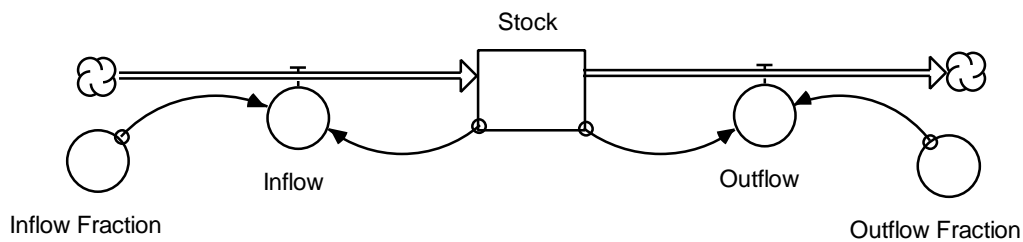# Beginner Modeling Exercises

Section 5

Mental Simulation of Combining Feedback
in First-Order Systems

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Prof. Jay W. Forrester

by
Laughton Stanley
Helen Zhu
May 29, 1996
Vensim
Examples added October 2001

Table of Contents

1. Introduction

Feedback loops are the basic structural elements of systems. Feedback in systems causes nearly all dynamic behavior. To use system dynamics successfully as a learning tool, one must understand the effects of feedback loops on dynamic systems. One way of using system dynamics to understand feedback is with simulation software on a computer. Computer simulation is a very useful tool for explaining systems; however one should also be able to use the other simulation tool of system dynamics: *mental simulation*. A strong set of mental simulation skills will enhance the ability to validate, debug, and understand dynamic systems and models.

It is assumed that the reader is familiar with the simulation of positive feedback, negative feedback, and adding constant flows.[1] To further develop these capabilities, it is necessary to study the behaviors associated with combined feedback loops in first-order systems. Here we examine four possible behaviors of this structure: equilibrium, exponential growth, asymptotic growth, and S-shaped growth. A set of exercises will reinforce understanding of these models. Solutions are included.

2. Example 1: Identical Constant Fractions

Eddie is creating a model of his nursery's tree production so that he can determine the policies that might best increase his inventory and sustain sales. Eddie looks at the behavior of the past five years and realizes that he has been using the combined feedback model shown in Figure 1. Eddie's yearly sales have been around 8% of his inventory, so his past policy was to replenish his inventory by planting the same 8% yearly.[2]

---

[1] The "Beginner Modeling Exercises, Mental Simulation" series is available in "Positive Feedback" (D-4487) by Joseph Whelan, "Negative Feedback" (D-4536) by Helen Zhu, and "Adding Constant Flows" (D-4546) by Alan Coronado. The series is part of Road Maps.

[2] All models are simplifications. The assumptions in this model about the Sales_Fraction are only an approximate method of modeling Sales. In the reality it is unlikely that Sales would remain a constant percentage of inventory, but for the purposes of this paper it is instructional to model Sales in this way.
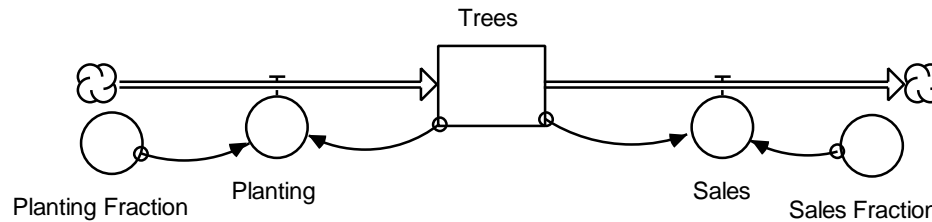
Figure 1: Eddie's Past Business Strategy

Using the mental simulation skills that he had learned from Road Maps, Eddie sets out to determine the results of his current strategy. He examines the model and recognizes the presence of both positive and negative feedback loops in the system. He ponders over how the two interact to produce the final system behavior. In attempting to figure this out, Eddie takes the following steps of mental simulation:

1. Determine the nature of loop behavior in the system.

Eddie looks at his model and realizes that as the number of Trees increases, so does the Planting rate. Over time, this generates exponential behavior due to positive feedback. The outflow loop, however, shows that as the number of Trees increases, Sales also increase, causing the Trees and Sales to drop again. Under just this second pattern of behavior, the number of Trees gradually approaches zero, its goal. This goal-oriented behavior is caused by negative feedback.

To determine the net behavior of the system, both flows must be considered. The two following equations give the values of Planting and Sales. The line below the equation gives the units for the equation.

Planting = Trees * Planting_Fraction = Trees * 0.08
      *trees/year = trees * (1/year)*
Sales = Trees * Sales_Fraction = Trees * 0.08
      *trees/year = trees * (1/year)*

In this case, the two flows are equally strong because the compounding fractions are equal. That means the inflow is always equal to the outflow; thus the net flow for the system is always zero, and the stock value is always constant.

2. *Determine the initial stock value.*

   Eddie notes that he starts out with 500 trees.

3. *Determine the final stock value.*

   What does the stock do after the initial point? The inflow at any time is the stock times the Planting_Fraction, and the outflow at any time is the stock times the Sales_Fraction. Both fractions happen to be 0.08. Eddie realizes that in this case the system is always in equilibrium because the inflow and outflow start out equal and never change. Thus the value of the stock is always 500 trees.

   Now that Eddie has a pretty good idea of what his Tree inventory looks like, he plots it in Figure 2.
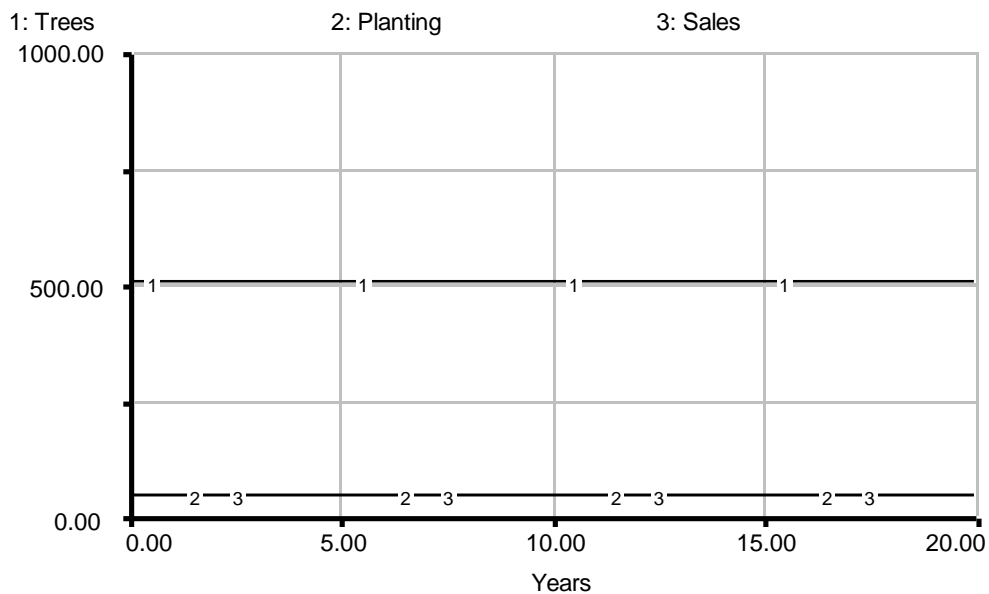
Figure 2: Eddie's Projected Inventory

3. Example 2: Different Constant Fractions

   Although business is stable, Eddie would like to design a strategy to gradually increase his inventory and expand business. He realizes that if the Planting_Fraction and the Sales_Fraction are equal, there will be no growth in his Sales. Therefore he decides to increase the Planting_Fraction in the hopes that it will give him more trees to sell. Eddie sticks with the same model structure, but changes the Planting_Fraction to 0.30 so that he

will plant 30% of his inventory each year. He then mentally simulates the system behavior using the same steps:

1. *Determine the nature of loop behavior in the system.*

    The behaviors of the individual loops are the same as in Example 1 because the fundamental relations between stocks and flows have not changed. The difference between the two models is the value of the Planting_Fraction.   Because the Planting_Fraction has changed, the relative strengths of the two loops are no longer equal.   The larger value for the Planting_Fraction makes the Planting loop considerably stronger. Because the dominant loop is a positive feedback loop, Eddie's tree inventory will increase exponentially.

2. *Determine the initial stock value.*

    Eddie's initial inventory is 500 trees.

