

Answers to Exercises for Chapter 17 INTRODUCTION TO DELAYS

from

INTRODUCTION TO COMPUTER SIMULATION

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Models translated to STELLA II 3.0 and Answers provided

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Vensim Examples added October 2001

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INTRODUCTION TO COMPUTER SIMULATION can be ordered from
Productivity Press, Inc., Dept. 315
P.O. Box 13390, Portland, OR 97213

The Chapter has also been provided under Road Maps Five in pdf format.

Please read Chapter 17: Introduction to Delays from *Introduction to Computer Simulation* by Nancy Roberts et. al. However, the chapter in the book has its models built using the DYNAMO simulation language, and as you are probably using Vensim, STELLA or iTHINK, we have translated the models for your convenience. This paper contains these translated versions of the models for the first seven exercises from the book. Later, we also suggest answers to these exercises for you to measure your answers against.

NOTE FROM THE TRANSLATOR: All the models in this paper have rates that flow in both directions. This is to encourage the readers to make the direction of the flow explicit in the equation, and not in the plumbing. In DYNAMO, there is no biflow-uniflow toggle. All rates are biflows.

Example 1: The Martan Chemical Company

We present the model of dumping of the Nobug pesticide by the Martan Chemical Company, as translated from DYNAMO to STELLA. A causal loop and flow diagram of the Martan case are shown in Figure 1.

The equations for the model are as follows:

$$\text{NOBUG}(t) = \text{NOBUG}(t - dt) + (\text{Dump_rate} - \text{Absorb_rate}) * dt$$
$$\text{INIT NOBUG} = 0$$

OUTFLOWS:

$$\text{ABSORB_RATE} = \text{NOBUG}/\text{NAT}$$

$$\text{NAT} = 2$$

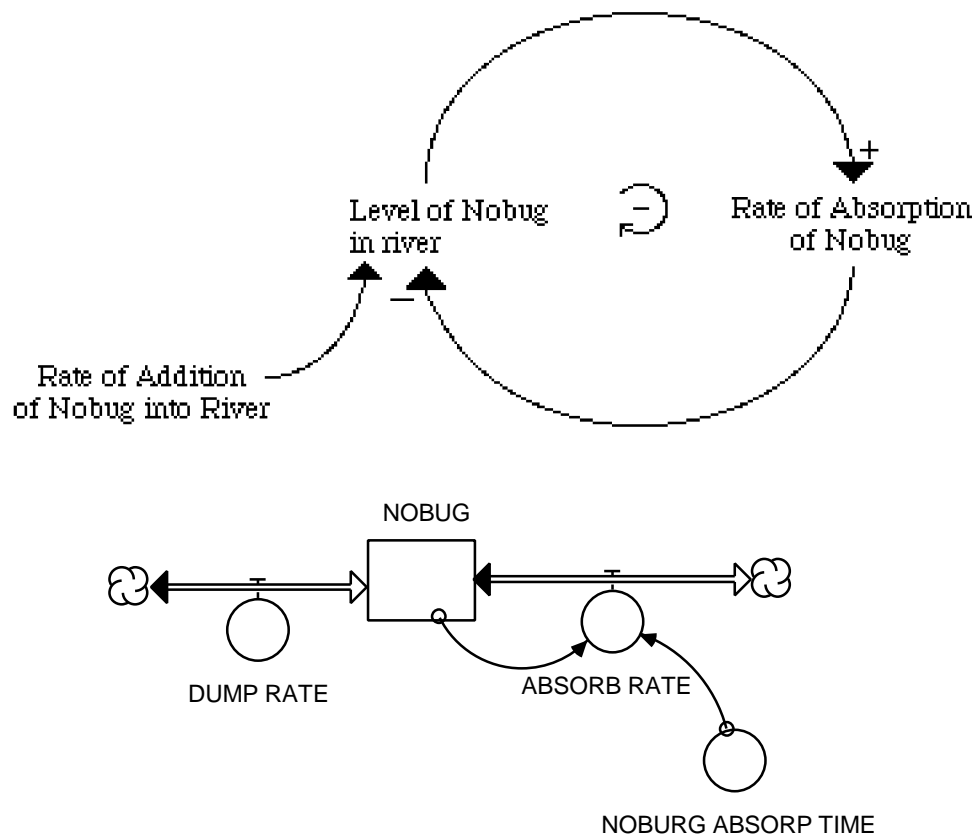


Figure 1. Diagram of the Martan Case

These equations show that the level of NOBUG in the Sparkill River is influenced by the DUMP RATE and the ABSORB RATE; and the ABSORB RATE is equal to the level of NOBUG divided by the Nobug Absorption Time NAT (2 days).

Exercise 1: Preliminary Nobug Model

a. Create a STELLA II model for the Nobug case, adding the DUMP RATE equation, and other needed specifications. Run the model, setting the initial level of NOBUG = 0 and choosing $DT = 0.25$ days. Set length of simulation (in the Time specs window under the Run menu) to 25 days, the range of Nobug (in the Range specs window under the Run menu) from 0 to 250 gallons and the range of the absorption rate from 0 to 100 gallons/day. What behavior does the model generate?

b. Rerun the model, setting the Nobug Absorption Time, $NAT = 4$. How do the results differ? Rerun the model, setting $NAT = 1$. How do these results differ?

USING THE PULSE FUNCTION TO REPRESENT THE DUMPING RATE

We use the corresponding STELLA functions instead of the DYNAMO functions in these exercises. The STELLA PULSE function permits modifying the model to represent the dumping of Nobug in once a week batches. The following rate equation indicates that 420 gallons of Nobug are dumped into the river on day 1 of the simulation, and 420 are dumped again at regular intervals of 7 days.

$$DUMP\ RATE = PULSE(420,1,7)$$

Figure 2 shows the dumping rate over the first 25 days of the simulation. Please see the chapter in the book for further reading.

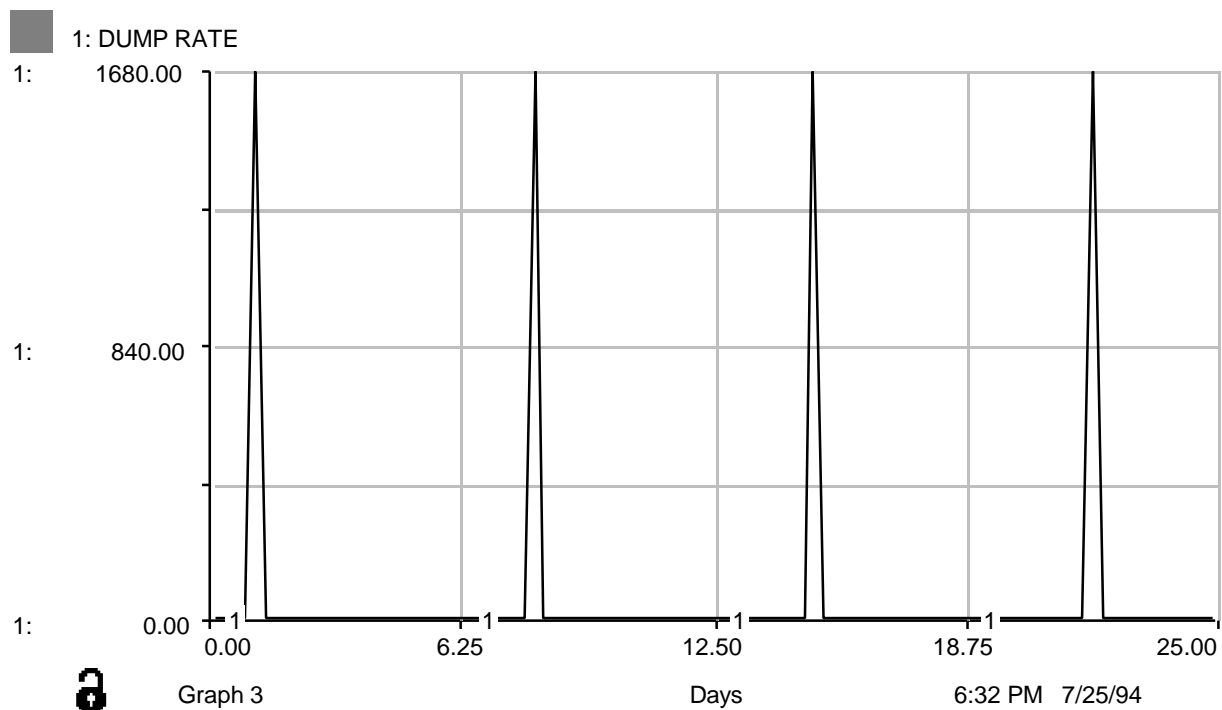


Figure 2. Dumping rate

The PULSE function can also be used to examine the response of the system

to the dumping of just one batch of NOBUG. It is possible to do this by making the following change in the equation for DUMP RATE.

$$\text{DUMP RATE} = \text{PULSE}(420,1,0)$$

This equation indicates that the dumping rate rises from zero to 1680 on day one (for only a quarter of the day) and stays at zero for the rest of the simulation.

Figure 3 shows a simulation of the Nobug system, with one batch of Nobug released into the river on day one. As can be seen, the level of NOBUG in the river rises sharply to 420 gallons on day one, and then drifts slowly toward zero.

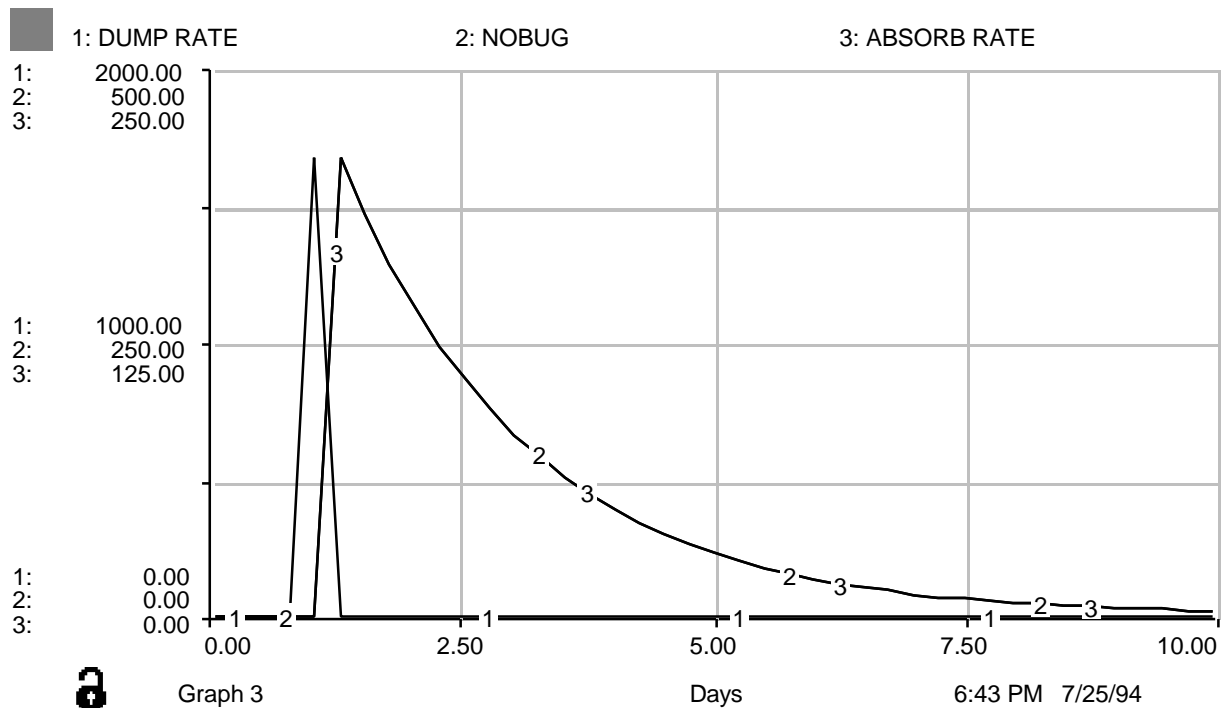


Figure 3. Release of one batch of NOBUG

The general form of the PULSE function is

$$\text{PULSE}(\text{AMOUNT}, \text{FIRST}, \text{INTERVAL})$$

where AMOUNT indicates the amount to be inputted in the pulse; FIRST indicates the time at which the first pulse occurs; and INTERVAL indicates the time interval between pulses. Setting INTERVAL to 0, indicates to STELLA that you only want

one pulse.

