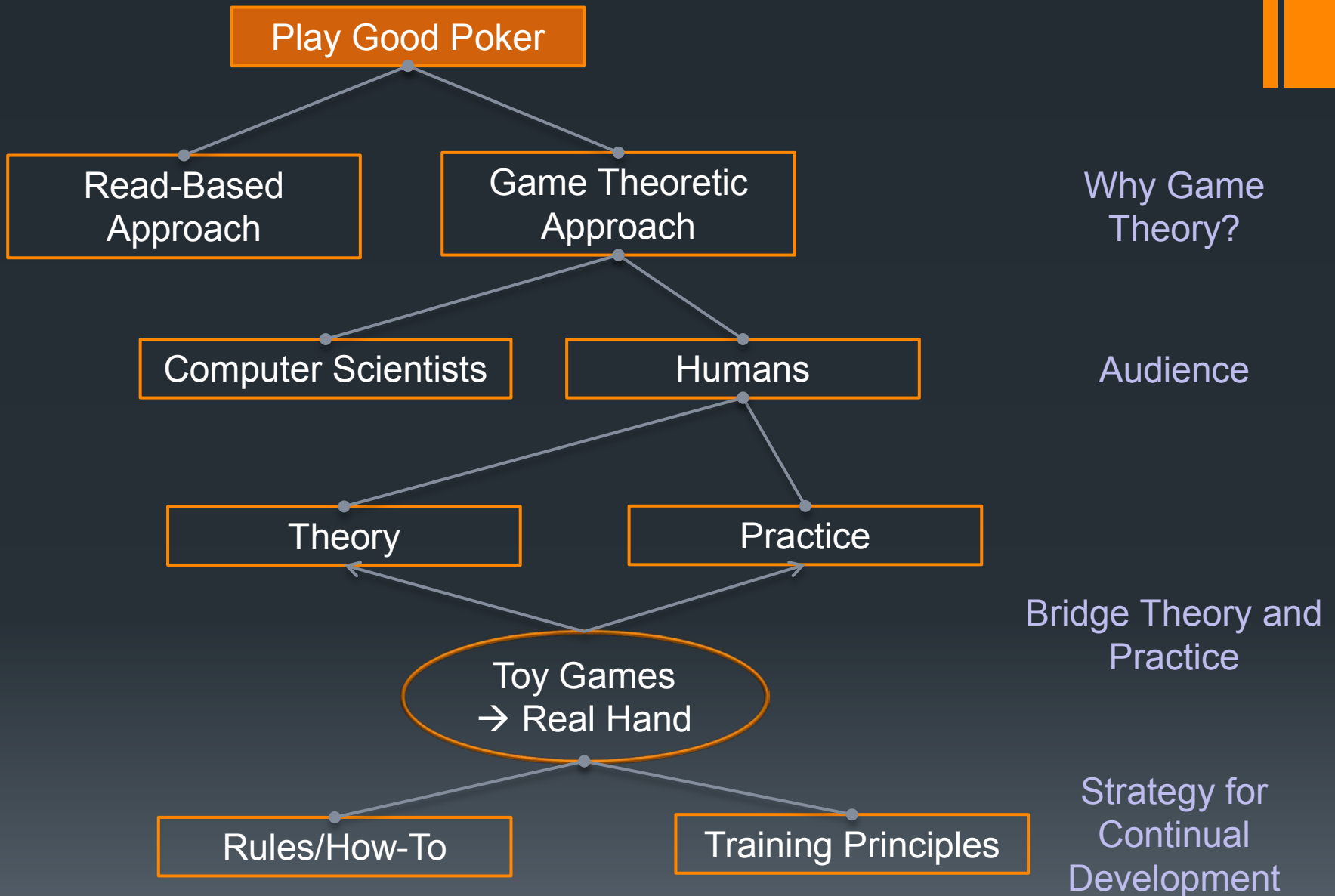




Game Theory for Humans

Matt Hawrilenko

MIT: *Poker Theory and Analytics*





For example . . .

