

Problem Set
Problems, including calculations for Columbia A

1) Consider two plants, A and B, both of which emit carbon, C , (where C is measured in units of carbon per cubic foot per hour.) The two plants have the following marginal cost of abatement schedules.

$$MCA_A = \$10.00 - .25C_A \quad MCA_B = \$8.00 - .20C_B$$

Where MCA_A is the Marginal Cost of Abatement at Plant A, C_A is the amount of carbon per hour being emitted at Plant A, and so on,

a) If the government sets a carbon tax of \$2.00/C, what is the quantity of carbon emitted per hour by the sum of the two plants?

b) What level of emissions fee would have to be imposed to reduce emissions to 44 units per hour?

c) Suppose that the government wanted to achieve the same goal as (b) using a cap and trade model instead of a carbon tax. Under this cap and trade model, each firm would be given the same number of permits and each permit would allow a firm to emit one unit of pollutant. How many permits would each firm initially receive? For each plant, discuss whether they would initially be a buyer or a seller of permits. When trading begins, describe the maximum amount the buying firm would be willing to pay for a permit and the minimum amount the selling firm would be willing to receive for a permit.

2) Consider a community's demand for public television. There are three people in the (small) community and their demands for hours of public television programming are given by the following three demand functions.

$$P_1 = \$150 - H$$

$$P_2 = \$200 - 2H$$

$$P_3 = \$250 - H$$

Where: P_i = the price the i 'th individual is willing to pay and H is the number of hours of public television the individual watches.

Hours of public television can be produced at a constant marginal cost of \$200 per hour.

a) Consider three varieties of television programming: public television that you can receive with a regular antenna, commercial television (i.e. with advertising) that you can receive with a

regular antenna, and cable television. Which, if any, of these varieties of television are pure public goods? Explain your reasoning.

b) From society's perspective, explain how you would determine the efficient number of hours of public television and calculate what that number is

c) If public television was sold to viewers who paid fees to the station, how many hours of public television would a competitive private market provide?

3) **(REWRITTEN)** Consider the stream of costs and benefits from a small commercial parking garage. You purchase the parking garage today for a price of \$880,000. You immediately begin to operate the parking garage and continue to operate it for ten years. At the end of the tenth year we assume (for simplicity) that the garage instantaneously collapses and there is no salvage value.

	Today	Year 1 of oper.	Year 2 of oper.	Year3 of oper.	Year 4 of oper.	etc. through year 10
Costs	\$880,000	\$40,000	\$40,000	\$40,000	\$40,000	\$40,000
Revenue	0	\$200,000	\$200,000	\$200,000	\$200,000	\$200,000

a) Familiarize yourself with the NPV function in Excel. Using this function, determine whether you would undertake the project if the annual interest rate were 5%. Then determine whether you would undertake the project if the annual interest rate were 10%? To use the NPV function, you may find it easier to collapse revenues and costs into a single "net revenue" figure for each year - i.e. \$160,000 for Year 1 of operation and so on.

In using the NPV function, note that Excel expects the first payment entered to occur at the end of the first period so you want to set up the problem as:

NPV of the project = $-\$880,000 + \text{Excel NPV function of (net revenue in year 1 of operation, in year 2 of operation, etc.)}$

b) The Army Corps of Engineers are responsible for building a significant number of dams and other federally funded construction projects. Before a project is approved, it must be justified by net present value calculations. Historically the Corps used a 3% discount rate at times when 30 year corporate bonds had interest rates of 5% or higher. What might be a justification for the 3% rate? What would be a criticism of this 3% rate?

c) Explain what your answers in (a) tell you about the parking garage's internal rate of return.

Use the IRR function in Excel to verify your answer (The IRR function in Excel expects the first entry to be at $t=0$ so you can the stream of costs and revenues as is).

d) Recall that in Mike Greenstone's discussion of estimating the social cost of carbon, the working group of which he was a member faced uncertainty over the proper value of the discount rate. As a result, they used a range of estimates in their work. While this parking garage is not an environmental project, we can use it to examine how you should deal with uncertainty over what discount rate you should be using in calculating net present value. Suppose you are unsure whether the correct interest rate (discount rate) on this project is 5% or 10%? This means that the project's net present value is uncertain. Consider two ways to calculate NPV under this uncertainty:

- i) Take the two values of NPV you calculated in (a) and average them - i.e. average the project's value calculated under each of the two interest rates.
- ii) Average the two interest rates themselves and calculate a single, new NPV value based on this average interest rate.

Explain what difference the choice of procedure makes in the case of assessing a large environmental cost that occurs 100 years into the future. (For the record, environmental economists agree that the correct way to deal with the problem is (i).

4) When a government or private company issues bonds, the bonds are usually sold at auction.

- The bond says it will pay to the holder fixed amounts of money on one or more specific dates – say \$10,000 on November 19, 2020. No interest rate is specified
- The bond is sold today at auction for a price based on supply and demand – we can call the price P_{2010}

On November 19, 2010, the city of Lynn, Massachusetts sells bonds at auction that promise to pay the bondholder \$1,000 on November 19, 2015 and \$5,000 on November 19, 2020. On the date of the auction, similar bonds sold for prices that imply a 5 percent rate of interest.

a) Compute the price these Lynn, Massachusetts bonds will sell for.

b) Financial news stories often contain the explanatory sentence – “Bond prices and interest rates move in opposite directions.” Use the kinds of calculations you made in (a) to demonstrate that proposition.

c) Suppose Lynn’s bonds were sold with a “call” provision. Under the provision, Lynn has the option of repurchasing the bonds at any future date between now and November 19, 2020 at the price the bond would bring on that date if interest rates remained at 5 percent. If Lynn wanted to exercise this call on November 19, 2016, what price would Lynn pay to repurchase a bond?

d) If Lynn issues its bonds with the call provision, will a bond’s initial auction price be higher or lower than if the price of the same bond with no call provision? (Think about the situation that would cause Lynn to exercise the call.) Explain your reasoning.

5) Work through the calculations needed to prepare the Commonwealth Development A case that can be downloaded from the website. While there is lots of detail, the overall structure of the problem is broadly similar to the parking lot problem. A developer is trying to calculate whether a development project offers a high enough rate of return to attract investors. This is the project’s structure.

a) Investors put up an initial amount of capital.

b) The invested capital together with a one year construction loan are used to finance the construction of the building.

c) Once the building is constructed, the investors take out a 30 year mortgage on the completed building. They use the mortgage to pay off the construction loan.

d) While the mortgage lasts for thirty years, the developer wants to estimate the value of the investment after 10 years of operation. Specifically, the developer will estimate what the building might be sold for after ten years. Assuming he actually sold the building at that price, he will calculate whether the rate of return on the investors’ initial capital at least equals a target rate of return he believes would be necessary to attract investors.

In sum, your job is to figure out which figures are relevant, to organize them appropriately, and then answer the developer’s question.

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11.202 Planning Economics
Fall 2010

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