phi$

Tθr

Tθφ

Appendix B

Vector and Operator Identities

$$\vec{A} \times \vec{B} \cdot \vec{C} = \vec{A} \cdot \vec{B} \times \vec{C} \tag{1}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) \tag{2}$$

$$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi \tag{3}$$

$$\nabla \cdot (\vec{A} + \vec{B}) = \nabla \cdot \vec{A} + \nabla \cdot \vec{B} \tag{4}$$

$$\nabla \times (\vec{A} + \vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B} \tag{5}$$

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi \tag{6}$$

$$\nabla \cdot (\psi\vec{A}) = \vec{A} \cdot \nabla\psi + \psi\nabla \cdot \vec{A} \tag{7}$$

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot \nabla \times \vec{A} - \vec{A} \cdot \nabla \times \vec{B} \tag{8}$$

$$\nabla \cdot \nabla\phi = \nabla^2\phi \tag{9}$$

$$\nabla \cdot \nabla \times \vec{A} = 0 \tag{10}$$

$$\nabla \times \nabla\phi = 0 \tag{11}$$

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2\vec{A} \tag{12}$$

$$(\nabla \times \vec{A}) \times \vec{A} = (\vec{A} \cdot \nabla)\vec{A} - 1/2\ \nabla(\vec{A} \cdot \vec{A}) \tag{13}$$

$$\nabla(\vec{A} \cdot \vec{B}) = (\vec{A} \cdot \nabla)\vec{B} + (\vec{B} \cdot \nabla)\vec{A} + \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) \tag{14}$$

$$\nabla \times (\phi\vec{A}) = \nabla\phi \times \vec{A} + \phi\nabla \times \vec{A} \tag{15}$$

$$\nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) + (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B} \tag{16}$$

Appendix C

Films

Developed for educational purposes with the support of the National Science Foundation at the Education Development Center, films cited fall in one of two series.

Produced by the National Committee for Fluid Mechanics Films and distributed by Encyclopedia Britannica Educational Corp., 425 N. Michigan Ave., Chicago, Illinois (60611) are:

(1) Channel Flow of a Compressible Fluid
(2) Current-induced Instability of a Mercury Jet
(3) Eulerian and Lagrangian Descriptions in Fluid Mechanics
(4) Flow Instabilities
(5) Fundamentals of Boundary Layers
(6) Low-Reynolds Number Flows
(7) Magnetohydrodynamics
(8) Pressure Fields and Fluid Acceleration
(9) Surface Tension and Fluid Mechanics
(10) Waves in Fluids

Produced by the National Committee for Electrical Engineering Films and distributed by Education Development Center, 39 Chapel Street, Newton, Mass. 02160 are:

(11) Complex Waves I and Complex Waves II
(12) Electric Fields and Moving Media

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