No Limit | \$12,000/\$24,000

ID:6476906

hoser4
\$477,000
Folds

hoser1
\$698,000
Folds

\$12,000

D

PokerHandReplays.com

Pot: \$119,000

\$24,000

villain
\$2,676,000

hoser3
\$555,000

\$65,000

A♣ A♥
hero
\$1,635,000

hoser2
\$885,000

PokerHandReplays.com

No Limit | \$12,000/\$24,000

ID:6476906

hoser4
\$477,000

hoser1
\$698,000

hoser3
\$555,000



lays.com

Pot: \$160,000

\$75,000

villain
\$2,560,000
Calls \$75,000

\$75,000

A
A
hero
\$1,560,000

hoser2
\$885,000

PokerHandReplays.com



No Limit | \$12,000/\$24,000

ID: 6476906

hoser4
\$477,000

hoser1
\$698,000

hoser3
\$555,000

Community cards: **D**
K♥, J♦, 8♠, 5♣

Pot: \$310,000

Stacks: \$205,000

villain
\$2,355,000
Calls \$205,000

hero
\$1,355,000
A♣, A♥

hoser2
\$885,000

PokerHandReplays.com

No Limit | \$12,000/\$24,000

ID: 6476906

hoser4
\$477,000

hoser1
\$698,000

hoser3
\$555,000



Pot: \$720,000

\$1,080,000

villain
\$1,275,000
Bets

A♣ A♥
hero
\$1,355,000

hoser2
\$885,000

PokerHandReplays.com

No Limit | \$12,000/\$24,000 ID:6476906

hoser4 \$477,000 Folds
 hoser1 \$698,000 Folds
 \$12,000

hoser3 \$555,000
 \$24,000
 villain \$2,676,000

PokerHandReplays.com
 \$119,000
 \$65,000

hero \$1,635,000
 hoser2 \$885,000

PokerHandReplays.com

No Limit | \$12,000/\$24,000 ID:6476906

hoser4 \$477,000
 hoser1 \$698,000

hoser3 \$555,000
 \$75,000
 villain \$2,560,000 Calls \$75,000

PokerHandReplays.com
 \$160,000
 \$75,000

hero \$1,560,000
 hoser2 \$885,000

PokerHandReplays.com

No Limit | \$12,000/\$24,000 ID:6476906

hoser4 \$477,000
 hoser1 \$698,000

hoser3 \$555,000
 \$205,000
 villain \$2,355,000 Calls \$205,000

PokerHandReplays.com
 \$310,000
 \$205,000

hero \$1,355,000
 hoser2 \$885,000

PokerHandReplays.com

No Limit | \$12,000/\$24,000 ID:6476906

hoser4 \$477,000
 hoser1 \$698,000

hoser3 \$555,000
 \$1,080,000
 villain \$1,275,000 Bets

Large Question Mark

hero \$1,355,000
 hoser2 \$885,000

PokerHandReplays.com



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*The trouble is, the other side
can do magic too*

-Cornelius Fudge, Minister for Magic



RULE #1

1. Forget her hand
 - 1 (a). Forget her range

1,000,000,000,000,000,000



*Apologies to John Nash and to the tank top I ripped off

The Clairvoyance Game

What would you do if you lived in a world where you always knew your opponent's hand . . .

Film still of [Harry Potter and Ron Weasley](#) staring into a crystal ball during Divination class. Image removed due to copyright restrictions.

. . . And he knew that you knew?



Coin Flip Clairvoyance

1. Each player antes \$1.
2. You flip a coin. Heads, you win. Tails, your opponent wins.

HOWEVER

3. Only you see the coin after the flip, then you can bet.
4. You choose to bet \$1 or check. Your opponent can only check/call or fold.

Scenario 1



Scenario 2



Images are in the public domain.



Two Questions

1. How often should she call?
2. How often should you bluff?

How often to call?

- Enough to make opponent indifferent to bluffing or giving up

$$E(\text{Bluffing}) = E(\text{Giving Up})$$

$$\text{Pot} * (1 - \% \text{ Call}) = (\text{amount bluffed}) * (\% \text{ Call})$$

$$P (1 - C) = 1 * C$$

$$C = P / (P + 1)$$

Amount
won by
bluffing

Amount
lost by
bluffing

How often to bluff?

- Enough to make opponent indifferent to calling or folding

$$E(\text{calling}) = E(\text{folding})$$

$$(\text{Ratio bluffs/value bets}) (\text{pot} + 1) - \text{value bets} = 0$$

$$b (P+1) - 1 = 0$$

$$b = 1 / (P + 1)$$

Generalizes to variable bet sizes

$$\text{Calling \%} = 1 / (1 + S)$$

$$\text{Bluff ratio} = S / (1 + S)$$

S = proportion of the pot bet



Q: What if it's not a repeated game?

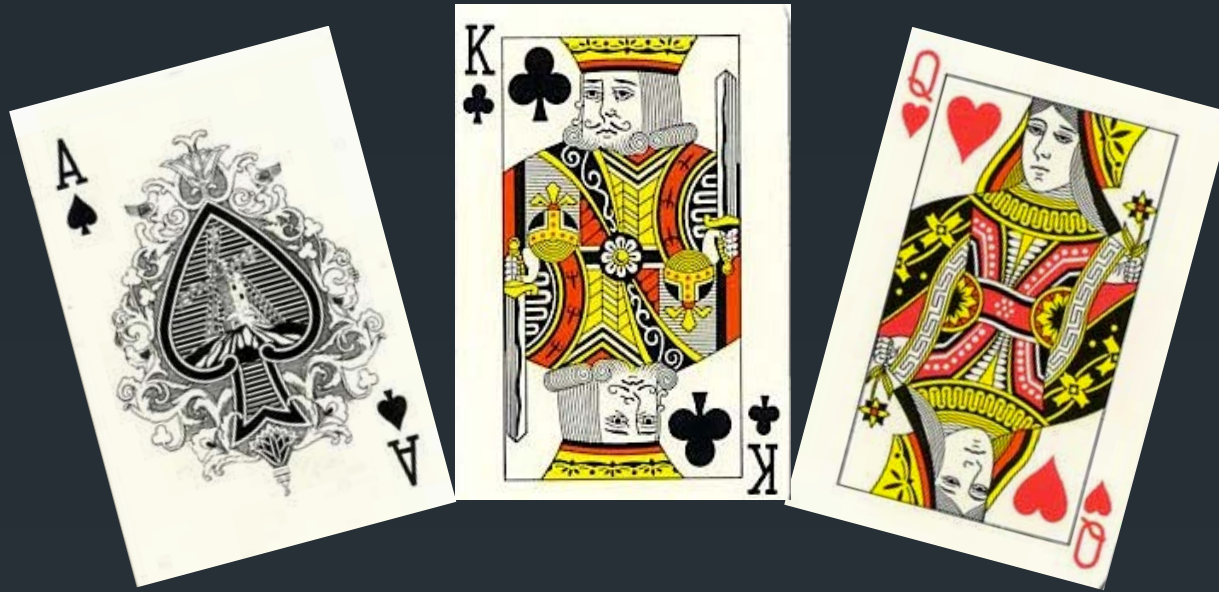
A: It's a repeated game.