3. *Determine the final stock value.*

    A system reaches equilibrium when the inflow and outflow are equal. Because the system produces exponential growth, it can never reach an equilibrium.   Eddie confirms this hypothesis by equating the flows:

    > Inflow = Outflow
    > Trees * Planting_Fraction = Trees * Sales_Fraction
    > *trees * (1/year) = trees * (1/year)*
    > Trees * 0.30 = Trees * 0.08
   > 0.30 = 0.08

    Can this be true? It cannot. Eddie is now certain the two flows are never equal (except for the trivial case where there are no trees). Figure 3 shows the computer-simulated system behavior over the next twenty years to illustrate the inequality of the flows. Notice that as the flows grow farther apart, the slope of the tree inventory line grows steeper.
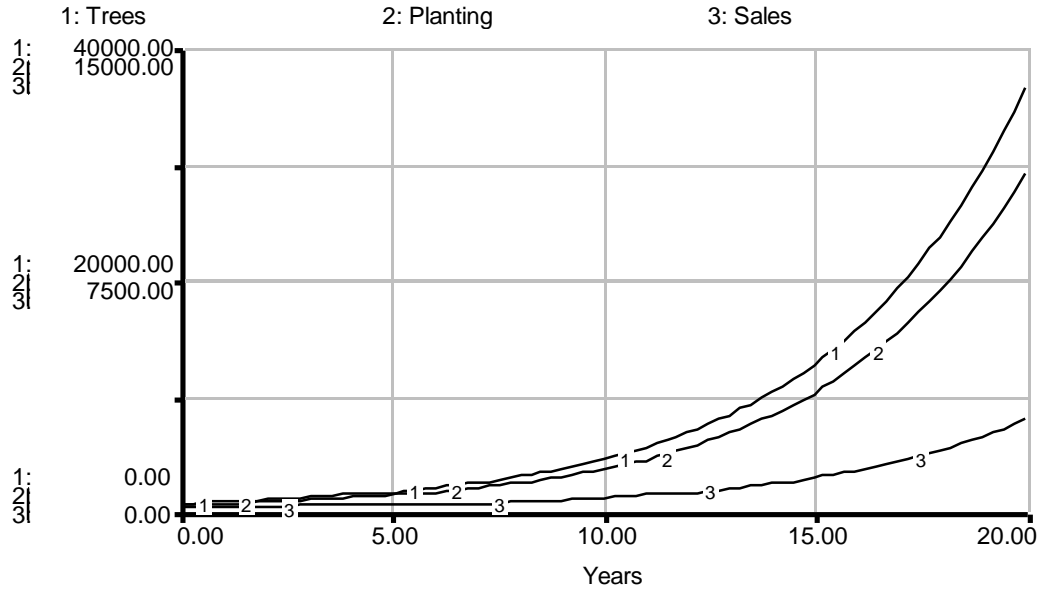
Figure 3: Model Behavior with Increased Planting_Fraction

4. Example 3: Linear Feedback

Looking over the results of his strategy, Eddie decides that it is not very practical for his nursery. He does not want his tree inventory to get out of hand because his trees could die from overcrowding and the lack of sunlight. After consulting with a botanist, Eddie decides that to make the most use out of his 6000 square yards of land, he should grow approximately 5800 trees. He proposes a hypothetical strategy designed to reach this goal over the next few years. Each year he will plant a number of trees equal to 30% of the difference between his goal of 5800 trees and his current inventory. He describes his strategy with the following model:
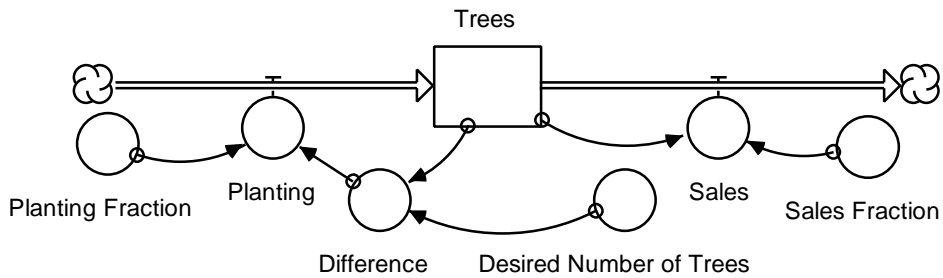
Figure 4: Eddie's Planting Regulation Strategy

Eddie uses his mental simulation steps as before to find out the results of his new strategy. He assumes that his inventory will not surpass his goal.

1. *Determine the nature of loop behavior in the system.*

   The structure of the Planting loop has changed. Now an increase in the number of trees closes the difference between the current inventory of Trees and Eddie's Desired_Number_of_Trees (5800 trees). Tree planting will slow as the inventory approaches the goal. This kind of goal-oriented behavior exemplifies a negative feedback loop. The Sales loop is also a negative feedback loop for the same reasons as before because the loop and its values have not changed.

   The general behavior of the system can be determined by examining the relative strengths of the feedback loops. Initially the Planting loop is stronger than the Sales loop. For instance, in the first year of his program Eddie will plant 1590 trees (Planting = 5300 * 0.3 = 1590) but he will sell only 40 trees (Sales = 500 * 0.08 = 40). This will not always be the case. As Eddie's tree inventory increases, the Difference between the inventory and his goal will decrease. Eventually he will be planting as many trees as he sells. At this point when inflow equals outflow, his stock will be at a stable equilibrium.

   Because the inflow loop starts off increasing the stock strongly and then gradually decreases until it reaches the strength of the negative loop, the system will show asymptotic behavior with a large initial slope that approaches zero as it goes toward the asymptote.

2. *Determine the initial stock value.*

   The initial value is still 500 trees.

3. *Determine the final stock value.*

   The final value occurs when the system is in equilibrium because the inflow and outflow are equal. Eddie quickly goes through the algebra:

   Planting = Sales
   trees/year = trees/year
   Planting_Fraction * Difference = Trees * Sales_Fraction

*(1/year) * trees = trees * (1/year)*

Substituting in numbers, he gets:

    0.30 * (5800 - Trees) = 0.08 * Trees
    5800 - Trees = 0.08 * Trees / 0.30
    5800 = Trees + Trees * (0.08 / 0.30)
    5800 = Trees * (1 + 0.08 / 0.30)
    5800 ˜ Trees * 1.267
    Trees ˜ 4578 at equilibrium

Although he set a goal of 5800 trees in his model, Eddie's model has an equilibrium that is much lower. The reason for this lies in the structure of the model. If there was no outflow at all, the inflow would eventually reach an equilibrium value very close to 5800. Because there is an 8% outflow, there must be an equivalent inflow to create equilibrium. This can only be true when the stock value is some distance, in this case about 1222[3] trees, away from the goal. The nature of this behavior can be seen graphically in Figure 5. Notice that the system reaches equilibrium as the two flows approach the same value.
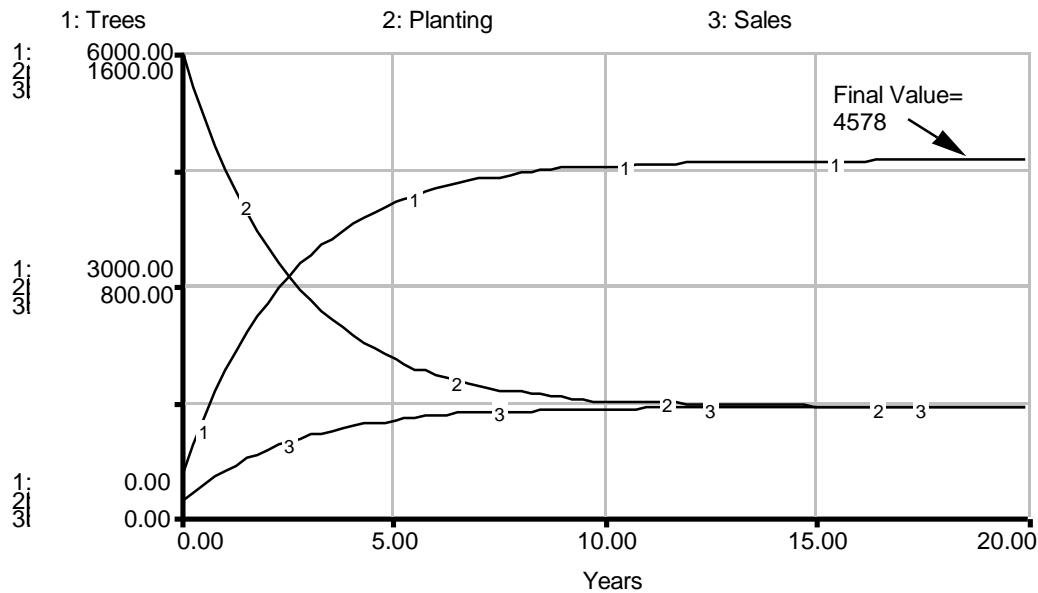

Figure 5: Stock and Flow Behavior of Eddie's Planting Regulation Strategy

---

[3] The goal is 5800, and equilibrium is reached at 4578 trees. Therefore, the stock is about 5800 - 4578 ? 1222 trees away from the goal.