Exercise 2: The Halving Time for NOBUG

Revise your STELLA equations to include a PULSE function for the DUMP RATE. Choose an interval of 0, in order to examine the effects of just one pulse.

- What is the halving time for the amount of NOBUG in the Sparkill River?
- Experiment with various values of NAT. How does the choice of NAT influence the halving time?
- Using your result for part (b), select a value of NAT to produce a halving time that corresponds to the data shown below in Figure 4.

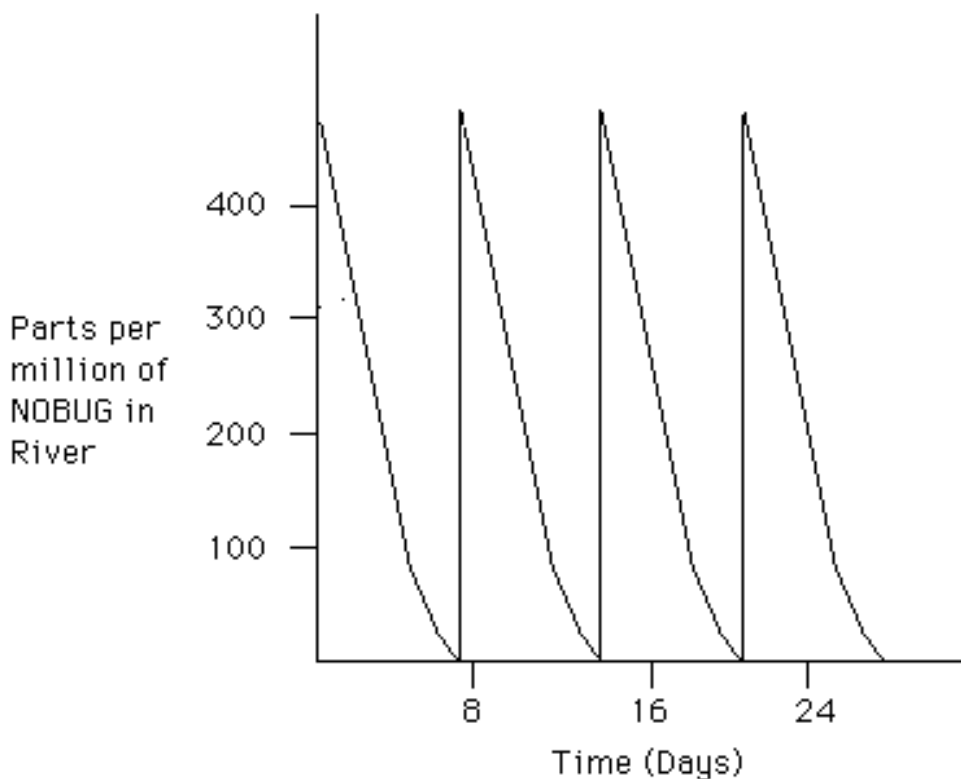


Figure 4. Amount of Nobug in the Sparkill River.

Exercises 3: Simulating Repeated Batches

- Select NAT equal to the value you determined in Exercise 2, part (c). Use the

PULSE function to simulate the effect of dumping 420 gallons of Nobug into the river at 7-day intervals. Compare your results with the data shown in , Figure 4 above.

b. Assume Martan Chemical Inc. changed the chemical composition of Nobug, such that the halving time of Nobug was lengthened to seven days, resulting in an increase in the Nobug absorption time NAT. Change the value of NAT in your model to reflect the change in the chemical composition of Nobug. How does the system respond?

NOBUG AND MATERIAL DELAYS

Please read the chapter for an introduction to the concept of a first-order material delay.

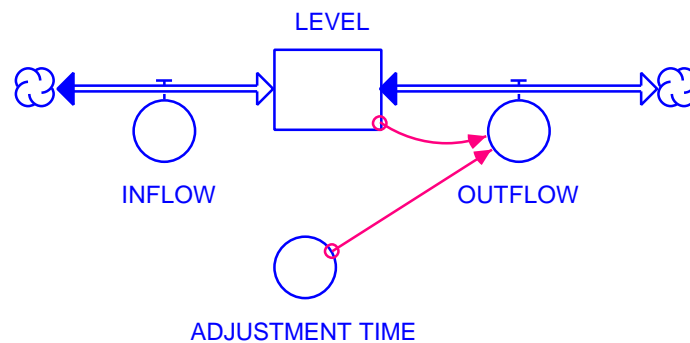


Figure 5. First-order material delay

When a first-order material delay is used in a model, there are two ways to write the equations. One approach is to simply to write out individual equations, exactly as was done in the Nobug case. But because first-order material delays are frequently used, a special STELLA function is available that can be used to replace the set of individual equations. Note that we are using the equivalent STELLA functions instead of the DYNAMO functions used in the book. The following equation can be used to indicate that the OUTFLOW rate is a first-order delayed response to the INFLOW rate, with an adjustment AT.

$$\text{OUTFLOW} = \text{SMTH1}(\text{INFLOW}, \text{AT})$$

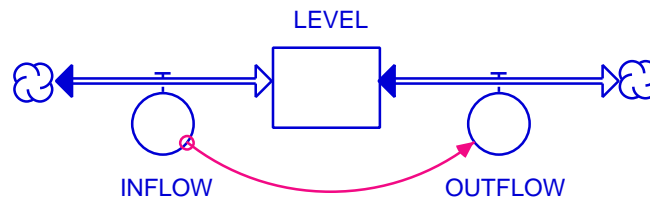


Figure 6. First-order material delay using SMTH1 function.

The SMTH1 function is simply a shorthand notation. When the model is run, STELLA will substitute the full level and rate formulation for a first-order delay, whenever the SMTH1 function appears in the model.

NOTE FROM THE TRANSLATOR:

Exercise 4 from the original text has been left out intentionally.

Exercise 5: The Mail Delay

The Nifty Department Store sends out bills to its charge-card customers once a month, and the credit department has learned that on the average, it takes about three days for the bills to arrive in the mail.

- a. Draw a causal-loop diagram, flow diagram, and equations for the Nifty Department Store case. (Hint: See figure 7). Assume that NIFTY has 1000 charge customers.
- b. Run the Model and examine the results.
- c. How many bills take more than six days to arrive?

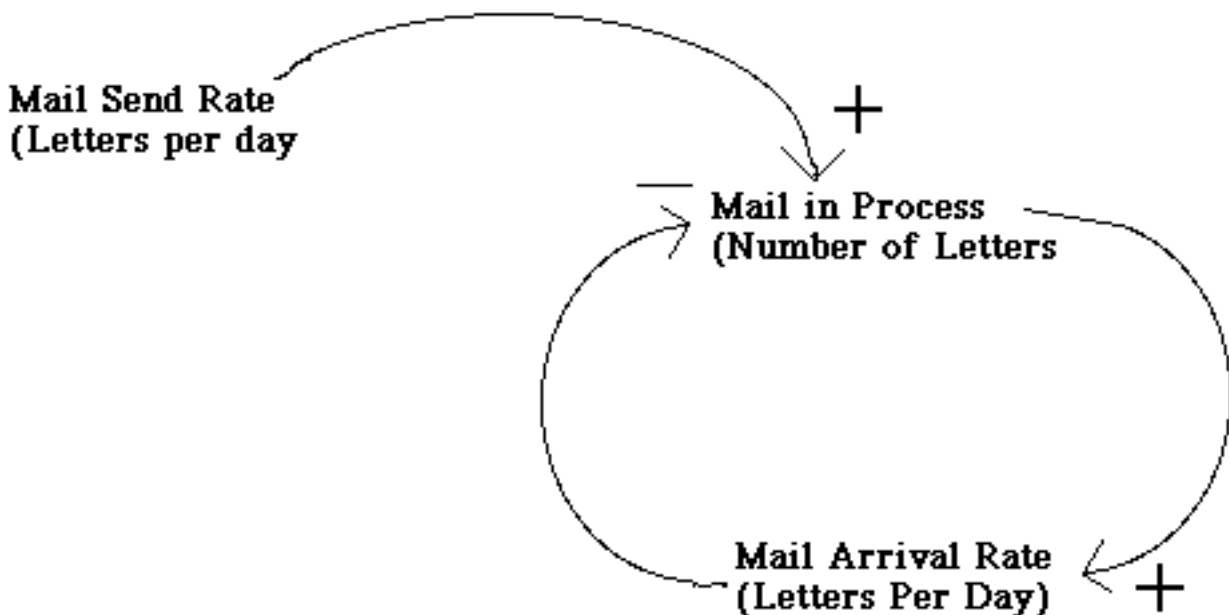


Figure 7. Causal loop representing a delay in mail delivery

EXAMPLE II: The GOAL-GAP FORMULATION, DELAYS AND CYCLES IN APARTMENTS

Please read the chapter for an introduction to this example. A causal-loop diagram and flow diagram for this system are shown in Figure 8.

Exercise 6: Apartments-Part I

- Build the apartment model shown in Figure 8 using STELLA. Assume that the desired number of apartments is 10,000 and the time required to respond to the gap is one year. (Do not include an explicit construction delay in your formulation-it will be added in the next exercise.)
- Run the model and examine the behavior. Determine the equilibrium value for the number of apartments.
- Start the model in equilibrium, and use a STEP function to test the response of the system to an increase in the number of apartments desired from 10,000 to 15,000.

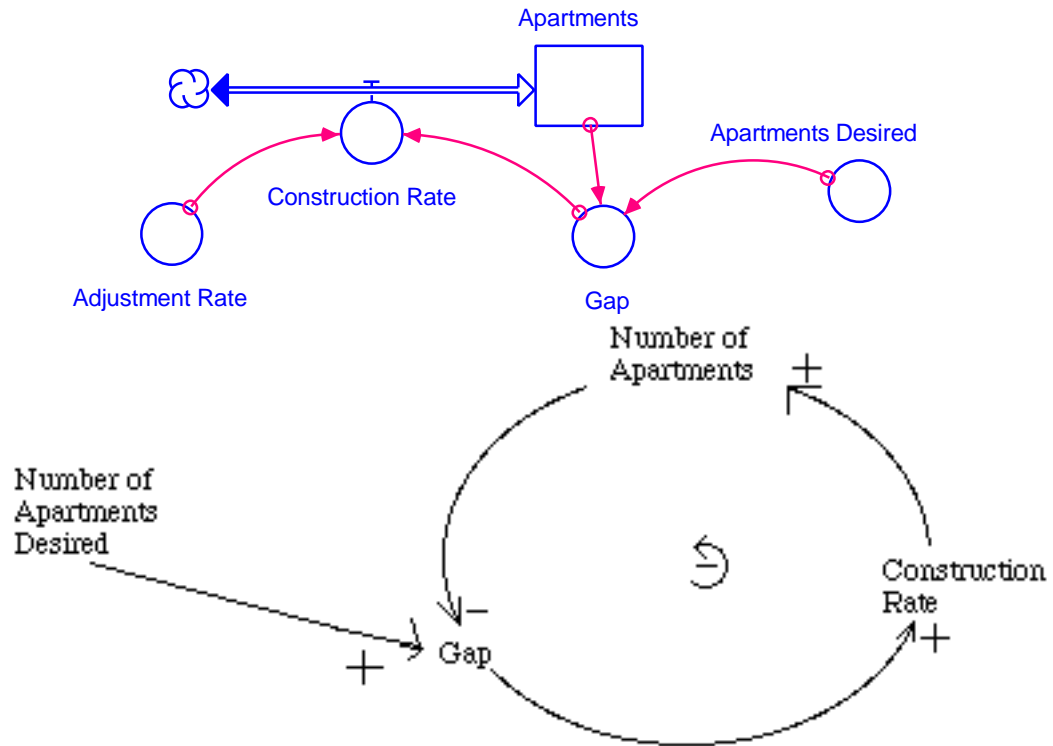


Figure 8. Diagrams expressing a housing gap

Please read the chapter for a discussion of why a delay should be added to the model. One way to do this is shown in figure 9.