Observations

- It's not about value-betting or bluffing, it's about the combination of the two
- We're trying to maximize the value of our entire set of hands, not just the hand we're currently playing

The Ace-King-Queen Game



RULES:

1. Each player antes \$1 and is dealt 1 card
2. Player 1 can check or bet
3. Player 2 can only check/call, or fold

Case 1

YOU



OPPONENT



CHECK OR BET?

Case 2

OPPONENT



YOU



CALL OR FOLD?

Case 3

OPPONENT



YOU



CALL OR FOLD?

Case 4

YOU



OPPONENT



CHECK OR BET?

Case 5

YOU



OPPONENT



CHECK OR BET?

Case 6

OPPONENT



YOU



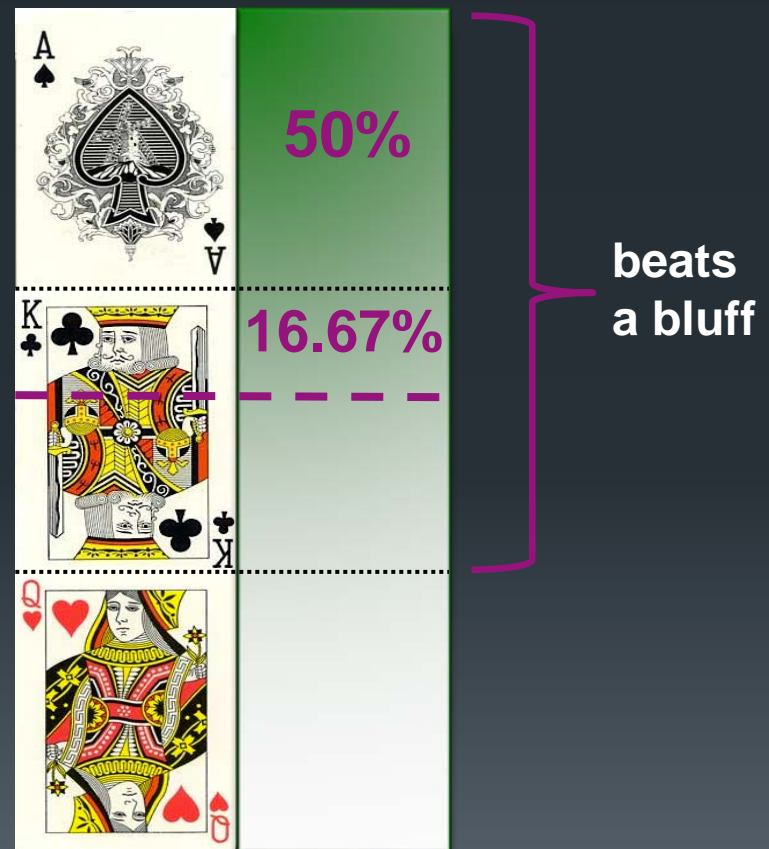
CALL OR FOLD?

How often to call?

Calling ratio = $1 / (1 + 0.5) = 2/3$
of hands that beat a bluff

Aces represent 50% of hands that beat a bluff

All aces + 1/3 of Kings = 2/3 of hands that beat a bluff





Observations

- It's not about value-betting or bluffing, it's about the combination of the two
- We're trying to maximize the value of our entire set of hands, not just the hand we're currently playing
- Useful to map hands as value, bluff catchers, and bluffs

You strategy for one hand determines your strategy for other hands

1. *Know Thyself*

2. *Nothing in Excess*

3. *Make a Pledge and
Mischief is Nigh*

Temple of Apollo at Delphi where people would go to visit the oracle. Image courtesy of [Pilar Torres](#) on Flickr. License: CC BY-NC-SA.



No Limit | \$12,000/\$24,000 ID:6476906

hoser4 \$477,000 Folds
 hoser1 \$698,000 Folds
 \$12,000

hoser3 \$555,000
 \$24,000
 villain \$2,676,000

PokerHandReplays.com
 \$119,000
 \$65,000

hero \$1,635,000
 hoser2 \$885,000

PokerHandReplays.com

No Limit | \$12,000/\$24,000 ID:6476906

hoser4 \$477,000
 hoser1 \$698,000

hoser3 \$555,000
 \$75,000
 villain \$2,560,000 Calls \$75,000

PokerHandReplays.com
 \$160,000
 \$75,000

hero \$1,560,000
 hoser2 \$885,000

PokerHandReplays.com

No Limit | \$12,000/\$24,000 ID:6476906

hoser4 \$477,000
 hoser1 \$698,000

hoser3 \$555,000
 \$205,000
 villain \$2,355,000 Calls \$205,000

PokerHandReplays.com
 \$310,000
 \$205,000

hero \$1,355,000
 hoser2 \$885,000

PokerHandReplays.com

No Limit | \$12,000/\$24,000 ID:6476906

hoser4 \$477,000
 hoser1 \$698,000

hoser3 \$555,000
 \$1,080,000
 villain \$1,275,000 Bets

?