5. Example 4: Nonlinear Feedback with Graph Function

Even though his previous model produces the goal-seeking behavior that he needs, Eddie is surprised to find the rate at which he must begin planting. Because he cannot afford to plant at a rate of 1600 trees per year without first increasing his sales, he sets out to find a different strategy. Eddie becomes curious as to what would happen if he used a nonlinear relationship to control his tree Planting. He hypothesizes that if he lowers his planting as the tree density increases, he will be able to successfully regulate the tree density. He constructs the following model:
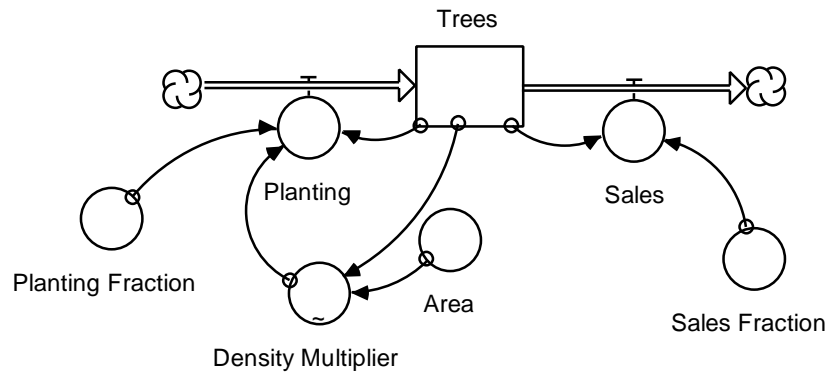
Figure 6: Model Regulating Tree Planting Rate

Eddie leaves his Sales loop the same as in Example 3, but he changes his Planting loop. Instead of having his planting be a fraction of the total number of trees, he scales this value by a multiplier that adjusts for the tree density. After studying the effects of overcrowding on tree growth, he designs the graph function in Figure 7 as his Density_Multiplier. The Density_Multiplier is designed to adjust Eddie's Planting based on the density of his trees so that he will plant fewer and fewer trees as he nears the carrying capacity of his property. The Area of his property is 6000 square yards.
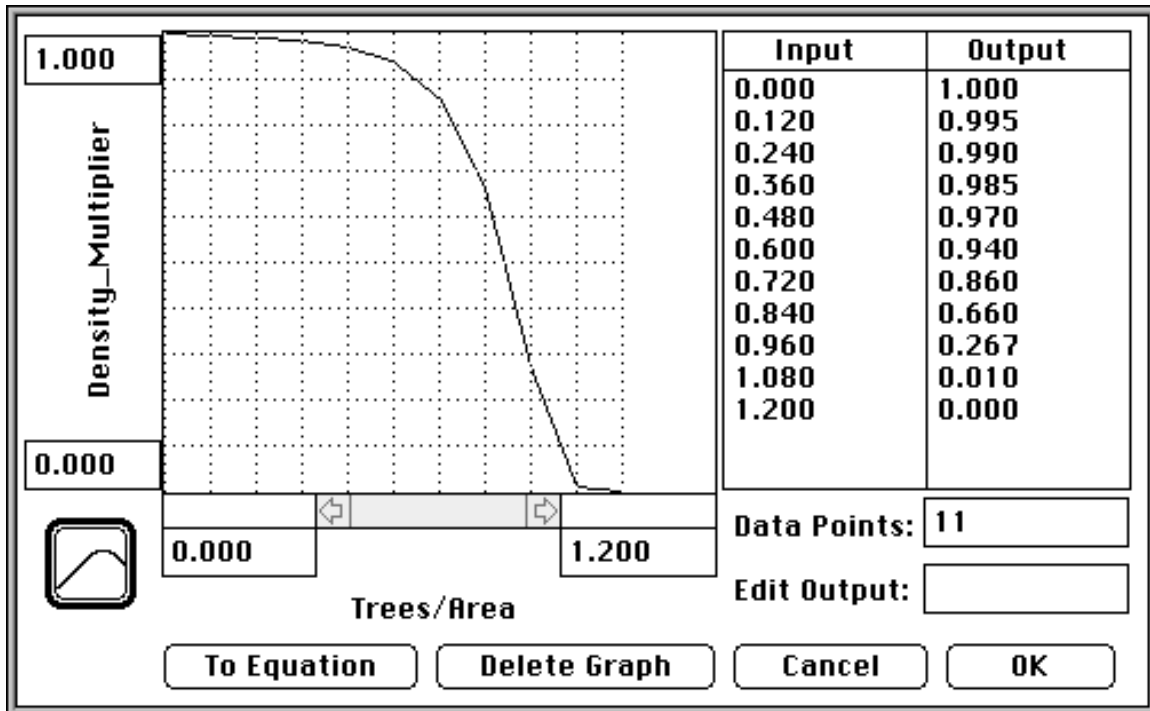
Figure 7: Graph of Eddie's Density_Multiplier

Using his mental simulation skills, Eddie looks at the model and begins to work out its behavior:

1. *Determine the nature of loop behavior in the system.*

   Eddie quickly recognizes that his Sales loop is unchanged and remains a negative feedback loop. After looking at the Planting loop, however, he is somewhat puzzled by the presence of both a Density_Multiplier and a Planting_Fraction in the loop. Looking back to his system dynamics text, he realizes that the extra elements in this loop can be seen as abstractions for a single element. In other words the Density_Multiplier and the Planting_Fraction may be lumped together into a single equation for the purposes of his calculations.

   Eddie sees that this loop has an unusual behavior because of his graph function. The 0.30 planting fraction has the tendency to increase exponentially the number of Trees planted. This is positive feedback. The Density_Multiplier, however, decreases as the number of trees increases. This is negative feedback. Being uncertain of which route the whole system will follow, Eddie writes out the possible behaviors.

Eddie assumes that at any given time, the planting loop will exhibit either positive or negative feedback behavior. Given the characteristics of the loop, Eddie envisions the following three possibilities:

?   The positive feedback will dominate the loop and characterize its behavior at all times.

When considering possibilities, it is often useful to consider extreme cases. Eddie sees that if his tree density grows to a value of 1.2 trees / square yard, the planting rate will become zero. This means that Eddie can discount the possibility of simple positive feedback generating exponential growth. There can be no growth at all with a planting rate of zero.

?   The negative feedback will dominate the loop and characterize its behavior at all times.

This possibility sounds plausible to Eddie. This could cause asymptotic behavior and bring his tree inventory to the desired level. Unfortunately this is very similar to example 3, which had the very large initial planting rate, and Eddie would like to find a more ideal strategy. The fact that the Density_Multiplier changes very little for densities up to .6 (trees/square yard) helps Eddie to decide that initially there will be room for exponential growth. If the Density_Multiplier is not changing up to .6 (trees/square yard) then it cannot be providing negative feedback at that point. In other words, the positive feedback portion of the loop will be stronger initially until the value of the Density_Multiplier makes a significant change.

?   The loop will initially have the characteristics of positive feedback, but the dominance will shift and the loop will show negative feedback characteristics.

Eddie likes this possibility almost immediately. This loop characteristic would allow for a low initial planting rate and an asymptotic final behavior.

After considering his choices, Eddie realizes that the switch from positive to negative feedback makes the most sense. The graph of the Density_Multiplier changes very little initially, so it is reasonable to expect the positive feedback to be dominant at first. As the density increases, the Density_Multiplier dives sharply. Here it is reasonable to assume that the negative feedback will dominate the loop because of the extremely

quick change in the value of the Density_Multiplier. Eddie concludes that the result
will be S-shaped growth as shown in Figure 8.

2. *Determine the initial stock value.*

This is still 500 trees.

3. *Determine the final stock value.*

The final value occurs when the system is in equilibrium because the inflow and
outflow are equal. As before, Eddie writes down the algebraic equations involved:

> Planting = Sales
> Planting_Fraction * Trees * Density_Multiplier = Trees * Sales_Fraction
>   *(1/year) * trees * (trees/trees) = trees * (1/year)*
> 0.30 * Trees * Density_Multiplier = Trees * 0.08
> Density_Multiplier = (Trees * 0.08) / (Trees * 0.30)
> Density_Multiplier = 0.08/0.30
> Density_Multiplier ˜ 0.267[4]

By looking back to his graph, Eddie is able to see that a Density_Multiplier value
of 0.267 corresponds to a density of 0.96 trees/square yard. At this density, his nursery,
with a 6000 square yard area, will hold about 5800 trees. Given the initial and final values
as well as a good idea of the graph shape, Eddie is able to sketch a model of the system
similar to that in Figure 8 which illustrates the behavior of the stock and flows over a span
of twenty years. Notice that the system reaches equilibrium as the flows reach identical
values.