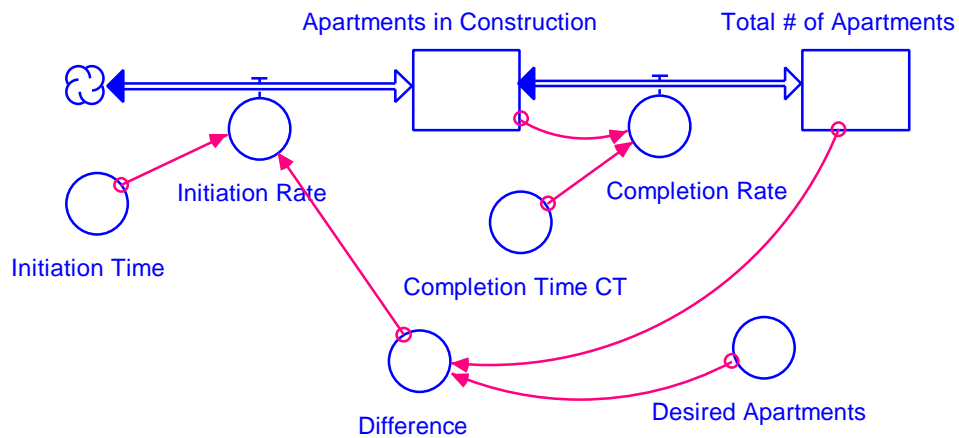


Figure 9. Apartment model with completion delay

Exercise 7: Apartments-Part II

For the questions in this exercise, please refer to the chapter.

INFORMATION DELAYS

Please read the chapter for an introduction to information delays.

EXAMPLE III: MARINA'S BAKE SHOP

Please read the chapter for an introduction to Marina and her Bake Shop.

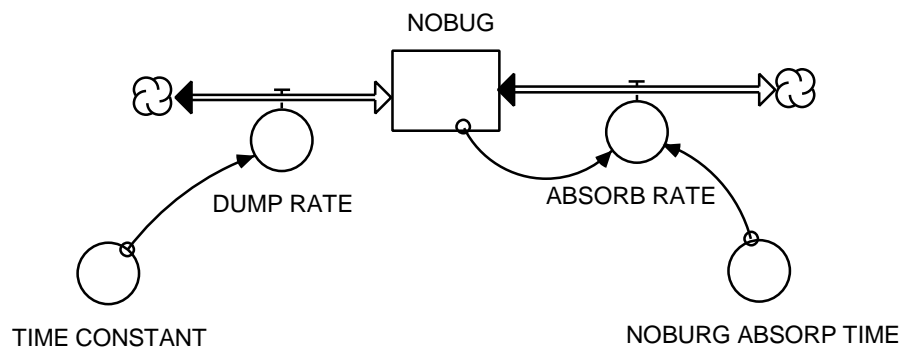
NOTE FROM THE TRANSLATOR: This paper presents only the first seven exercises from Chapter 7 in the book. We will discuss higher order delays later in Road Maps.

Now, we present our suggestions for the answers to the exercises.

Answers to Exercises

Exercise 1

a.



INFLOWS:

$$\text{DUMP_RATE} = 60/\text{TIME_CONSTANT}$$

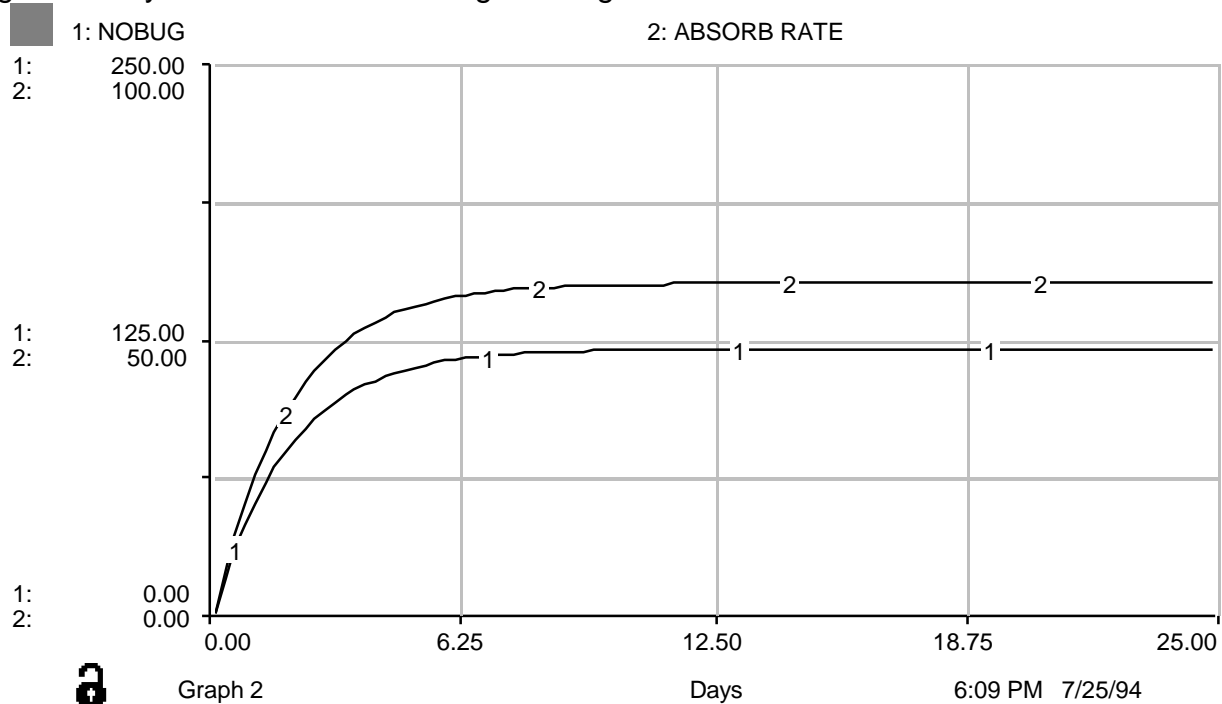
DOCUMENT: The DUMP RATE is how fast Nobug is dumped into the river. In this case it is 60 gallons per day, which was given in the text.

Units: gallons/day

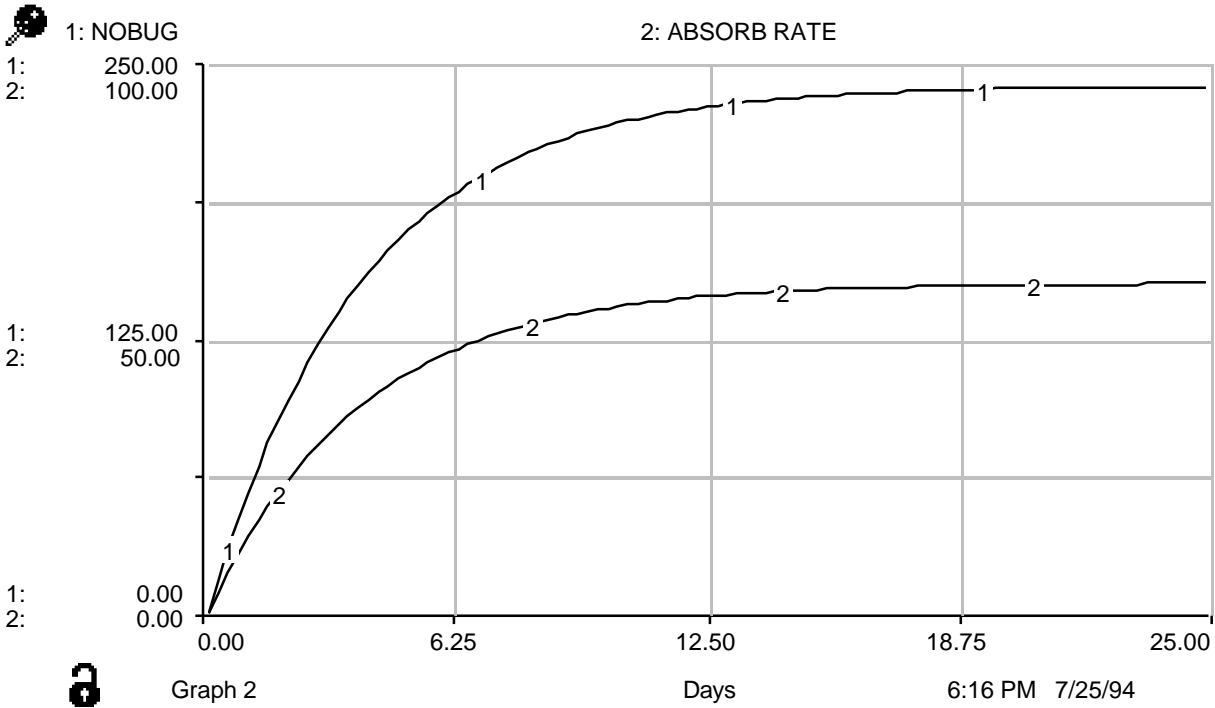
$$\text{TIME_CONSTANT} = 1$$

DOCUMENT: The time constant is the time over which the dump rate is calculated.
 Units: days

The DUMP RATE is not shown in the graph below because it is constant. As the content of NOBUG increases (because of the DUMP RATE), the ABSORB RATE also increases since it is dependent on the content of NOBUG in the river. The system reaches equilibrium when the ABSORB RATE equals the DUMP RATE at 60 gallons/day and the level of Nobug is 120 gallons.