PokerHandReplays.com
 \$720,000
 \$1,080,000

hero \$1,355,000
 hoser2 \$885,000

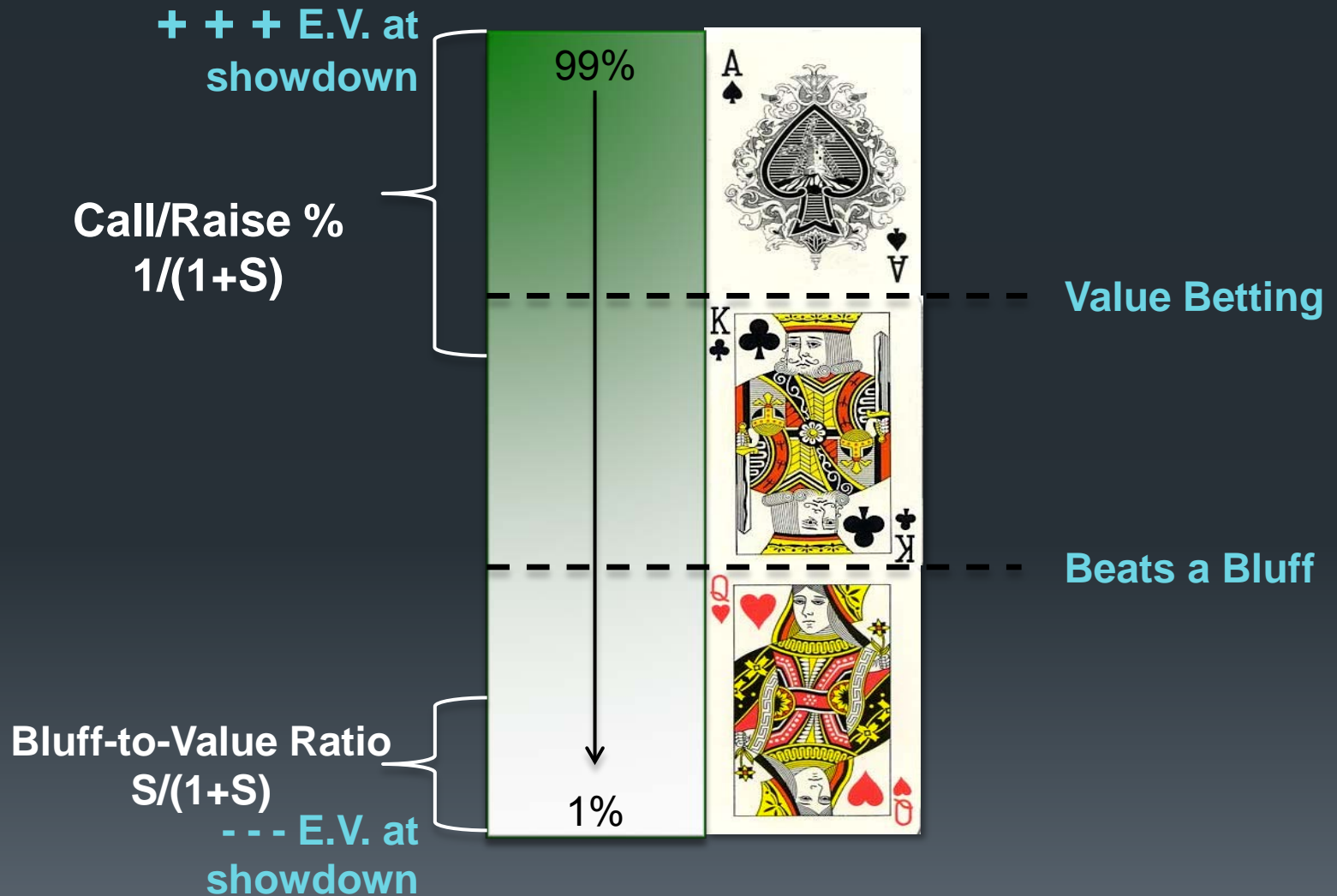
PokerHandReplays.com

Three (exploitive) Strategies

1. My hand vs. your hand
2. My hand vs. your distribution
 - Distribution**: *the frequency distribution of hands a player might hold, given all the action that has occurred*
3. My distribution vs. your distribution



Mapping the AKQ Game





Reading Your Own Hand

- What you do with one hand depends on what you'd do with your other hands
- Most important skill in poker
- Two updates for each street:
 - Account for card removal
 - Account for your action

Preflop Distribution



suited conn

Hand	Combos	Category	Hand	Combos
AK	16	no gap	AK	16
KQ	16		KQ	16
QJ	16		QJ	16
JT	16		JT	16
86s	4	one gap	86s	4
97s	4		97s	4
T8s	4		T8s	4
J9s	4		J9s	4
QT	16		QT	16
KJ	16		KJ	16
AQ	16		AQ	16
KT	16	2 gaps	KT	16
AJ	16		AJ	16
K9s	4	3 gaps	K9s	4
AT	16		AT	16
A2s-A9s	32		A2s-A9s	32
total	310		total	310

Flop Card Removal Update



Hand	Combos	Hand	Combos
22	6	AK	12
33	6	KQ	12
44	6	QJ	12
55	6	JT	12
66	6	86s	3
77	6	97s	4
88	3	T8s	3
99	6	J9s	3
TT	6	QT	16
JJ	3	KJ	9
QQ	6	AQ	16
KK	3	KT	12
AA	6	AJ	12
T9s	4	K9s	3
98s	3	AT	16
87s	3	A2s-A9s	31
76s	4		
65s	4	total	263

Flop Action Update

Villain checks,
hero bets 75,000



Hand	Combos	Hand	Combos
22	6	AK	12
33	6	KQ	12
44	6	QJ	12
55	6	JT	12
66	0	86s	0
77	0	97s	0
88	3	T8s	0
99	0	J9s	3
TT	0	QT	16
JJ	3	KJ	9
QQ	6	AQ	0
KK	3	KT	12
AA	6	AJ	12
T9s	4	K9s	3
98s	0	AT	0
87s	0	A2-A9s	0
76s	4		
65s	4		
		total	160