---

[4] To solve this equation, Eddie has to solve for the value of the graph output and then to determine the
value of the graph's input based on its output value. This method is necessary when working with a graph
function that is not defined mathematically.
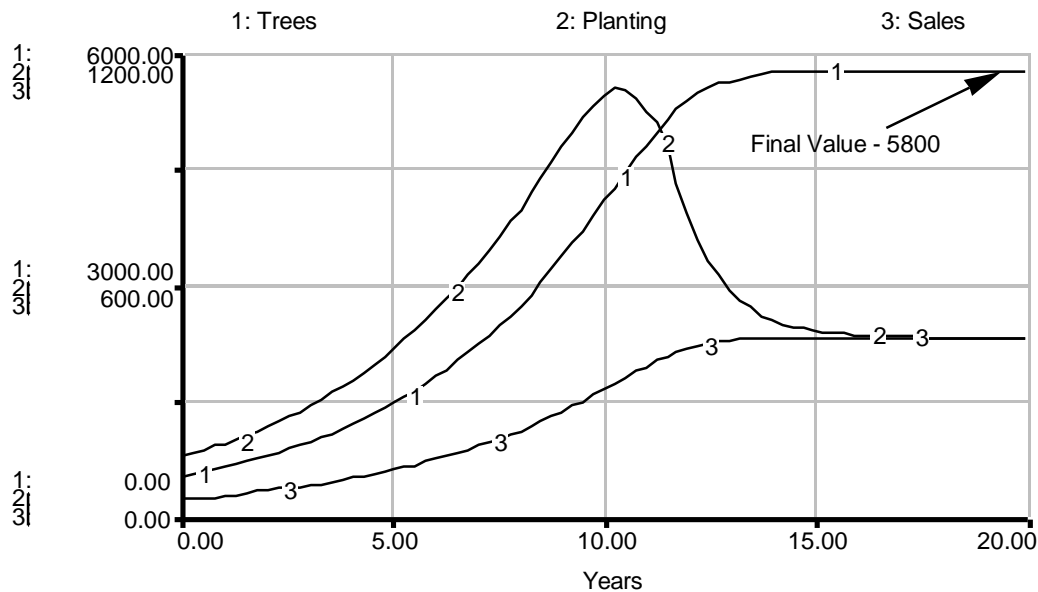
Figure 8: Behavior of Non-Linear Regulation of Planting

Completely satisfied with the simulation results tested using system dynamics, Eddie proceeds to follow his newest strategy. Sure enough, it turns out to be a success!

6. Review

Before moving on to the exercises, here is a summary of the steps of mental simulation for combined feedback in first order systems:

1. *Determine the nature of loop behavior in the system.*

   Recognize the positive and negative feedback loops and how they affect the stock behavior over time.

2. *Determine the initial stock value.*

   This is usually given.

3. *Determine the final stock value.*

   This can be done by equating the rates and solving the resulting equation for the stock value. The stock will have a non-zero solution only when there is an equilibrium.

7. Exercises

## 7.1 Exercise 1: Running a Marathon

Henry is preparing to run the Boston Marathon. In preparation, he has devised a model for the water level in his body during the race. Henry assumes that he will sweat 3% of his available water every hour. To make up for this loss, he plans to consume water at the rate of 3% of his total water content per hour.
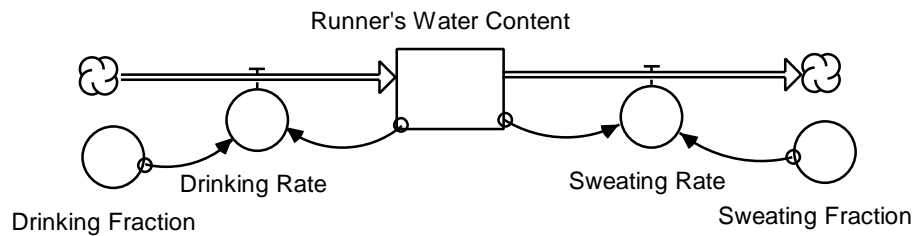
A. What are the feedback loops in this system?

B. What is the qualitative behavior of the stock over time?

C. Is the qualitative stock behavior dependent on the initial stock value?

7.2 Exercise 2: Unread Books

Francisco is a philosophy major who loves to read. His affluent Uncle Midas tries to give him enough books to keep him busy every month. Uncle Midas would like Francisco to have 15 unread books on hand at all times. When buying books, Uncle Midas makes a list of new books for Francisco and buys the books from his list that are being sold at the local bookstore. Some of the books that Uncle Midas selects are rare, however, and he often is able to find only 75% of them. Even so he manages to keep up a good flow of books for his nephew. Francisco, determined to make a dent in the shelf full of his uncle's contributions, has decided that he will read half of the books each month, no matter how many that may be. Here is a model of the number of Francisco's unread books.

Francisco's Unread Books
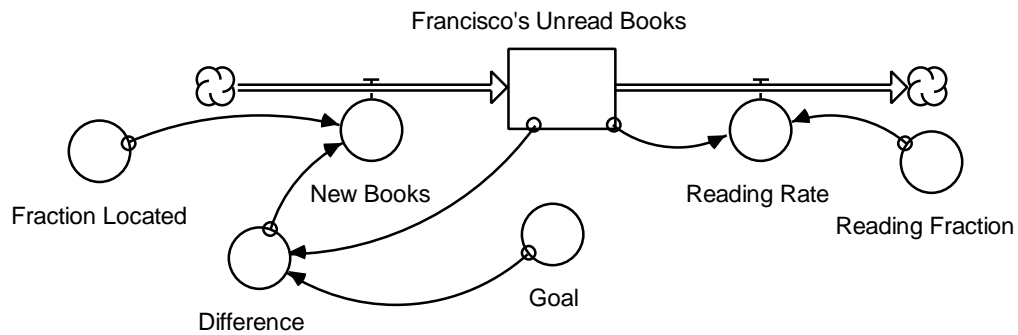
The Fraction_Located represents the average percentage of the books on the list that Uncle Midas can find in the store each month.

New_Books = Difference * Fraction_Located
*books/month = books * (1/month)*
Difference = Goal - Francisco's_Unread_Books
books = books - books
Goal = 15 *(books)*
Fraction_Located = 0.75 *(1/month)*
Reading_Rate = Francisco's_Unread_Books * Reading_Fraction
*books/month = books * (1/month)*
Reading_Fraction = 0.50 *(1/month)*
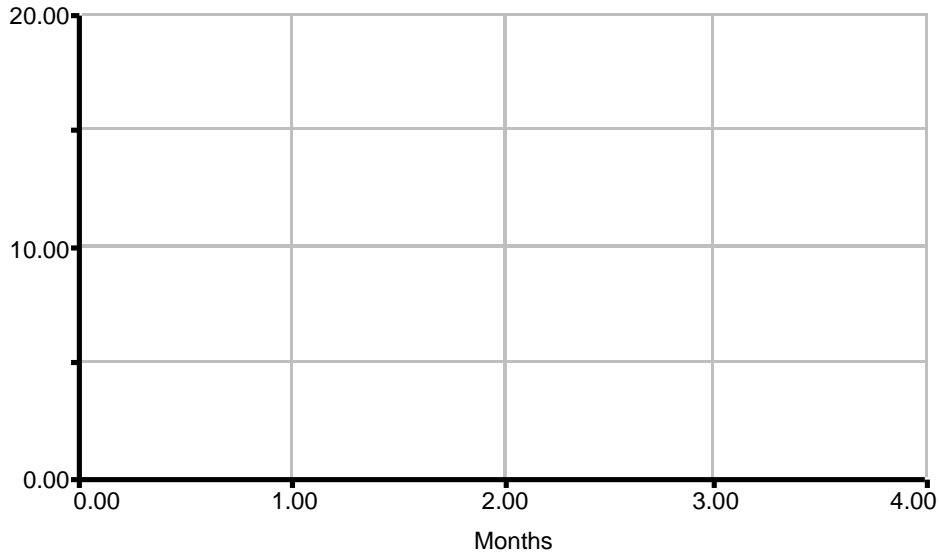Initial number of Francisco's_Unread_Books = 20 *(books)*

