Manipulating numbers numerically

Take these two numbers (they are called a conjugate pair):

$$z_1 = 1 + i\sqrt{3}$$

$$z_2 = 1 - i\sqrt{3}$$

- Firstly, what are the magnitudes and phase angles?

  $|z_1| = \sqrt{\mathbf{1^2} + (\sqrt{\mathbf{3}})^{\mathbf{2}}} = \mathbf{2}$ $\qquad$ $|z_2| = \sqrt{\mathbf{1^2} + (\sqrt{\mathbf{3}})^{\mathbf{2}}} = \mathbf{2}$

  $\angle z_1 = \tan^{-\mathbf{1}} \sqrt{\mathbf{3}} = \frac{\pi}{\mathbf{3}} = \mathbf{60}\,\text{deg}$ $\qquad$ $\angle z_2 = \tan^{-\mathbf{1}} -\sqrt{\mathbf{3}} = -\frac{\pi}{\mathbf{3}} = \mathbf{-60}\,\text{deg}$

- How about their sum?

$$z_1 + z_2 = (\mathbf{1} + \mathbf{1}) + \mathbf{i}(\sqrt{\mathbf{3}} - \sqrt{\mathbf{3}}) = \mathbf{2}$$

- And the product?
  Use either the individual components, or the magnitude and phase:
  $|z_1 z_2| = |z_1||z_2|$, $\angle(z_1 z_2) = \angle z_1 + \angle z_2$

  The product has magnitude 4 ($|z_1 z_2|$) and phase angle 0 ($\angle z_1 + \angle z_2$). So $z_1 z_2 = 4$.

Algebraic expressions

- If A is a constant, find:

$$1 - \frac{1}{1 + iA} = \frac{\mathbf{1 + iA}}{\mathbf{1 + iA}} - \frac{\mathbf{1}}{\mathbf{1 + iA}} = \frac{\mathbf{iA}}{\mathbf{1 + iA}}$$

- If ω, R, L and C are constants, and:

$$z_1 = R + i\omega L \qquad \text{and} \qquad z_2 = \frac{1}{i\omega C}$$

  Find the following:

$$\frac{z_1 z_2}{z_1 + z_2} = \frac{\frac{\mathbf{R + i\omega L}}{\mathbf{i\omega C}}}{\mathbf{R + i\omega L} + \frac{\mathbf{1}}{\mathbf{i\omega C}}}$$

$$\frac{z_1 z_2}{z_1 + z_2} = \frac{\mathbf{R + i\omega L}}{\mathbf{1 - \omega^2 LC + i\omega RC}}$$

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