Turn Card Removal Update

**Villain checks,
hero bets 205,000
Villain calls**



Hand	Combos	Hand	Combos
22	6	AK	12
33	6	KQ	12
44	6	QJ	12
55	3	JT	12
88	3	J9s	3
JJ	3	QT	16
QQ	6	KJ	9
KK	3	KT	12
AA	6	AJ	12
T9s	4	K9s	3
76s	4		
65s	3		

total 156

Turn Action Update

**Villain checks,
hero bets 205,000
Villain calls**



Hand	Combos	Hand	Combos
22	0	AK	12
33	0	KQ	12
44	0	QJ	12
55	3	JT	0
88	3	J9s	0
JJ	3	QT	16
QQ	6	KJ	9
KK	3	KT	12
AA	6	AJ	12
T9s	4	K9s	3
76s	4		
65s	0		

**How's our
proportion of
bluffs here?**

total 120

River Card Removal Update



Hand	Combos
55	3
88	3
JJ	3
QQ	6
KK	1
AA	6
T9s	4
76s	4
AK	8
KQ	8
QJ	12
QT	16
KJ	6
KT	8
AJ	12
K9s	2
Total	102

River Decision



$$S = 1,080,000 / 720,000 \\ = 1.5$$

$$\text{Call} = 1.5 / (1 + 1.5) \\ = 40\%$$

of hands that beat a bluff

Know
Thyself!



beats a bluff

KK	1%
KJ	9%
JJ	13%
88	17%
55	21%
AK	31%
KQ	41%
KT	51%
K9s	54%
AA	62%
QQ	69%
AJ	85%
QJ	100%
QT	
T9s	
76s	



Call
 $1/(1+S)$



Nothing
in
excess!



Recap

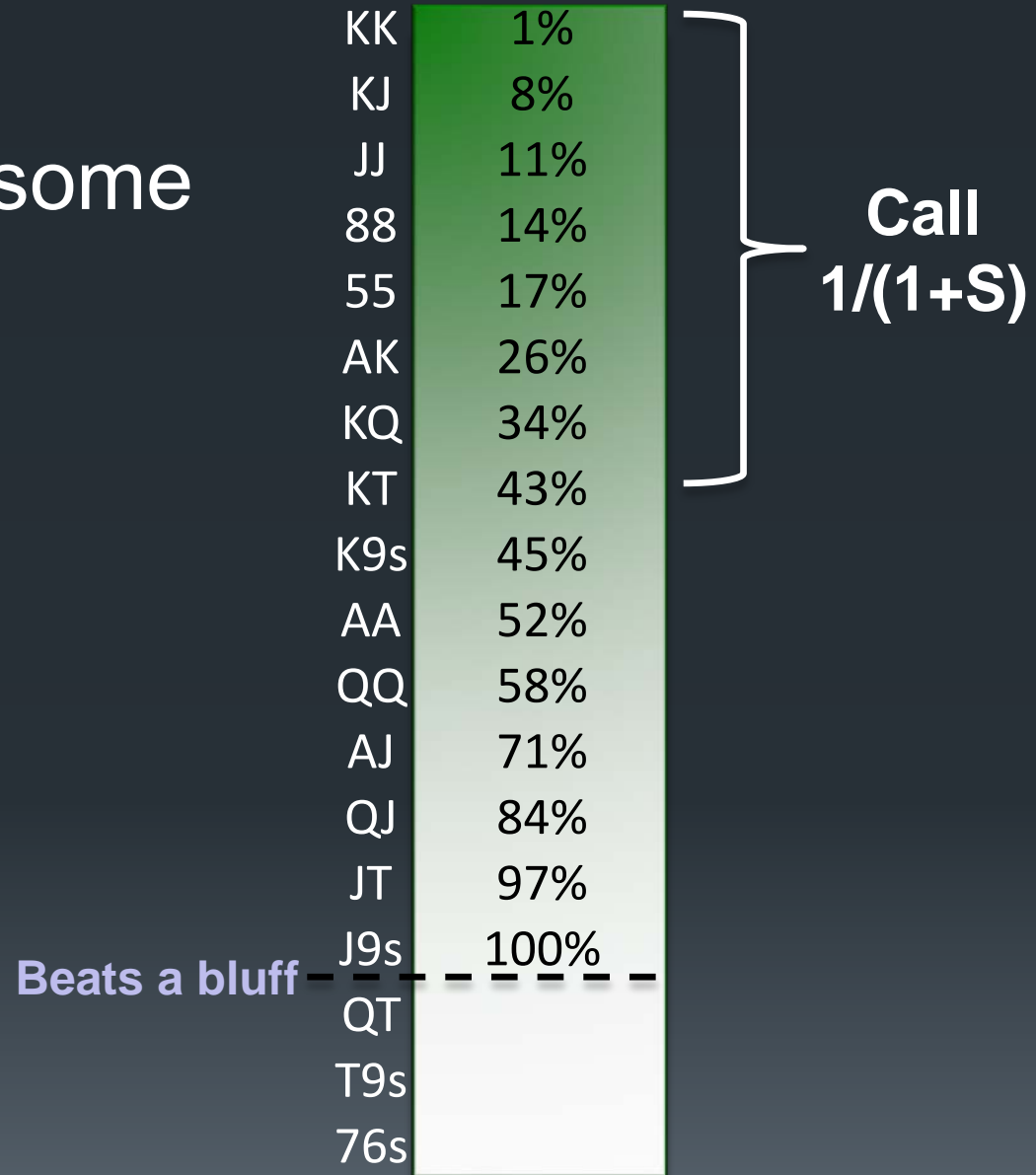
- Solved! Fold AA, Even fold KT

Gut check: Do we want a distribution where we have to fold trips?