A. Identify the positive and negative feedback loops in this system.

B. What is the final value of the stock?

C. Plot the general behavior of the stock on the following graph:

1: Francisco's Unread Books

7.3 Exercise 3: Galt Development Company

Milton Head Island was just purchased by the Galt Development Company, which hopes to turn it into a major golf resort. The island has a total area of 1000 acres that the company would like to develop into golf courses. The company first develops two test holes on 10 acres of land to insure terrain quality. At this point, Galt Development conducts extensive market research into the growth of successful golf developments. Their statistician, fresh out of MIT and raving with the latest statistical modeling strategies, comes to the conclusion that development should be directly proportional to the product of the developed and undeveloped areas. The strategy is modeled below:
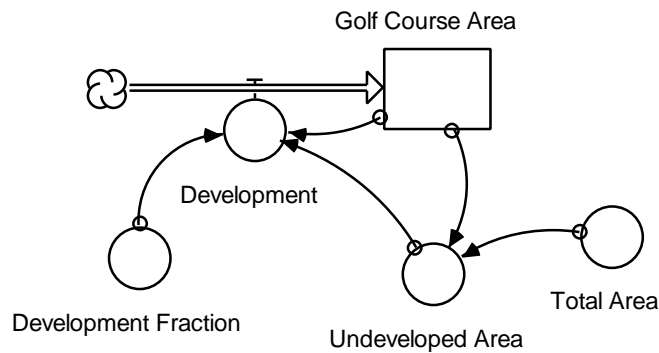
Development = Development Fraction * Undeveloped Area *
Golf Course Area

*acres/month = (1/(month * acre))* acres * acres*

Development Fraction = 0.0004 *1/(month * acre)*

Undeveloped Area = Total Area - Golf Course Area

acres   = acres ó acres

Total Area = 1000 *acres*
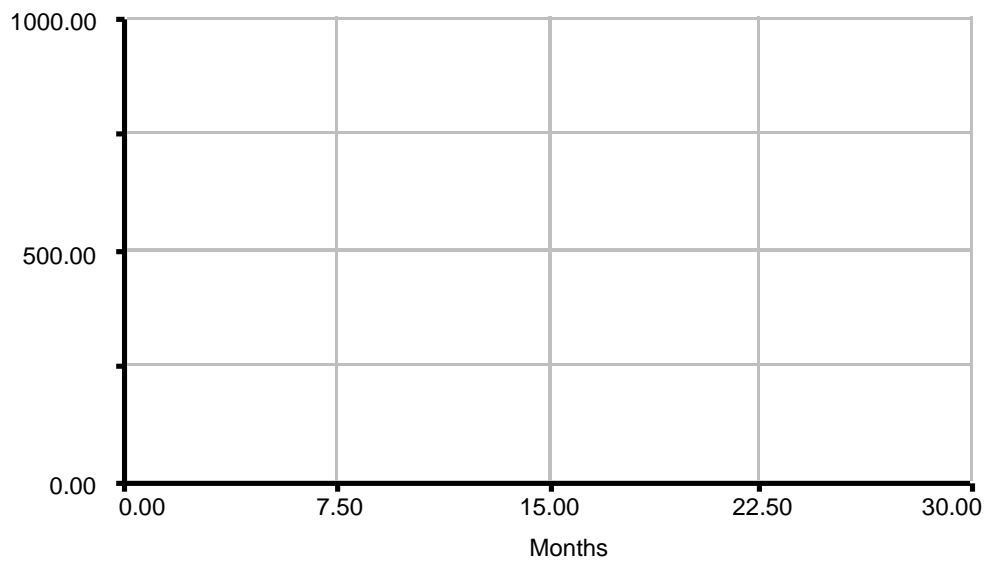
initial Golf Course Area = 10 *acres*

A. What kind of feedback behavior is contained in this system?

B. What is the final stock value?

C. What kind of behavior does the stock exhibit?

D. Plot the stock behavior on the following graph pad:

1: Golf Course Area

8. Solutions to Exercises

8.1 Solution to Exercise 1

A.   Recognize the presence of a positive and negative feedback loop in this system. The Drinking_Rate is a positive feedback loop because the inflow of water into the runner's system increases the water in his system and thus increases the rate at which he drinks water. The Sweating_Rate is a negative feedback loop because sweating takes away water from the runner and thereby decreases his Sweating_Rate. Although these assumptions may sound unreasonable in the real world, it is the model that we are interested in understanding.

B.   The important fact to notice here is that the Drinking_Fraction and the Sweating_Fraction are the same. Thus the system will start at equilibrium and never divert.

C.   No. The qualitative behavior of the system (equilibrium throughout) is not dependent upon the initial value of the stock. It is caused merely by the equivalence of the inflow and the outflow.

8.2 Solution to Exercise 2

A.   There are two feedback loops in this model. The feedback system of Uncle Midas's giving exhibits negative feedback because an increase in the number of Francisco's_Unread_Books will cause a decrease in Uncle Midas's New_Books rate. The Reading_Rate loop is also a negative feedback system. An increase in the Reading_Rate will decrease the number of Francisco's_Unread_Books and decrease the Reading_Rate.

B & C. The equations given show that:

New_Books = Difference * Fraction_Located
*books/month = books *(1/month)*

Difference = Goal - Francisco's_Unread_Books
  books = books - books

Goal = 15 *books*

Fraction_Located = 0.75 *(1/month)*

Reading_Rate = Francisco's_Unread_Books * Reading_Fraction
  *books/month = books * (1/month)*

Reading_Fraction = 0.50 *(1/month)*

Initial number of Francisco's_Unread_Books = 20 *(books)*

In order to figure out the final value of the stock, we equate the system's flows:

Inflow = Outflow
  books/month = books/month

New_Books = Reading_Rate
  books/month = books/month

Difference * Fraction_Located = Reading Fraction *
      Francisco's_Unread_Books
  *books * (1/month) = (1/month) * books*

(Goal - Francisco's_Unread_Books) * Fraction_Located =
                    Reading Fraction * Francisco's_Unread_Books
  *(books - books) * (1/month) = (1/month) * books*

(15 - Francisco's_Unread_Books) * 0.75 = 0.50 *
                                        Francisco's_Unread_Books
  *(books - books) * (1/month) = (1/month) * books*

(15 - Francisco's_Unread_Books) = [0.50 *
                                Francisco's_Unread_Books] / 0.75
  *(books - books) = [(1/month) * (books)] / (1/month)*

Francisco's_Unread_Books = 15 **-**
                    [0.50 * Francisco's_Unread_Books] / 0.75
  *books = books — [(1/month) * (books)] / (1/month)*

Francisco's_Unread_Books = **-**Francisco's_Unread_Books * (2/3) + 15
  books = books + books

Francisco's_Unread_Books + (2/3) * Francisco's_Unread_Books = 15
  books + books = books

(5/3) * Francisco's_Unread_Books = 15
  books = books

Francisco's_Unread_Books = 15 * (3/5)
  books = books

Francisco's_Unread_Books = 9
  books = books

1: Francisco's Unread Books   2: New Books   3: Reading Rate

Months
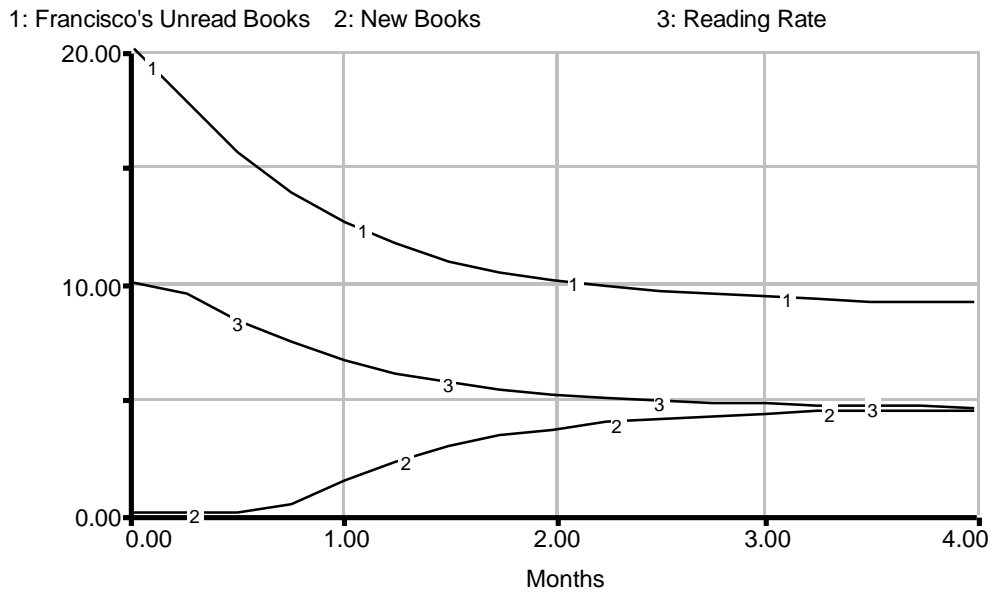
8.3 Solution to Exercise 3

A.   By inspection, there are two feedback loops acting upon the system, one positive and one negative. The greater golf course area, the greater the rate of development, in turn further increasing golf course area. At the same time, stock is inversely related to the undeveloped area, which hinders the development rate.