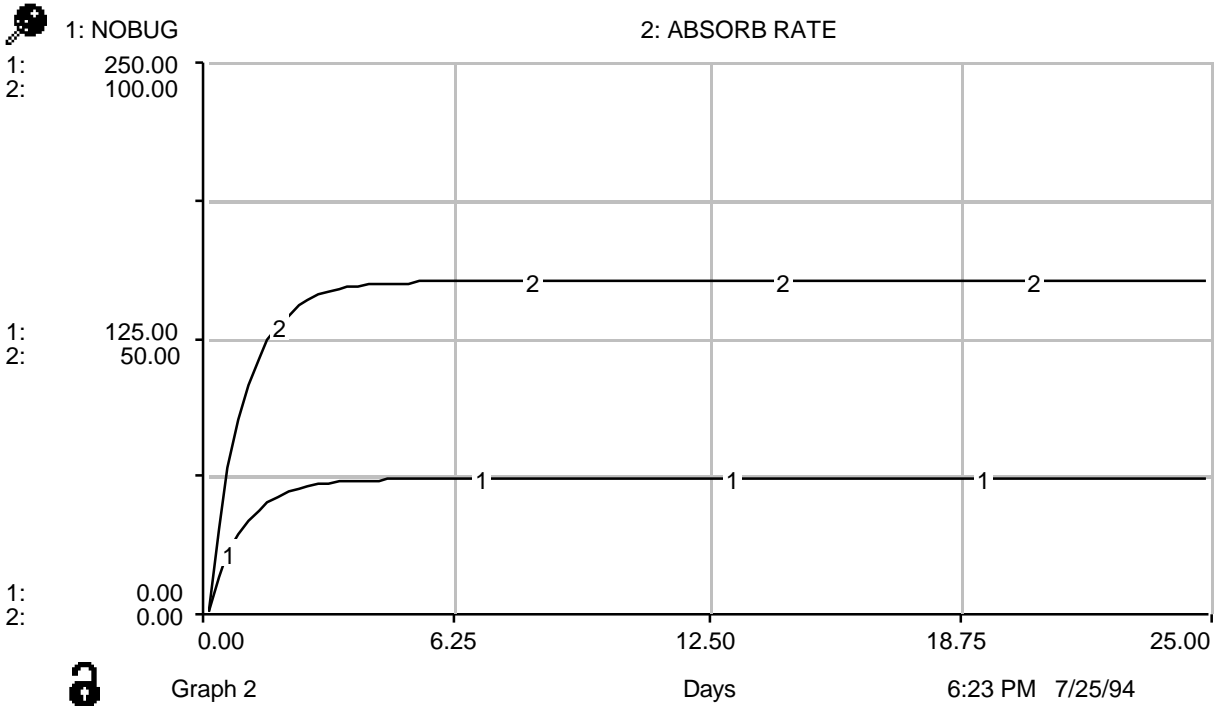


b. For NAT=4, the behavior is shown below.



When NAT=4, it takes twice as long to reach equilibrium, because the ABSORB RATE is twice as small as in part a. Also, the new equilibrium level of Nobug is at 240 gallons.

For NAT=1, the behavior is shown below.



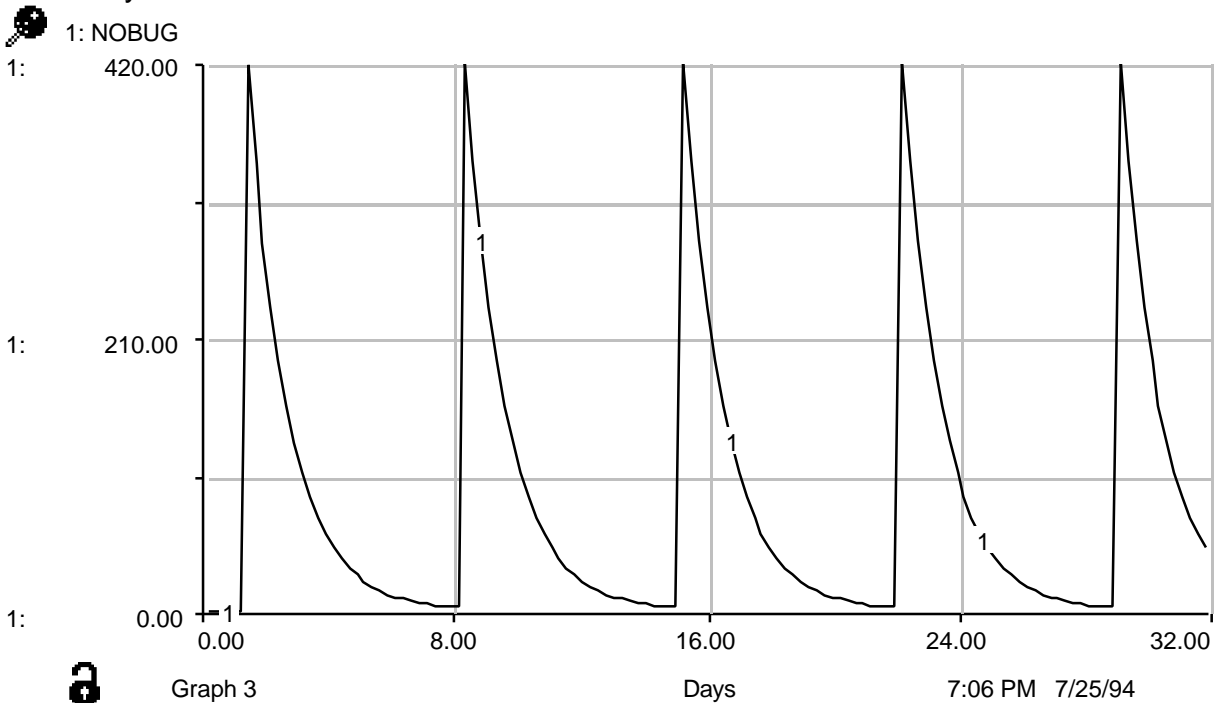
When NAT=1, equilibrium is reached in half the time it takes in part a. That is because the ABSORB RATE is twice as large as in part a. The new equilibrium level of Nobug is at 60 gallons.

Exercise 2

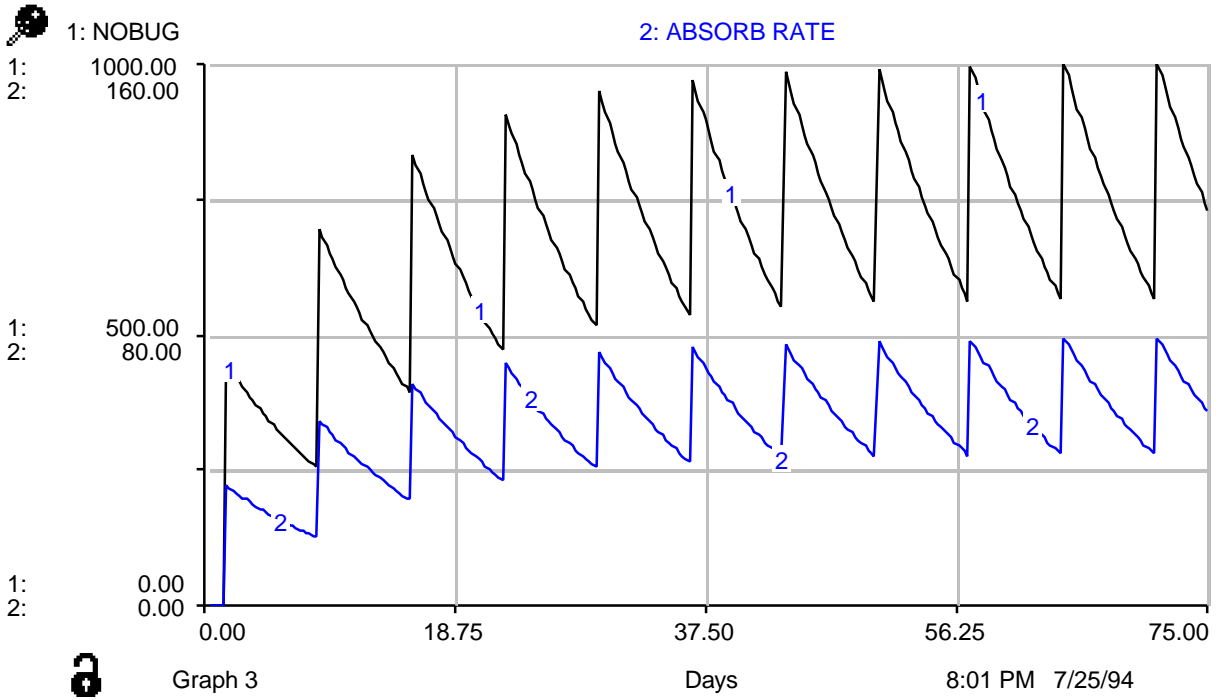
- a.** The level of NOBUG drops from 420 gallons at day 1 to about 210 gallons in 2.5 days. The halving time is then about 1.5 days.
- b.** After experimentation, you should find that an increase or decrease in the Nobug Absorption Time correspondingly increases or decreases the halving time.
- c.** Figure 4 shows a halving time for NOBUG of about 1 day (ie. half-life = 1 day). The value of NAT that produces this behavior can be calculated as $(\text{half-life})/.69$ or 1.4 days. With this value for NAT the outcome matches Figure 4.

Exercise 3

- a.** When the model is run with 420 gallons dumped by MARTAN every 7 days, with a NAT of 2 days, the output is a good match. Note that a run of NAT = 1.4 would give essentially the same run.

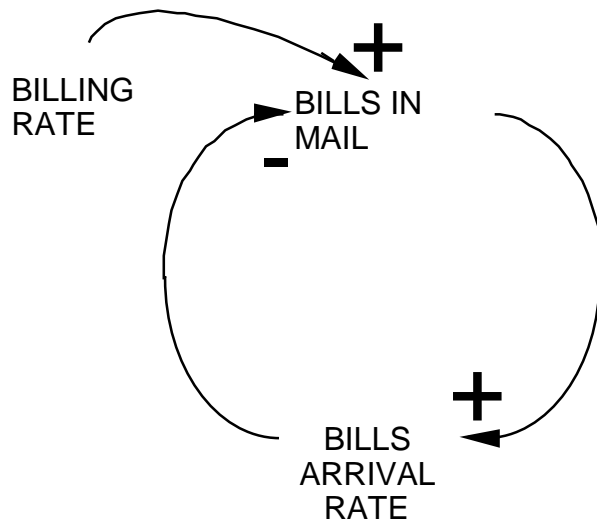


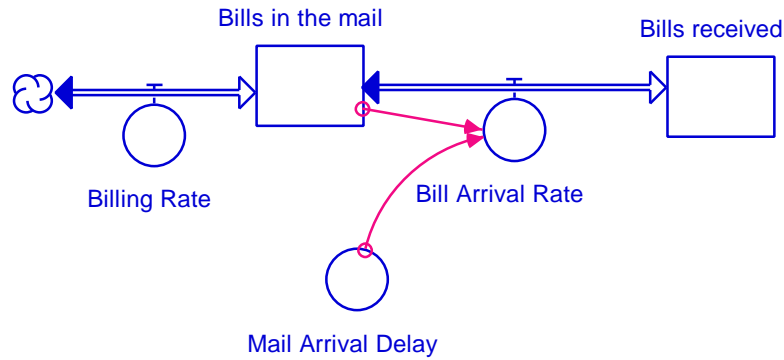
- b.** With the halving time of Nobug lengthened to seven days, the new value of NAT is 12 days (ie. half-life/.69). Since halving time is so long, Nobug is not eliminated from the river within the 7 days. As a result, content of Nobug continues to increase with each successive dumping of Nobug. However, the level of Nobug stabilizes again! The level stabilizes when the quantity of Nobug dumped into the river (ie. 420 gallons once a week) is equal to the quantity absorbed (ie. the area under the curve of the ABSORB RATE) during the week.



Exercise 4

a.





$$\text{Bills_in_the_mail}(t) = \text{Bills_in_the_mail}(t - dt) + (\text{Billing_Rate} - \text{Bill_Arrival_Rate}) * dt$$
 INIT Bills_in_the_mail = 0
 DOCUMENT: Initially there are no bills in the mail.
 Units: bills

INFLOWS:

Billing_Rate = PULSE(1000,1,0)

DOCUMENT: 1000 bills are sent out on the first day of the month. The interval was set to 0 because in this case we are only interested in studying the first pulse.

