Chapter 11

Chapter IV: Consistency, Completeness, and Geometry

I have obtained some results which are "repugnant to the nature of a straight-line." – Saccheri

11.1 Abstract

The preceding Dialogue is explicated to the extent it is possible at this stage. This leads back to the question of how and when symbols in a formal system acquire meaning. The history of Euclidean and non-Euclidean geometry is given, as an illustration of the elusive notion of "undefined terms". This leads to ideas about the consistency of different and possibly "rival" geometries. Through this discussion the notion of undefined terms is clarified, and the relation of undefined terms to perception and thought processes is considered. *GEB pp. ix*

11.2 Questions

1. Give some examples of complex isomorphisms. What machinary is Hofstadter (DRH) talking about?

2. Why does DRH keep apologizing about his use of the term "isomorphism"?

3. Give an example of when a word can have multiple meanings. When you say something, how do you communicate the intended meaning?

4. DRH asks that we assume that every reader of English uses more or less the same "isomorphism" in sucking meaning from the marks on paper. What does he mean by this? In what ways could two readers of English use different isomorphisms in extracting meaning from the same piece of writing? Hint: Think about the conceptual web you attach to even very simple words. As an exercise pick any noun from the dialogue and create your own personal concept web attached to that noun. For example if you picked the noun "present" write it in the middle of a blank piece of paper and draw two lines to separate areas of the paper and then write "now" at the end of one line and "gift" at the other line's end. Branch off of each of these concepts ("Santa Clause", "My Birthday", etc.) and give yourself a point for every concept you attach to your initial word. Identify levels of meaning if you're keen.

5. What's the problem with interpreting mathematical objects? In the case of the modified pq-system? In the case of Euclidean geometry? What's wrong with our interpretation of a "straight line"?

6. Draw pictures of Euclid's five postulates. Do they seem like a safe truths to assume?

7. Do you think mathematics is the same all possible worlds? Consider carefully what you consider to be mathematics.

8. Do you think logic is the same in all possible worlds? What then constitutes a possible world?

9. Pushing possible worlds aside, What logic does our universe obey? Consider if our universe obeys **The Law of the Excluded Middle** which states that either P or not-P must be true, there is no room between true and false. What does quantum mechanics say about the logic of the universe?

10. *GEB pp. 102* Answer DRH's question about the record and the record-player. How can they both pass their respective tests?

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