B.   When the system comes to equilibrium, undeveloped area should be 0, so all of the area should be developed. The final stock value is, therefore, 1000 acres.

C.   In order to figure out what kind of behavior the stock exhibits, we need to figure out which feedback loop dominates first. In this case, at time = 0, the positive feedback loop dominates. Goal oriented behavior does not dominate until developed area becomes larger than the undeveloped area. Since the system is initially dominated by positive feedback but the dominance shifts to negative feedback, the system exhibits S-shaped behavior.

D.
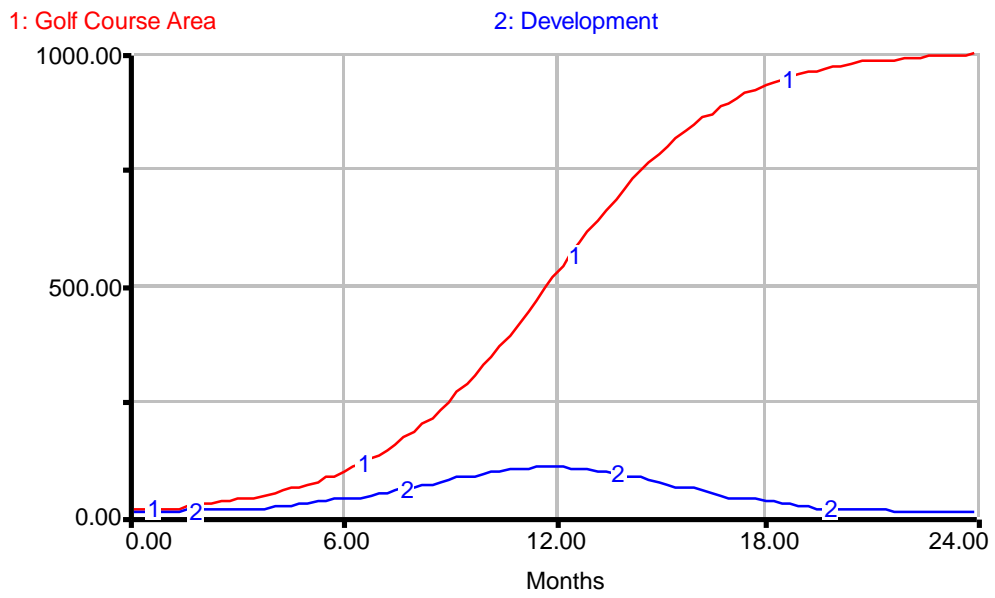
9. Appendix: Exponentiation

In Exercise 3, it is necessary to perform operations on the exponents so that Boat Speed may be solved for explicitly. The step is shown below:

$$(\text{Boat Speed} / 20)^{1.8} = \text{Force} / 500$$
$$\text{Boat Speed} / 20 = (\text{Force} / 500)^{5/9}$$

The change of exponents is an operation which may be performed on both sides of an equation just as one would do with addition or multiplication. In this case, the left side of the equation is raised to the 1.8 power. Since 1.8 is the equal to 9/5, it may also be said that the left side is raised to the 9/5 power. In order to solve for a variable, such as Boat Speed, explicitly, it is necessary to have the variable raised only to the first power. (Any variable raised to the first power is the variable itself.) To change $(\text{Boat Speed} / 20)^{9/5}$, one may raise that quantity to another power. In this case raising it to the 5/9 power is needed. Since raising one power to another power is equivalent to multiplying the two powers, and (9/5)*(5/9) is 1, the result is $(\text{Boat Speed} / 20)^1$ or just Boat Speed / 20. Since everything done to one side of the equation must also be done to the other side, the right hand side is raised to the 1*5/9 or 5/9 power.

Vensim Examples:
Beginner Modeling Exercises Section 5: Mental Simulation of
Combining Feedback in First-Order Systems
By Lei Lei & Nathaniel Choge
October 2001

Example 1: Identical Constant Fractions
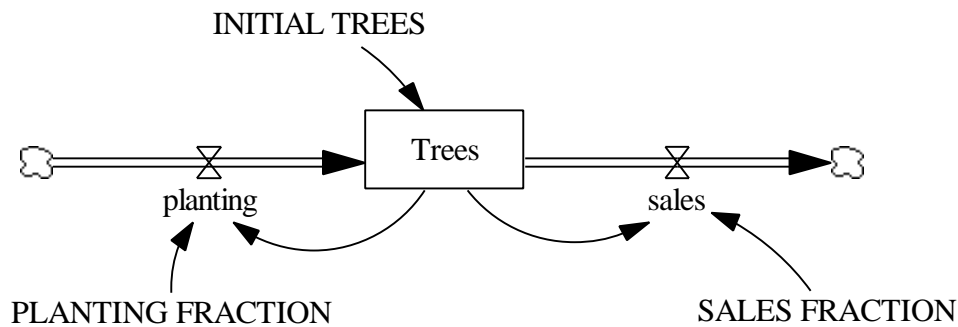
Figure 9: Vensim Equivalent of Figure 1: Eddie's Past Business Strategy

Documentation for Identical Constant Fractions Model

(01)   FINAL TIME = 20
          Units: year
          The final time for the simulation.

(02)   INITIAL TIME = 0
          Units: year
          The initial time for the simulation.

(03)   INITIAL TREES = 500
          Units: trees

(04)   planting = Trees* PLANTING FRACTION
          Units: trees/year

(05)   PLANTING FRACTION = 0.08
          Units: 1/year

(06)   sales = Trees* SALES FRACTION
          Units: trees/year
(07)   SALES FRACTION = 0.08
          Units: 1/year

(08)   SAVEPER = TIME STEP
         Units: year
         The frequency with which output is stored.

(09)   TIME STEP = 0.125
         Units: year
         The time step for the simulation.

(10)   Trees = INTEG (planting-sales, INITIAL TREES)
         Units: trees

Graph of Trees, Planting, and Sales

Figure 10: Vensim Equivalent of Figure 2: Eddie's Projected Inventory

Graph of Trees, Planting and Sales with Different Constants

40,000	trees
15,000	trees/year
15,000	trees/year

0	trees
0	trees/year
0	trees/year

Time (year)

Trees : Current ————1————1————1————1————1————1————1————1————1————1————+ trees
planting : Current ————2————2————2————2————2————2————2————2————2————2 trees/year
sales : Current ——3————3————3————3————3————3————3————3————3————3 trees/year

Figure 12: Vensim Equivalent of Figure 3: Model Behavior with increased PLANTING FRACTION