Units: bills/day

OUTFLOWS:

Bill_Arrival_Rate = Bill_Arrival_Rate/Mail_Arrival_Delay

DOCUMENT: This is a first order material delay, where bills arrive on average 3 days after they are sent.

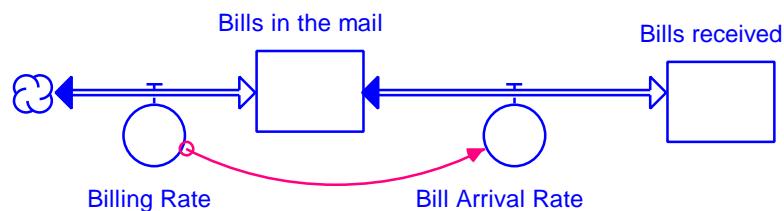
Units: bills/days.

Mail_Arrival_Rate = 3

Document : Time it takes for mail to arrive to destination.

Units: days

Another way of modeling the same behavior using the SMTH1 function is shown below. The behavior of this model should be the same as the graph below.

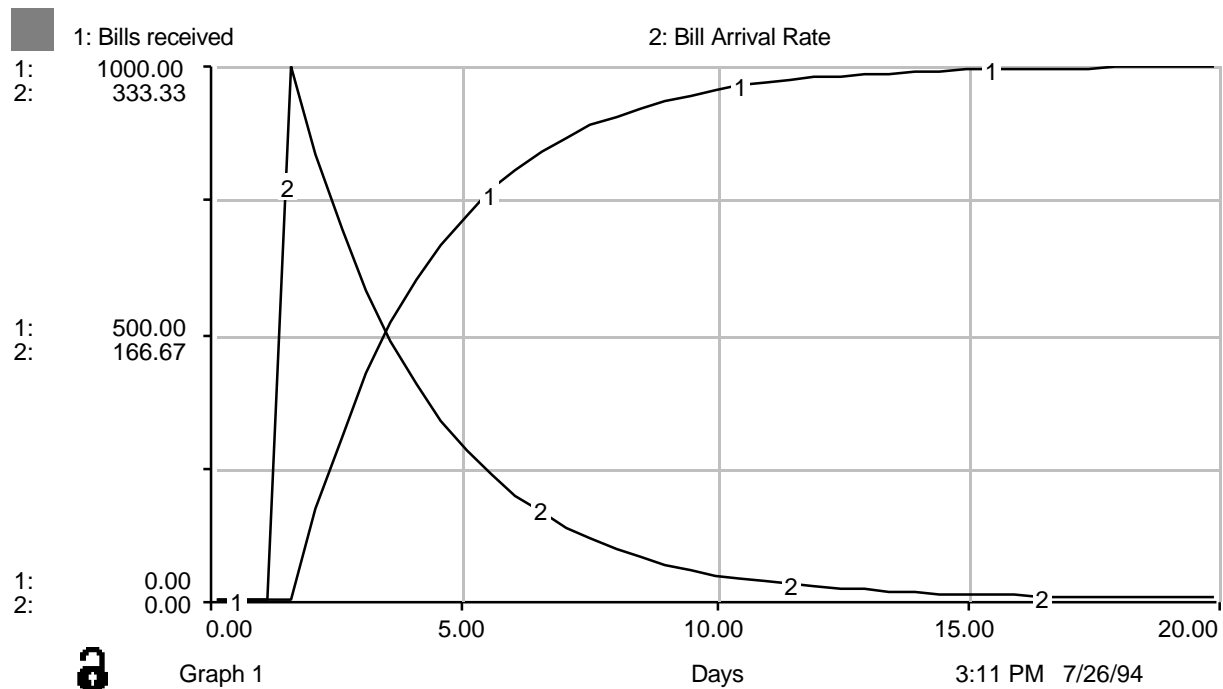


Billing_Rate = PULSE(1000,1,0)

Bill_Arrival_Rate = SMTH1(Billing_Rate, 3)

DOCUMENT: The adjustment time or delay is 3 days.

b.

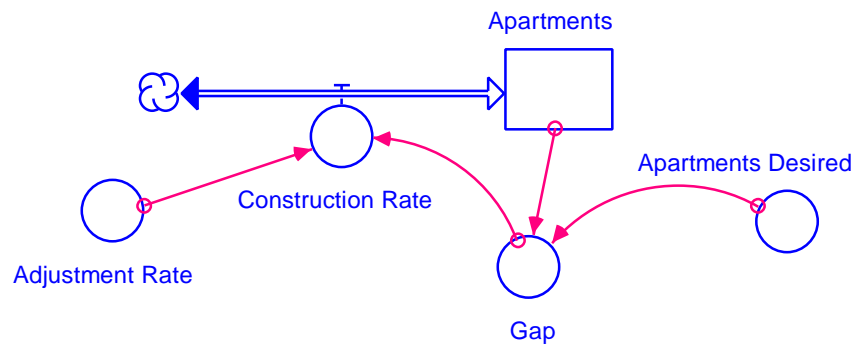


The pattern in the output is a mail arrival rate that decays exponentially from an initial peak just after the mail is sent. This is an unrealistic modeling of the mail process since no mail will be received immediately after the mail is sent.

c. From the tabular output, at the end of 6 days, 806 bills have been received or 194 bills (ie. 1000-806) will take more than 6 days to arrive.

Exercise 6

a.



$$\text{Apartments}(t) = \text{Apartments}(t - dt) + (\text{Construction_Rate}) * dt$$

INIT Apartments = 0

DOCUMENT: Apartments built so far.
 Units: apartments

INFLOWS:
 Construction_Rate = Gap/Adjustment_Rate

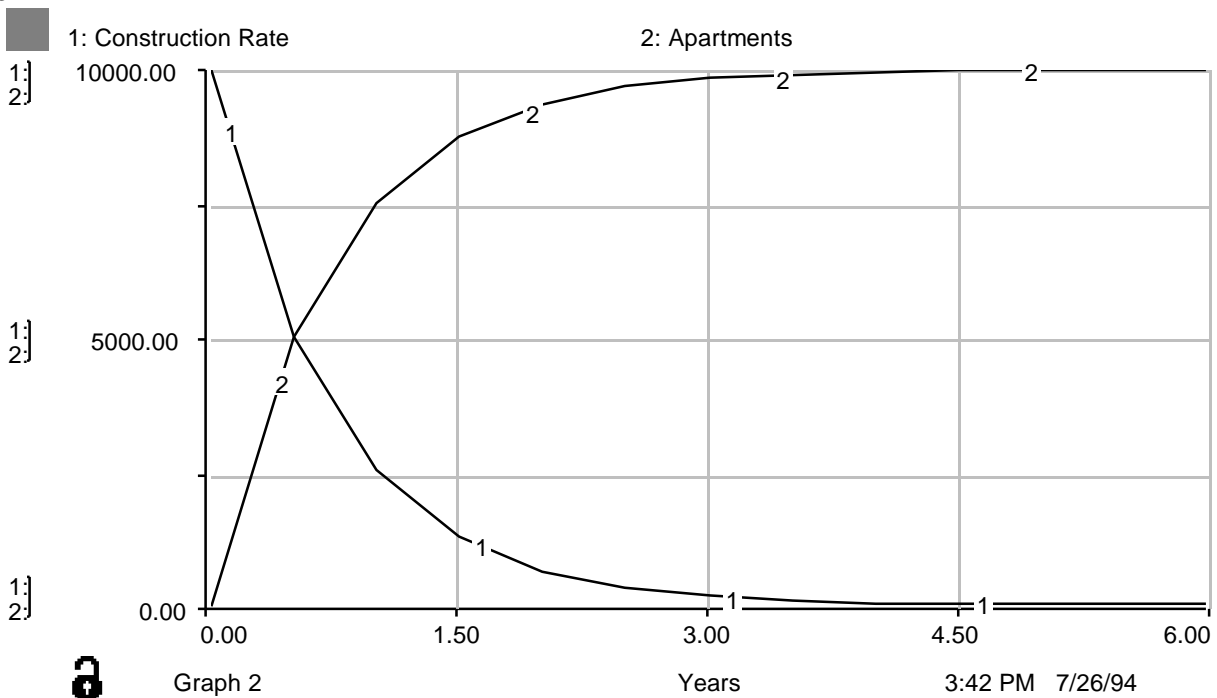
DOCUMENT: Apartments per year constructed.
Units: apartments/year

Adjustment_Rate = 1
DOCUMENT: Time to respond to the gap is one year.
Units : years

Apartments_Desired = 10000
DOCUMENT: Units : apartments

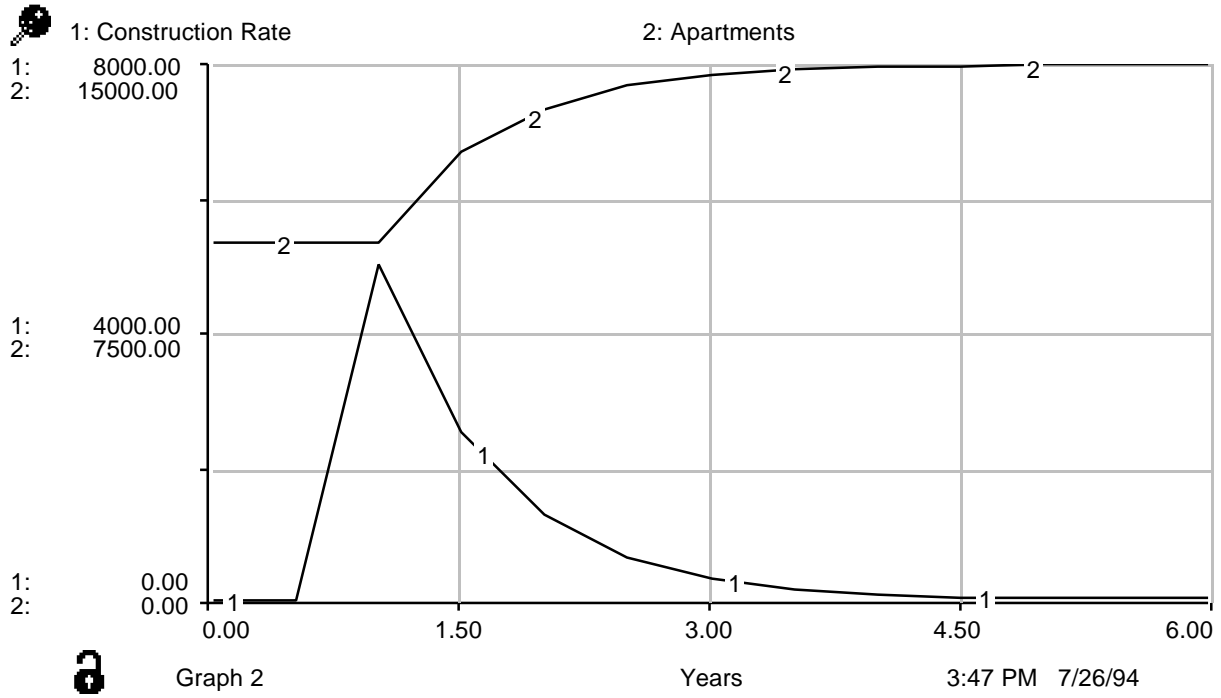
Gap = Apartments_Desired - Apartments
DOCUMENT: Difference between the number of desired apartments and actual apartments.
Units: apartments.

b.