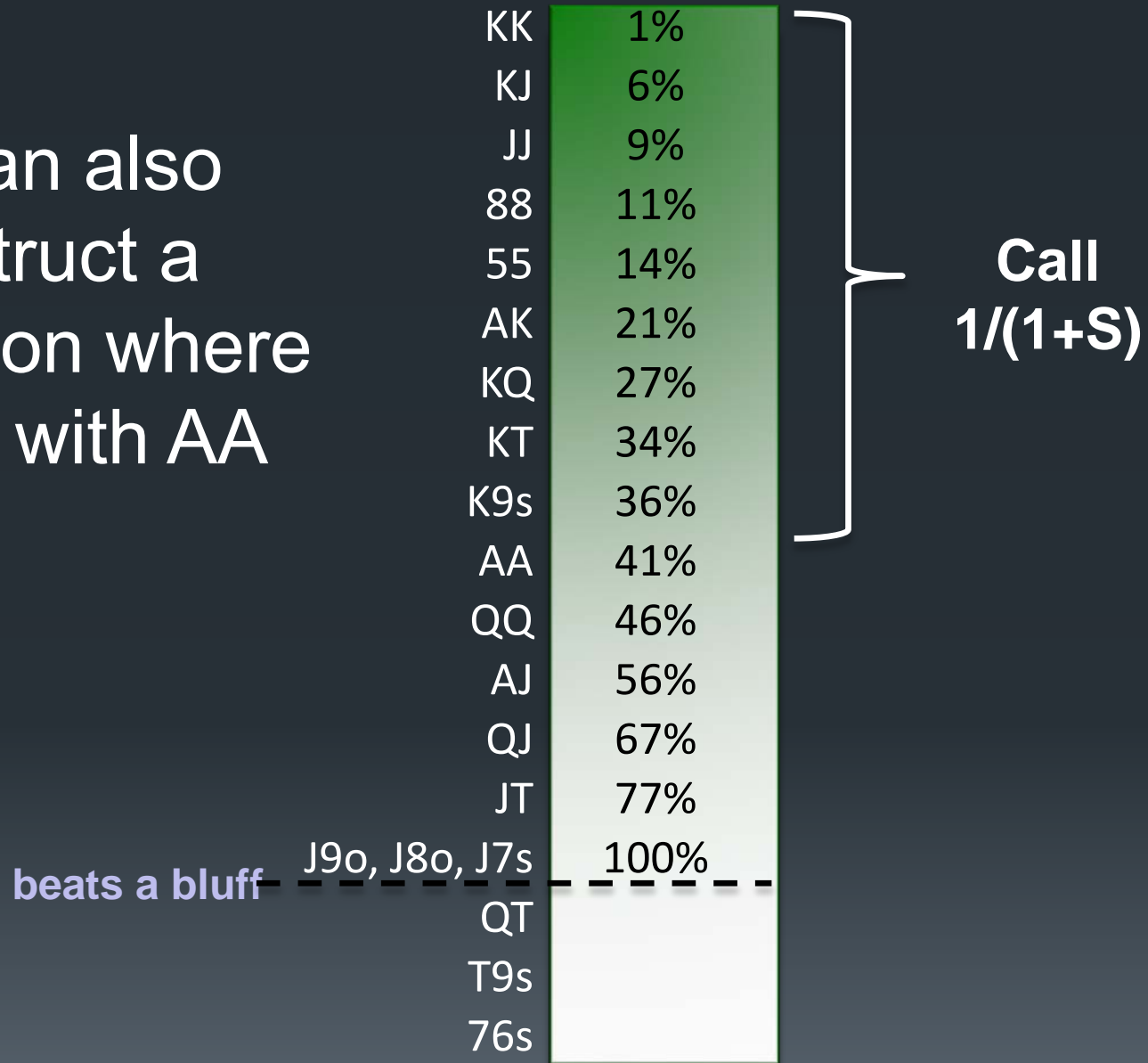
KK	1%
KJ	9%
JJ	13%
88	17%
55	21%
AK	31%
KQ	41%
KT	51%
K9s	54%
AA	62%
QQ	69%
AJ	85%
QJ	100%
QT	
T9s	
76s	

Rule of thumb:
If you'd bet it for value, you
want a distribution where you
don't have to fold it

We can add some hands in . . .