Example 3: Linear Feedback

INITIAL TREES

Trees

planting

sales

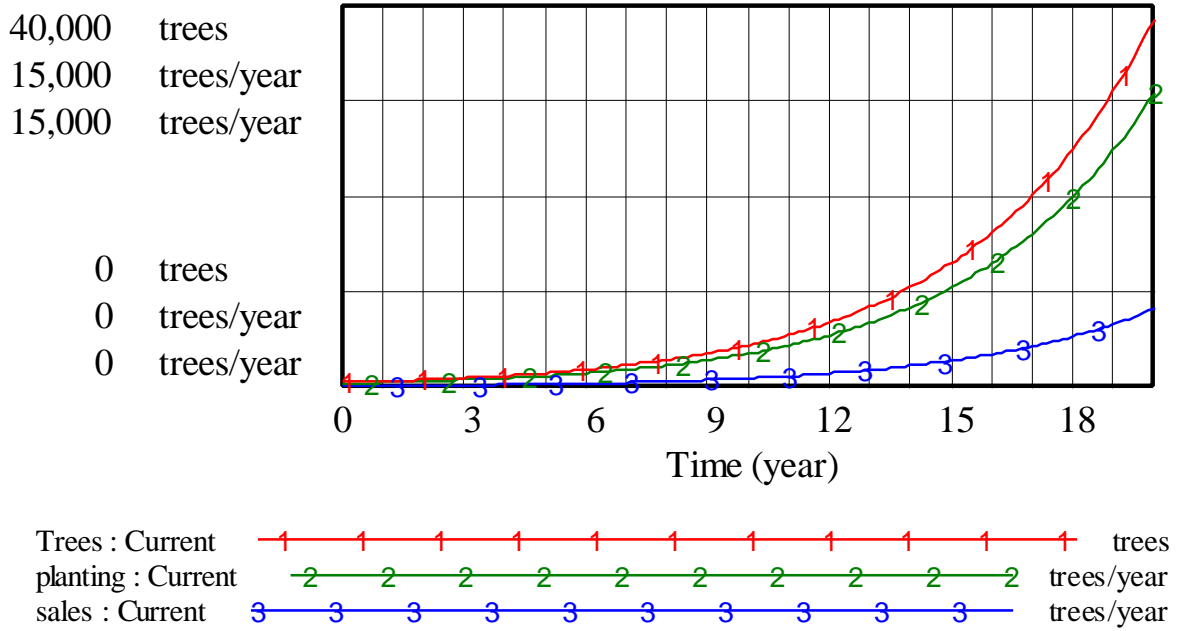difference

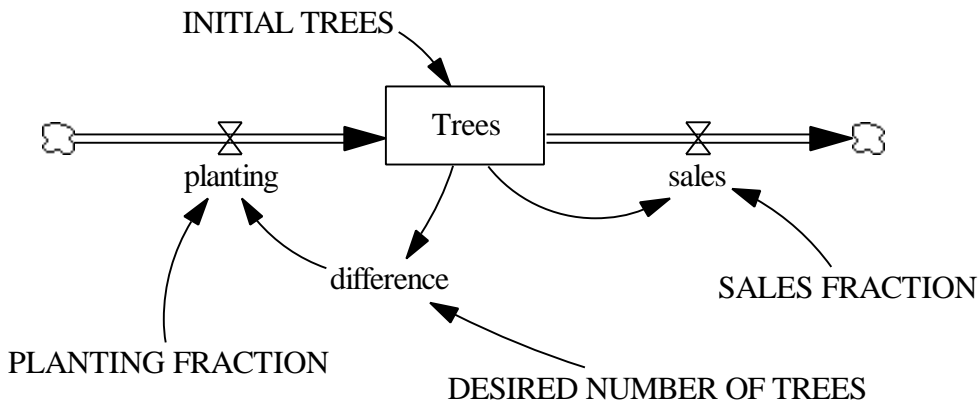PLANTING FRACTION

SALES FRACTION

DESIRED NUMBER OF TREES

Figure 13: Vensim Equivalent of Figure 4: Eddie's Planting Regulation Strategy

Documentation of Nonlinear Feedback Model

(01)   DESIRED NUMBER OF TREES = 5800
         Units: trees

(02)   difference = DESIRED NUMBER OF TREES-Trees
         Units: trees

(03)   FINAL TIME = 20
         Units: year
         The final time for the simulation.

(04)   INITIAL TIME = 0
         Units: year
         The initial time for the simulation.

(05)   INITIAL TREES = 0
         Units: trees

(06)   planting = difference* PLANTING FRACTION
         Units: trees/year

(07)   PLANTING FRACTION = 0.3
         Units: 1/year

(08)   sales = Trees* SALES FRACTION
         Units: trees/year

(09)   SALES FRACTION = 0.08
         Units: 1/year

(10)   SAVEPER = TIME STEP
         Units: year
         The frequency with which output is stored.

(11)   TIME STEP = 0.125
         Units: year
         The time step for the simulation.

(12)   Trees = INTEG (planting-sales, INITIAL TREES)
         Units: trees

Graph of Trees, Planting, and Sales for Linear Feedback

6,000	trees
1,600	trees/year
1,600	trees/year
0	trees
0	trees/year
0	trees/year

Time (year)

Trees : Current ——1——1——1——1——1——1——1——1——1——1—— trees
planting : Current ——2——2——2——2——2——2——2——2——2—— trees/year
sales : Current ——3——3——3——3——3——3——3——3——3—— trees/year
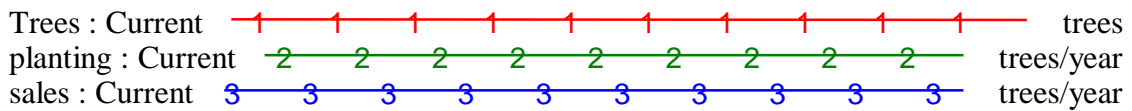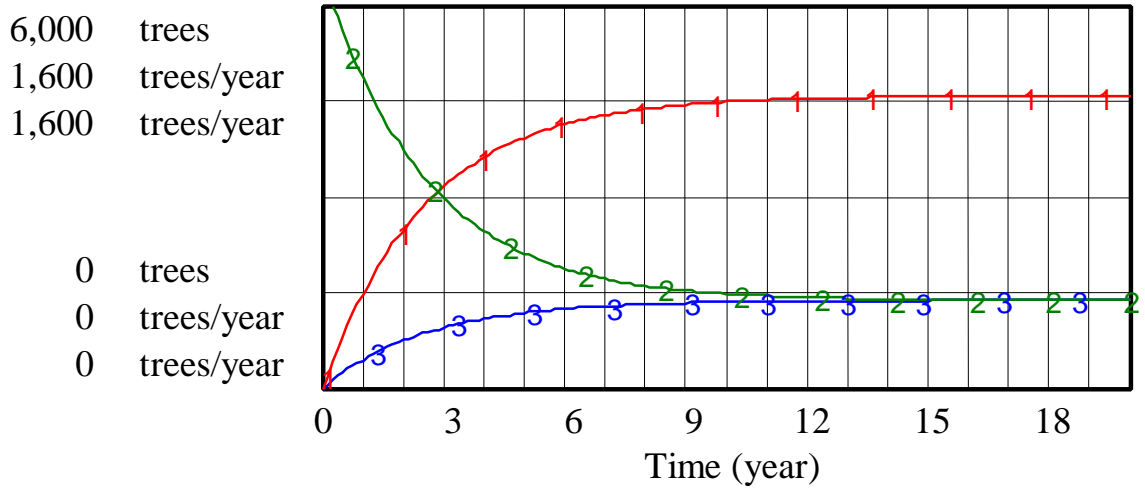
Figure 14: Vensim Equivalent of Figure 5: Stock and Flow Behavior of Eddie's Planting
          Regulation Strategy
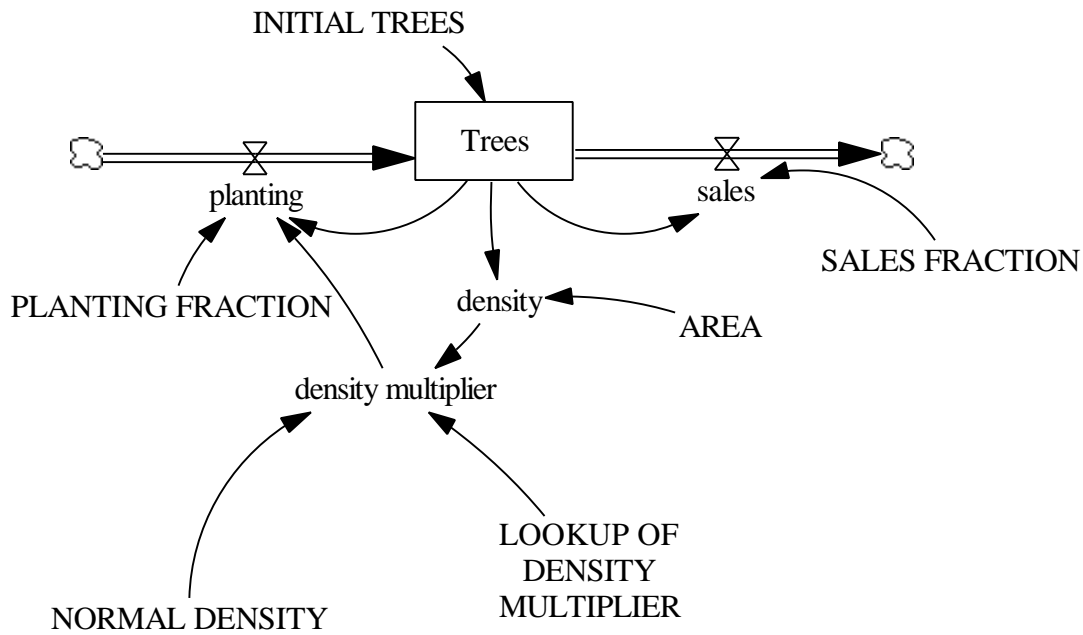
Example 4: Nonlinear Feedback with Graph Function

Figure 15: Vensim Equivalent of Figure 6: Model Regulating Tree Planting Rate

Documentation for Nonlinear Feedback with Graph Function Model

(01)   AREA = 6000
        Units: yard* yard

(02)   density = Trees/AREA
        Units: trees/(yard*yard)

(03)   density multiplier = LOOKUP OF DENSITY MULTIPLIER(density/NORMAL
        DENSITY)
        Units: dmnl
        This is the equation for the table function.