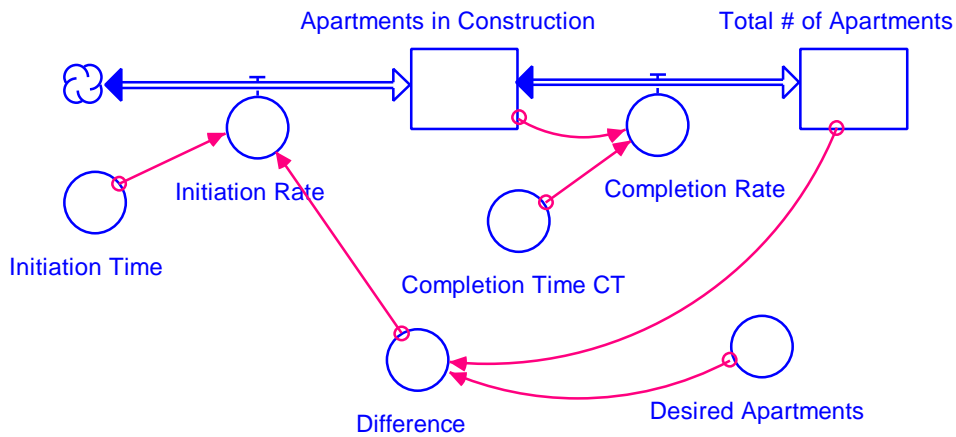
From the plot, the number of apartments at equilibrium will be 10000, as expected for this simple model with no demolition process.

c. The equation for desired apartments should be changed to:
$$\text{Apartments_Desired} = 10000 + \text{STEP}(5000, 1)$$



Exercise 7

a.



$$\text{Apartments_in_Construction}(t) = \text{Apartments_in_Construction}(t - dt) + (\text{Initiation_Rate} - \text{Completion_Rate}) * dt$$

INIT Apartments_in_Construction = 0

INFLOWS:

$$\text{Initiation_Rate} = \text{Difference} / \text{Initiation_Time}$$

OUTFLOWS:

$$\text{Completion_Rate} = \text{Apartments_in_Construction} / \text{Completion_Time_CT}$$

$$\text{Total_#_of_Apartments}(t) = \text{Total_#_of_Apartments}(t - dt) + (\text{Completion_Rate}) * dt$$

INIT Total_#_of_Apartments = 10000

INFLOWS:

Completion_Rate = Apartments_in_Construction/Completion_Time_CT

Completion_Time_CT = 4

Desired_Apartments = 10000+STEP(5000,1)

DOCUMENT: Units : apartments

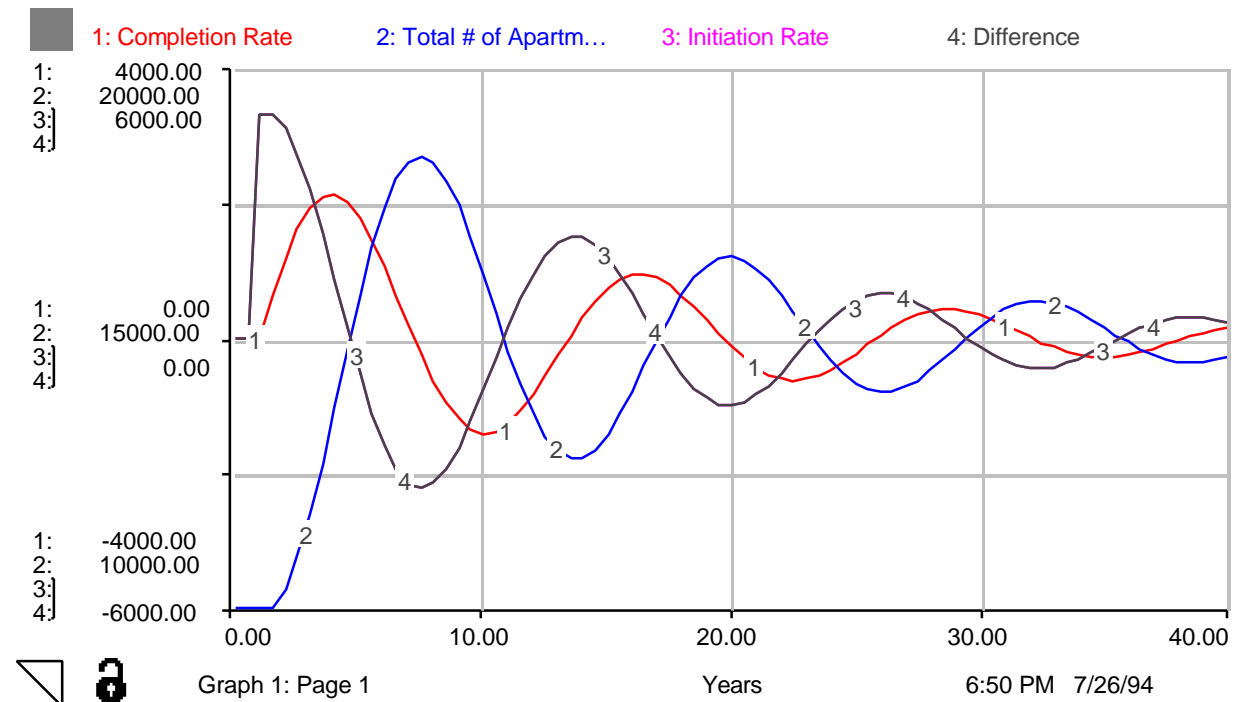
Difference = Desired_Apartments-Total_#_of_Apartments

DOCUMENT: Difference between the number of desired apartments and actual apartments.

Units: apartments.

Initiation_Time = 1

b& c. Set the Total # of Apartments equal to 10,000 to start the model in equilibrium. The STEP function is used to increase the Desired Apartments from 10,000 to 15,000. The stock for the Total # of Apartments must not be left checked as a non-negative stock in the dialog box for this stock. This is because it is possible to have a negative construction rate. See part (f) for further explanation.



c. The system generates oscillating behavior. The oscillations in this case are damped oscillations. The Total # of Apartments oscillates around 15,000 apartments and eventually converges to this value in equilibrium.

d. A delay in a negative feedback loop leads to oscillation. There are different types of oscillatory behavior- sustained, expanding and damped oscillations. The linkage between the stock for the Total # of Apartments and it's outflow causes the system to exhibit damped oscillations.

- e.** Try three runs with Completion Times CT of 3, 4, and 6 years. In all cases, the number of apartments will oscillate about the equilibrium value, the main effect of changing the construction delays time is to change the period of oscillation. An increase in the Completion Time increases the period of oscillation and makes it take longer for the system to achieve equilibrium. In all cases, the oscillations are damped, becoming smaller and smaller in amplitude from each cycle to the next.
- f.** A negative apartment construction rate appears to be physical nonsense. However, it could correspond to the destruction of buildings or to the cancellation of contracts to build, even after construction has started. The prediction was to be made for a period twice as long as your run. Since you have made several runs, you have several possibilities. Consider the case of CT (construction delay time) being 6 years. The period of oscillation for this value of CT is about 15 years so you could expect new maxima in the rate of completion of apartments at 35 and 50 years but much smaller in size than those at 5 and 20 years. A run for 50 years shows just that pattern.

Vensim Examples: Introduction to Computer Simulation

By Lei Lei and Samitha Samaranayake
October 2001

Example 1A: Martan Chemical Company

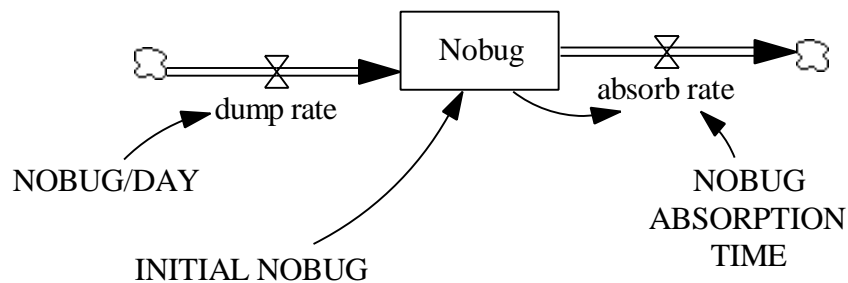


Figure 10: Vensim Equivalent of Diagram of the Martan Case without Pulse Function

Documentation for Martan Chemical Company Model

- (01) $\text{absorb rate} = \text{Nobug} / \text{NOBUG ABSORPTION TIME}$
Units: gallons/day
- (02) $\text{dump rate} = \text{"NOBUG/DAY"}$
Units: gallons/day
The dump rate is how fast Nobug is dumped into the river.
- (03) $\text{FINAL TIME} = 25$
Units: days
The final time for the simulation.
- (04) $\text{INITIAL NOBUG} = 0$
Units: gallons
The Initial amount of Nobug is 0 gallons.
- (05) $\text{INITIAL TIME} = 0$
Units: days
The initial time for the simulation.
- (06) $\text{Nobug} = \text{INTEG} (+\text{dump rate} - \text{absorb rate}, \text{INITIAL NOBUG})$
Units: gallons
- (07) $\text{NOBUG ABSORPTION TIME} = 1$