We can also
construct a
distribution where
we call with AA





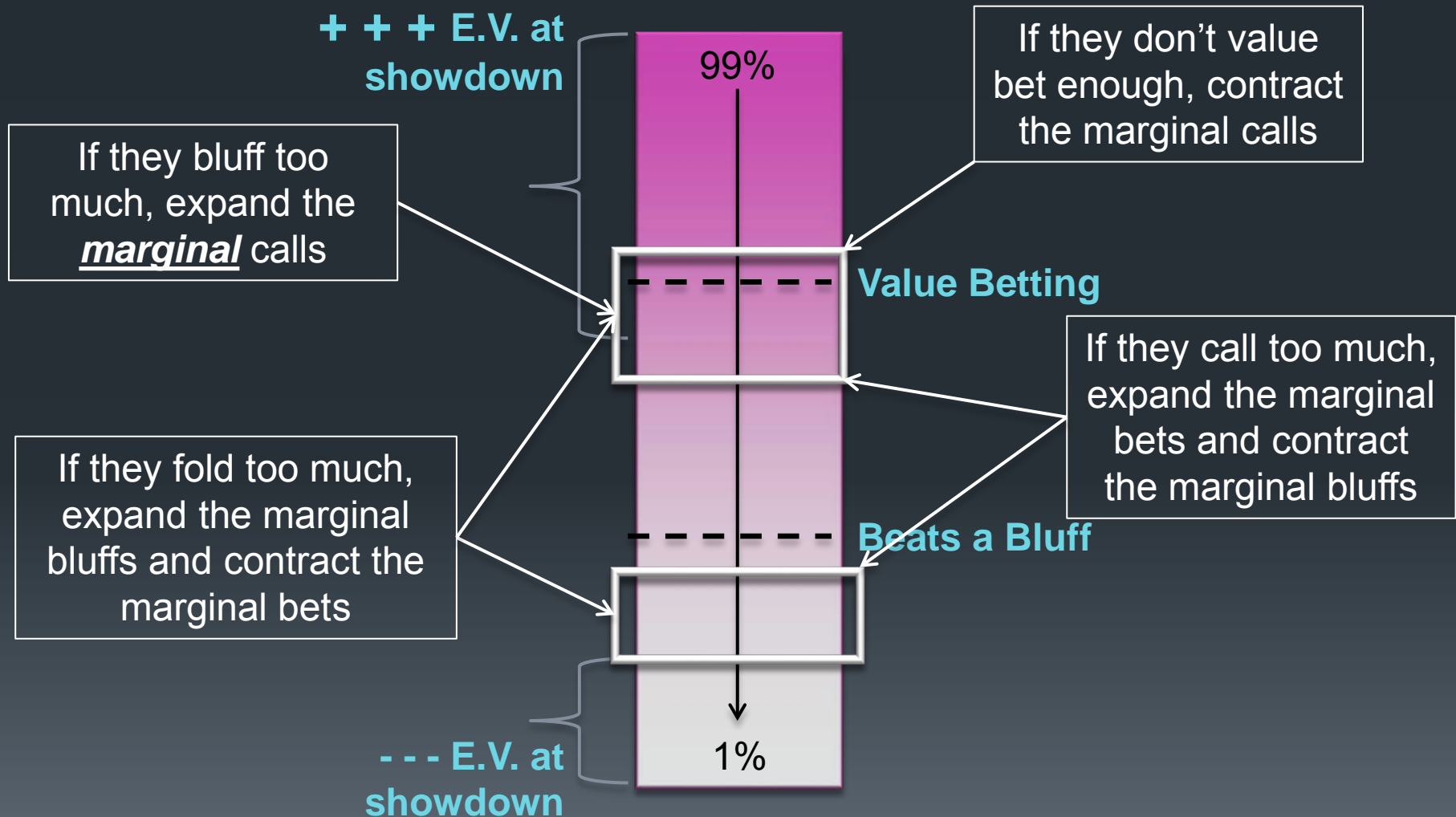
RYOH Redux

- Check for balance on all streets
- Don't overthink it: focus on the glaring errors
- Don't needlessly bifurcate your distribution
- Identify situations where you tend to become imbalanced, then watch opponents for the same tendency