(04)   FINAL TIME = 20
        Units: year
        The final time for the simulation.

(05)   INITIAL TIME = 0
        Units: year
        The initial time for the simulation.

(06)   INITIAL TREES = 500
        Units: trees

(07)   LOOKUP OF DENSITY MULTIPLIER
        [(0,0)-(20,1)],(0,1),(1,1),(1.44,0.995),(2.88,0.985),(4.32,0.99),
        (5.76,0.97),(7.2,0.94),(8.64,0.86),(10.08,0.66),(11.52,0.267),
        (12.97,0.01),(14.41,0)
        Units: dmnl

(08)   NORMAL DENSITY = 0.0833
        Units: trees/(yard* yard)
        This is the initial, or normal density of trees. It is taken by dividing the intial
        number of Trees, 500 trees, by the AREA, 6000 square yards.

(09)   planting = Trees* density multiplier* PLANTING FRACTION
        Units: trees/year

(10)   PLANTING FRACTION = 0.3
        Units: 1/year

 (11)   sales = Trees* SALES FRACTION
        Units: trees/year

(12)   SALES FRACTION = 0.08
        Units: 1/year

(13)   SAVEPER = TIME STEP
          Units: year
          The frequency with which output is stored.

(14)   TIME STEP = 0.125
          Units: year
          The time step for the simulation.

(15)   Trees = INTEG (planting-sales, INITIAL TREES)
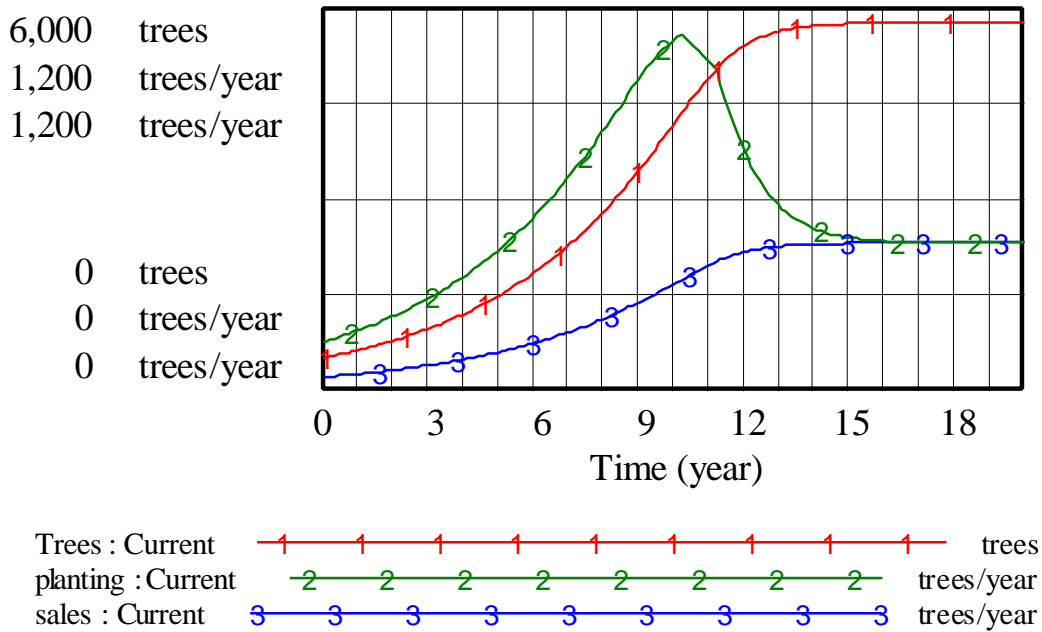   Units: trees

Figure 16: Vensim Equivalent of Figure 8: Behavior of Non-Linear Regulation of
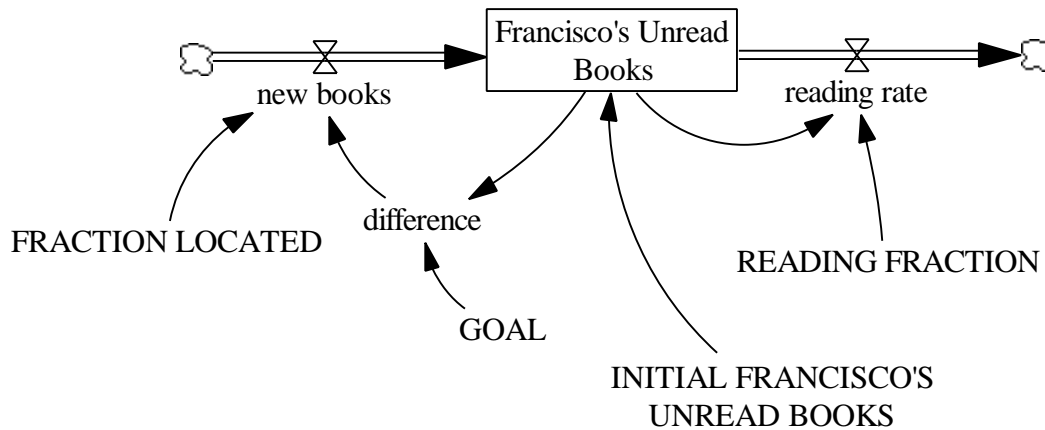   Planting

Exercise 2: Unread Books

Figure 17: Vensim Equivalent of Diagram of Francisco's Unread Books

Documentation for Unread Books Model

(01)   difference = GOAL-Francisco's Unread Books
        Units: books

(02)   FINAL TIME = 4
        Units: Month
        The final time for the simulation.

(03)   FRACTION LOCATED = 0.75
        Units: 1/Month

(04)   Francisco's Unread Books = INTEG (new books-reading rate,
        INITIAL FRANCISCO'S UNREAD BOOKS)
        Units: books

(05)   GOAL = 15
        Units: books

(06)   INITIAL FRANCISCO'S UNREAD BOOKS = 20
        Units: books

(07)   INITIAL TIME = 0
        Units: Month
        The initial time for the simulation.

(08)   new books = difference* FRACTION LOCATED

Units: books/Month

(09)   READING FRACTION = 0.5
        Units: 1/Month

(10)   reading rate = Francisco's Unread Books* READING FRACTION
        Units: books/Month

(11)   SAVEPER = TIME STEP
        Units: Month
        The frequency with which output is stored.

(12)   TIME STEP = 0.125
        Units: Month
        The time step for the simulation.

Graph of Unread Books, New Books and Reading Rate

Figure 18: Vensim Equivalent of Simulation of Book Model

Exercise 3: Galt Development Company
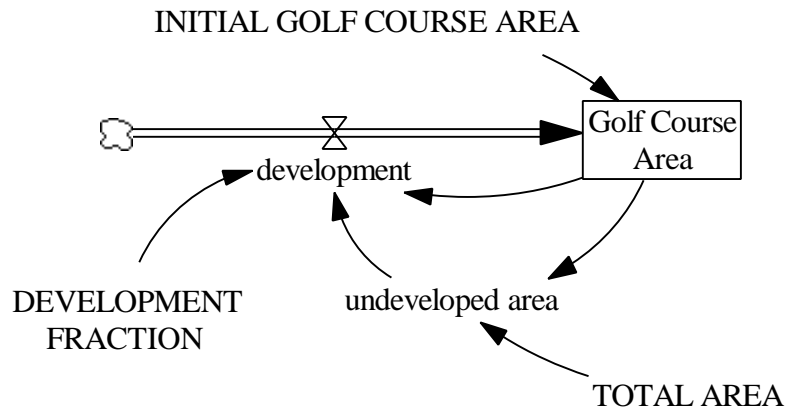

Figure 19: Vensim Equivalent of Golf Course Development Model

Documentation for Galt Development Company Model

(01)   development = DEVELOPMENT FRACTION* undeveloped area*
        Golf Course Area
        Units: acres/Month

(02)   DEVELOPMENT FRACTION = 0.0004
        Units: 1/(Month* acres)

(03)   FINAL TIME = 24
        Units: Month
        The final time for the simulation.

(04)   Golf Course Area = INTEG (development, INITIAL GOLF COURSE AREA)
        Units: acres

(05)   INITIAL GOLF COURSE AREA = 10
        Units: acres

(06)   INITIAL TIME = 0
        Units: Month
        The initial time for the simulation.

(07)   SAVEPER = TIME STEP
        Units: Month
        The frequency with which output is stored.

(08)   TIME STEP = 0.0625

Units: Month
The time step for the simulation.

(09)   TOTAL AREA = 1000
         Units: acres

(10)   undeveloped area = TOTAL AREA-Golf Course Area
         Units: acres

Graph of Golf Course Area and Development

Figure 20: Vensim Equivalent of Simulation of Golf Course Area and development