Graph of Nobug and Absorb Rate for NAT=4

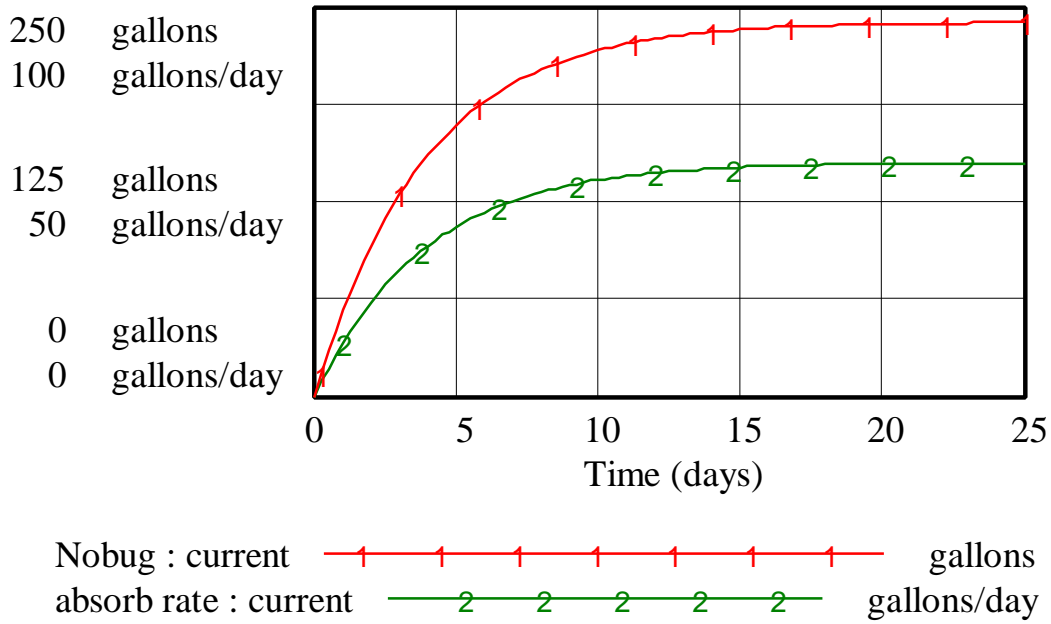


Figure 12: Vensim Equivalent of Simulation for NAT = 4

Graph of Nobug and Absorb Rate for NAT=1

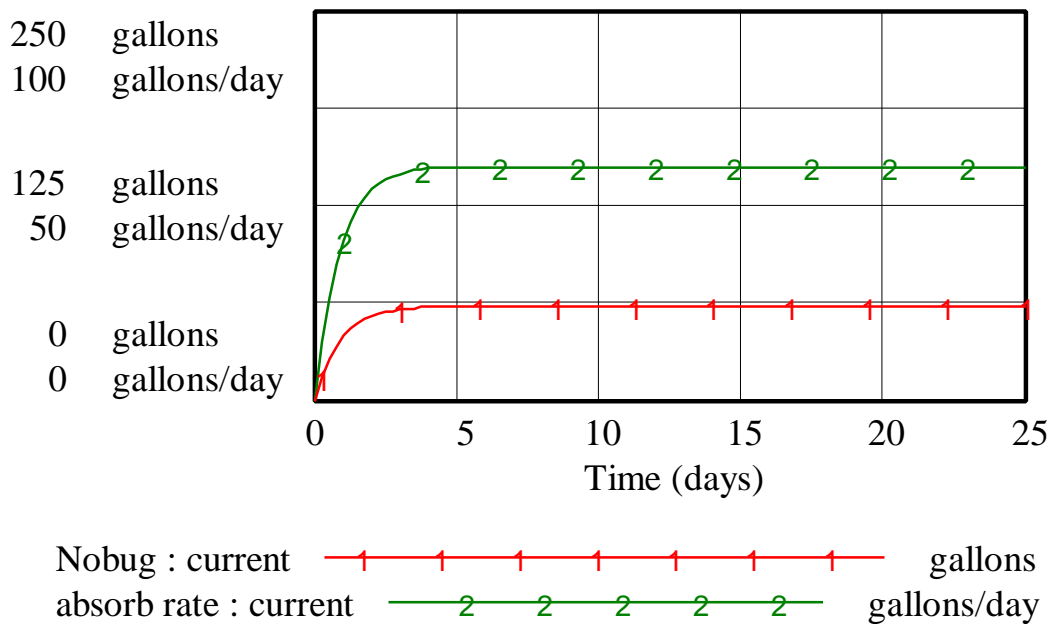


Figure 13: Vensim Equivalent of Simulation for NAT = 1

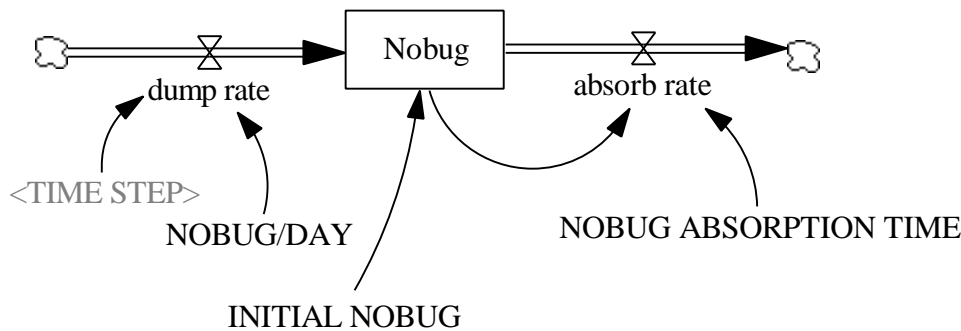
Example 1B: Dump Rate With Pulse(s)

Figure 14: Vensim Equivalent of Figure 1: Diagram of Martan Case

Documentation for Dump Rate With Pulse(s) Model

- (01) $\text{absorb rate} = \text{Nobug} / \text{NOBUG ABSORPTION TIME}$
Units: gallons/day
- (02) $\text{dump rate} = \text{"NOBUG/DAY"} * (\text{PULSE}(1, \text{TIME STEP}) + \text{PULSE}(8, \text{TIME STEP}) + \text{PULSE}(15, \text{TIME STEP}) + \text{PULSE}(22, \text{TIME STEP}))$
Units: gallons/day
420 gallons of Nobug are dumped each quarter of a day, so 1680 gallons of Nobug are dumped on the first day of each week

Note: This formulation is used because Vensim does not have a repeated pulses function. We have inserted individual pulses at intervals of 7 days instead.

- (03) $\text{FINAL TIME} = 25$
Units: days
The final time for the simulation.
- (04) $\text{INITIAL NOBUG} = 0$
Units: gallons
- (05) $\text{INITIAL TIME} = 0$
Units: days
The initial time for the simulation.
- (06) $\text{Nobug} = \text{INTEG} (+\text{dump rate} - \text{absorb rate}, \text{INITIAL NOBUG})$
Units: gallons
- (07) $\text{NOBUG ABSORPTION TIME} = 7$
Units: day
- (08) $\text{"NOBUG/DAY"} = 1680$
Units: gallons/day

There are 1680 gallons of Nobug dumped per day.

- (09) $\text{SAVEPER} = \text{TIME STEP}$
 Units: days
 The frequency with which output is stored.
- (10) $\text{TIME STEP} = 0.25$
 Units: day
 The time step for the simulation.

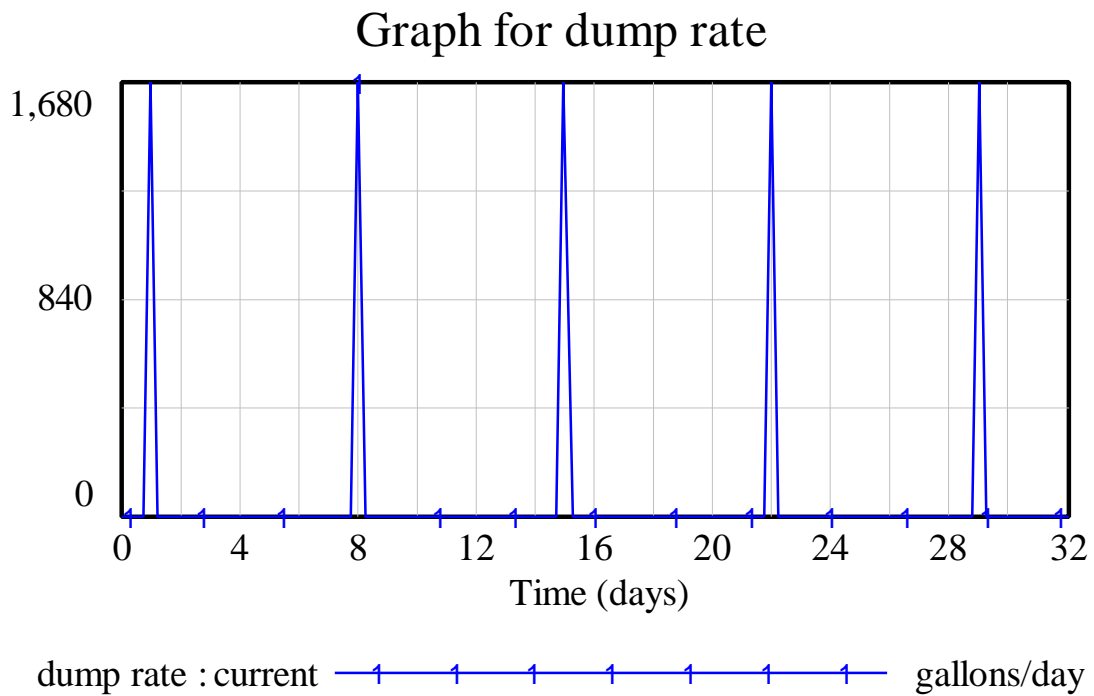


Figure 15: Vensim Equivalent of Figure 2: Simulation for dump rate

Graph of Nobug and Absorb Rate for NAT = 12

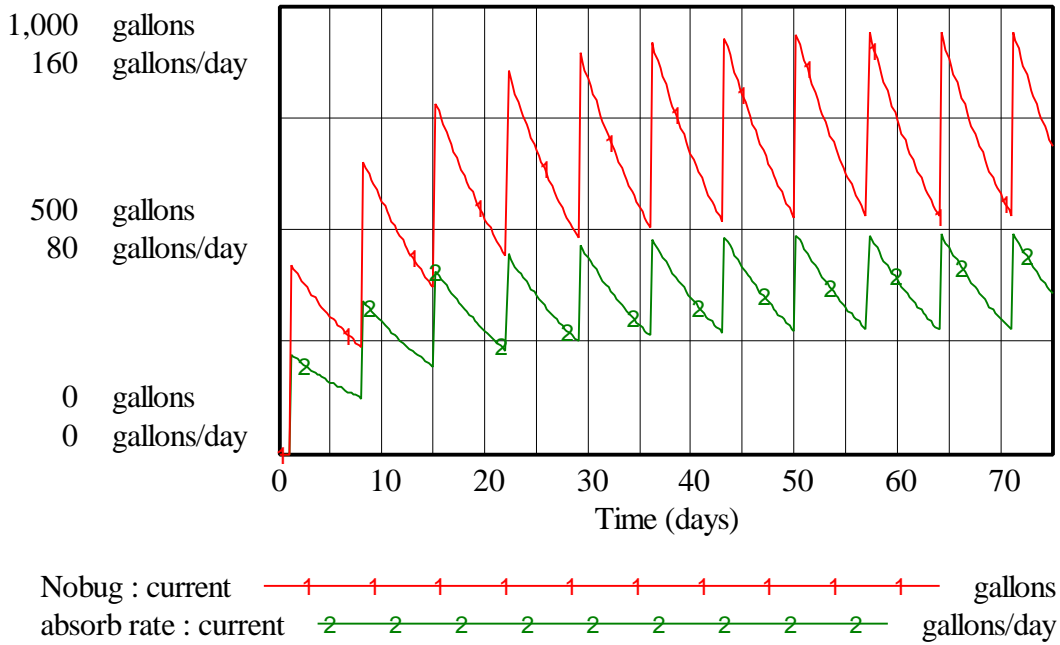


Figure 18: Vensim Equivalent of Simulation of Nobug and Absorb Rate for NAT = 12

Exercise 5: The Mail Delay

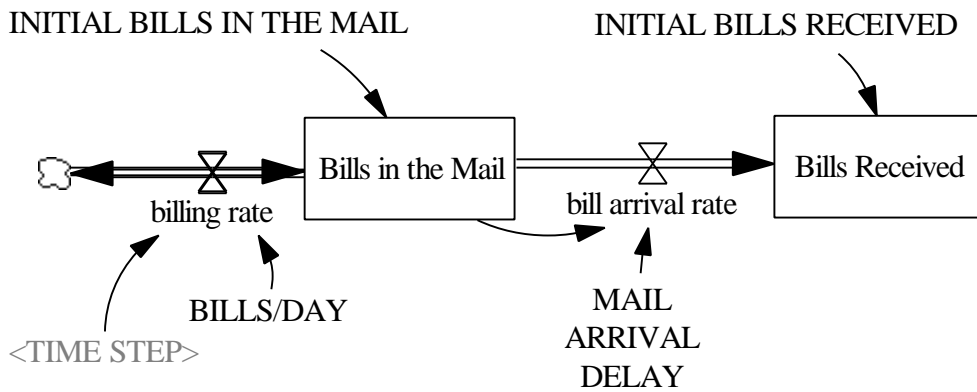


Figure 19: Vensim Equivalent of Mail Delay Model

Documentation for Mail Delay Model

- (01) $bill\ arrival\ rate = \frac{Bills\ in\ the\ Mail}{MAIL\ ARRIVAL\ DELAY}$
 Units: bills/day
 This is a first order material delay, where bills arrive on average 3 days after they are sent.

