15.997 Practice of Finance: Advanced Corporate Risk Management
Spring 2009

Problem Set #2.

Risk neutral pricing of payoffs tied to the copper price.

References

This assignment requires that you apply the principles of risk-neutral pricing to the binomial tree tree from Problem Set #1. A relevant reference is Parsons and Mello, Lecture Notes on Advanced Corporate Financial Risk Management, Chapter 9.2: Risk-Neutral Pricing.

Binomial Tree

(1) Recall the binomial model developed in homework one, with T = 2 years and N=2, i.e. using a two-step tree, one step for each year. The expected rate of appreciation in the price is 10%, the annual volatility is 28%, the risk-free rate to 5%, and the copper spot price starts at $2.65/pound. Now assume that the appropriate discount rate is 10%.

a) What are the risk-neutral probabilities? Is the risk-neutral probability of an up move higher or lower than the actual probability, and why?
b) What is the expected price of copper in two years? What is the expected price using the risk-neutral probabilities?
c) What is the value of a contract which delivers one unit of copper in two years? Do this once by discounting the future value of the copper delivery using the risk-neutral methodology – i.e., using risk neutral probabilities and the risk-neutral discount rate. Do this a second time discounting the future value of the copper delivery using a risk-adjusted discount rate – i.e., using the actual probabilities and some discount rate greater than the risk-neutral discount rate. What risk-adjusted discount rate gives you the same answer?
d) What should be the price of a forward contract delivering one unit of copper 2 years from now?
e) Now find the value of an option to buy copper two years from now at a strike price of $2.50/pound. First, use the risk-neutral approach – risk-neutral probabilities and a risk-free discount rate. Second, use the risk-adjusted discounting approach, using the risk-adjusted discount rate derived earlier. Compare the results. Third, suggest an alternative risk-adjusted rate.

(2) Change the appropriate risk-adjusted discount rate to 15%. Adjust your spreadsheet and redo all of the calculations above.

(3) Apply the 10% discount rate to the 10-step binomial tree.

a) What is the current forward price for contracts delivering copper in each of the different years, t=1,2,3,…10? Graph the term structure of the forward price.
b) What is the expected price of copper in 10 years? What is the expected price using the risk-neutral probabilities?
c) Find the value of an option to buy copper at a fixed strike price of $2.50/pound with the maturity date of t=1,2,3…10. Graph the term structure of the call prices.
d) What is the value of a contract that takes delivery of 1 unit of copper in every year, with the price collared at $3 and $7? With a collar price, the price is equal to the market price unless the market price is below $3, in which case the contract price is $3, or the market price is above $7, in which case the contract price is $7.

(4) Apply the 15% discount rate to the 10-step binomial tree.
a) What is the forward price for contracts delivering copper in each of the different years, t=1,2,3,…10? Graph the term structure of the forward price and compare it to the term structure when the discount rate is 10%.
b) What is the term structure of call prices at different maturities for an exercise price of $2.50? Graph the term structure and compare it to the term structure when the discount rate is 15%.?