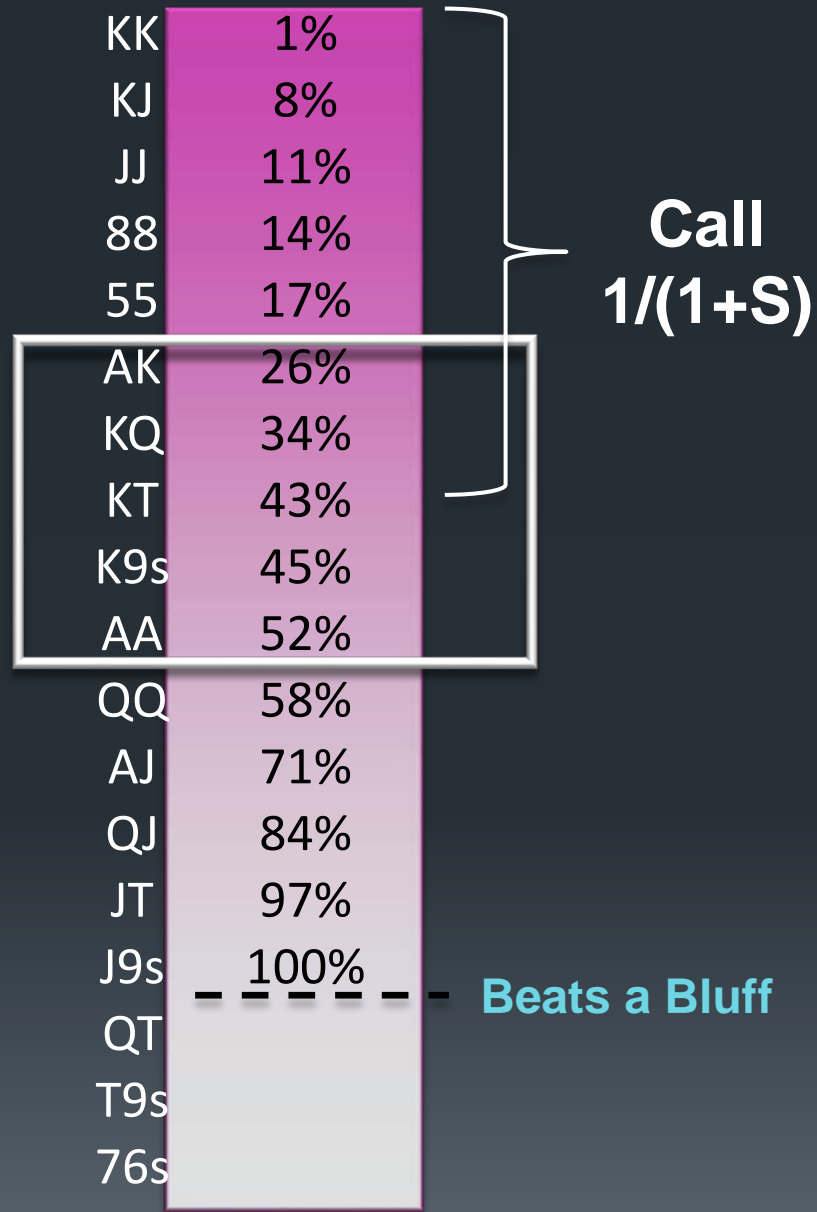


INSINCERE APOLOGY
+
BRIEF MONOLOGUE

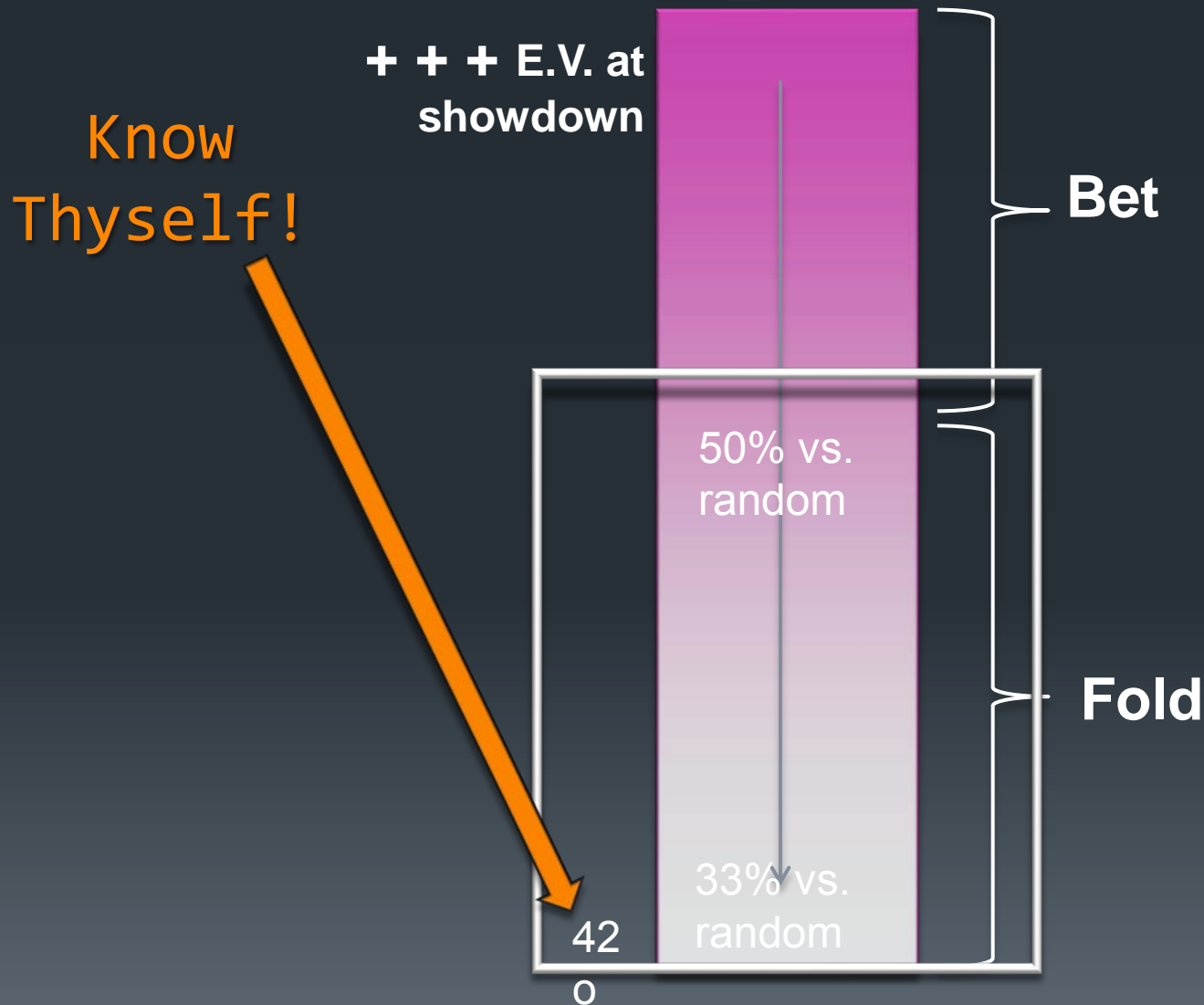
Exploitive Play: Foundations



With our Example Hand



With my Shamefully Exploitive Hand



Advanced Exploitive Play

- Don't forget about this part of the equation!

$$\text{Bluff-to-Value Ratio} = S / (1 + S)$$


Four Principles

1. Know thyself
2. Nothing in excess
3. Mischief
4. Exploit at the margins



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References

 Chen, B. & J. Ankenman. *The Mathematics of Poker*. ConJelCo, 2006.

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15.S50 Poker Theory and Analytics

January IAP 2015

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