- (02) $\text{billing rate} = \text{"BILLS/DAY"} * \text{PULSE}(1, \text{TIME STEP})$
Units: bills/day
1000 bills are sent out on the first day of the month. The interval was set to 0 because in this case we are only interested in studying the first pulse.
- (03) $\text{Bills in the Mail} = \text{INTEG} (+\text{billing rate} - \text{bill arrival rate}, \text{INITIAL BILLS IN THE MAIL})$
Units: bills
Initially there are no bills in the mail.
- (04) $\text{Bills Received} = \text{INTEG} (\text{bill arrival rate}, \text{INITIAL BILLS RECEIVED})$
Units: bills
- (05) $\text{"BILLS/DAY"} = 1000$
Units: bills/day
1000 bills are sent out on the first day of the month.
- (06) $\text{FINAL TIME} = 20$
Units: day
The final time for the simulation.
- (07) $\text{INITIAL BILLS IN THE MAIL} = 0$
Units: bills
There are initially 0 Bills in the Mail.
- (08) $\text{INITIAL BILLS RECEIVED} = 0$
Units: bills
There are initially 0 bills received.
- (09) $\text{INITIAL TIME} = 0$
Units: day
The initial time for the simulation.
- (10) $\text{MAIL ARRIVAL DELAY} = 3$
Units: day
Time it takes for mail to arrive to destination.
- (11) $\text{SAVEPER} = \text{TIME STEP}$
Units: day
The frequency with which output is stored.
- (12) $\text{TIME STEP} = 0.0625$
Units: day
The time step for the simulation.

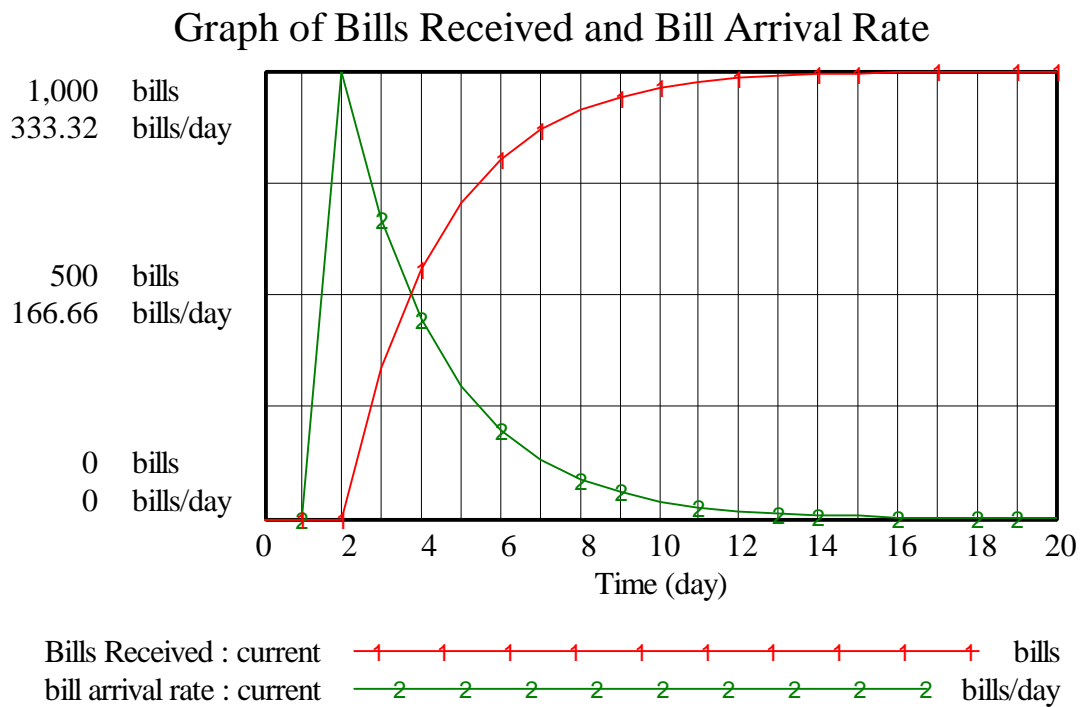


Figure 20: Vensim Equivalent of Simulation of Bills Received and Bill Arrival Rate

Exercise 6A: Apartments-Part 1

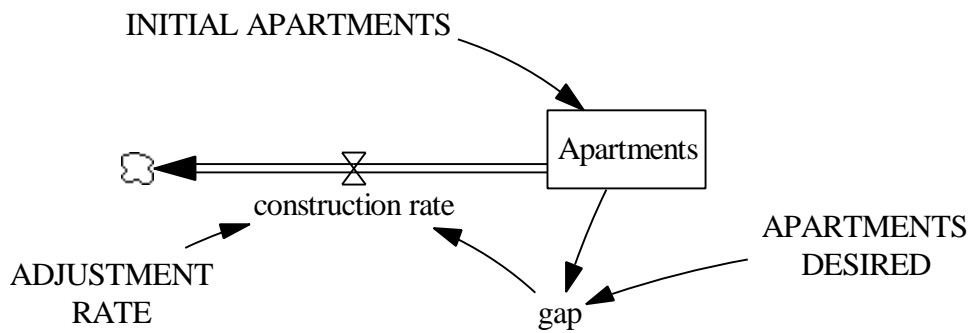


Figure 21: Vensim Equivalent of Apartments Model-Part 1

Documentation for Apartments Model-Part 1

- (01) ADJUSTMENT RATE = 1
Units: year
Time to respond to the gap is one year.
- (02) Apartments = INTEG (construction rate, INITIAL APARTMENTS)

Units: apartments
Apartments built so far.

- (03) APARTMENTS DESIRED = 10000
Units: apartments
- (04) construction rate = gap/ADJUSTMENT RATE
Units: apartments/year
- (05) FINAL TIME = 6
Units: year
The final time for the simulation.
- (06) gap = APARTMENTS DESIRED - Apartments
Units: apartments
Difference between the number of desired apartments and actual Apartments.
- (07) INITIAL APARTMENTS = 0
Units: apartments
There are 0 apartments initially.
- (08) INITIAL TIME = 0
Units: year
The initial time for the simulation.
- (09) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.
- (10) TIME STEP = 0.0625
Units: year
The time step for the simulation.

Graph for construction rate and Apartments With Step Function

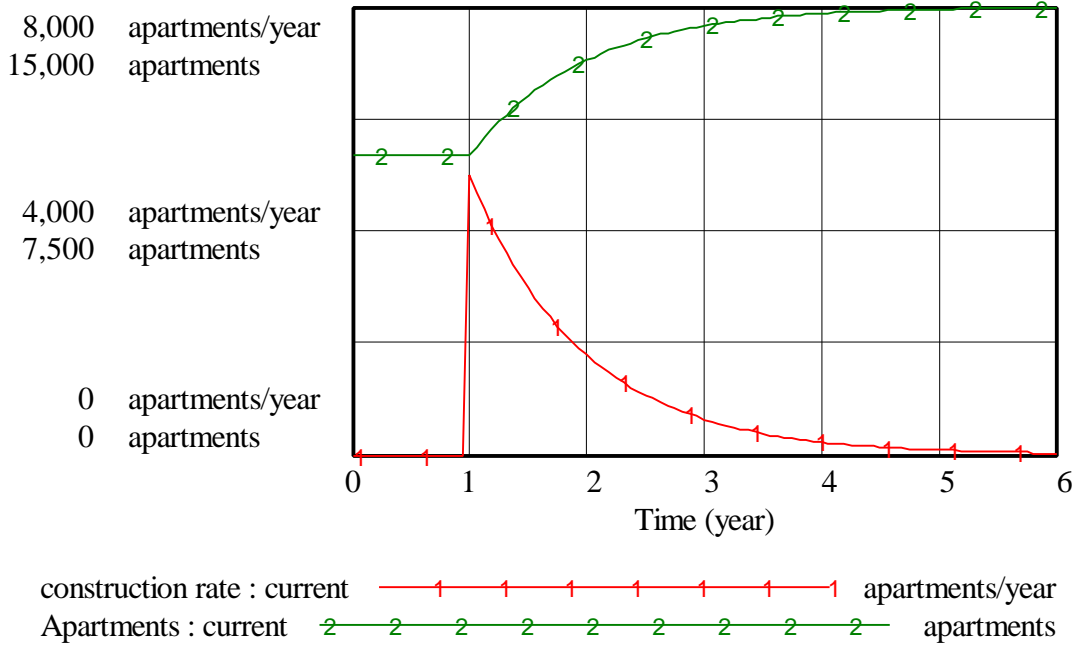


Figure 22: Vensim Equivalent of Simulation with Step Function

Exercise 6B: Apartments –Part 2

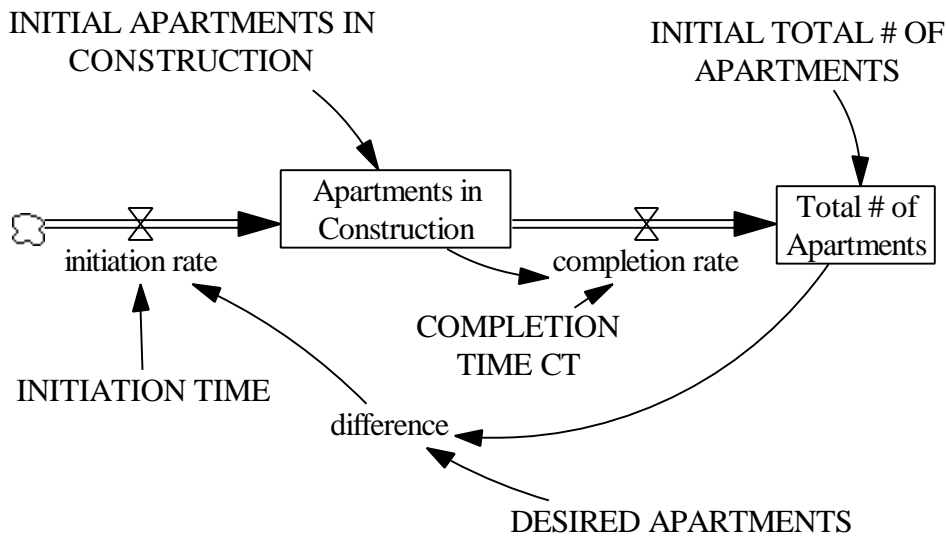


Figure 23: Vensim Equivalent of Apartments Model-Part 2

Documentation for Apartments Model-Part 2

(01) Apartments in Construction = INTEG (+initiation rate-completion rate, INITIAL APARTMENTS IN CONSTRUCTION)

Units: apartments

(02) completion rate = Apartments in Construction/COMPLETION TIME CT

Units: apartments/year

(03) COMPLETION TIME CT = 4

Units: year

(04) DESIRED APARTMENTS = 10000+STEP(5000,1)

Units: apartments

(05) difference = DESIRED APARTMENTS-"Total # of Apartments"

Units: apartments

Difference between the number of desired apartments and actual apartments.

(06) FINAL TIME = 40

Units: year

The final time for the simulation.

(07) INITIAL APARTMENTS IN CONSTRUCTION = 0

Units: apartments

There are 0 apartments initially.

(08) INITIAL TIME = 0

Units: year

The initial time for the simulation.

(09) INITIAL TOTAL # OF APARTMENTS= 10000

Units: apartments

There are 10000 apartments initially.

(10) initiation rate = difference/INITIATION TIME

Units: apartments/year

(11) INITIATION TIME = 1

Units: year

(12) SAVEPER = TIME STEP

Units: year

The frequency with which output is stored.

(13) TIME STEP = 0.0625

Units: year

The time step for the simulation.

(14) Total # of Apartments = INTEG (completion rate, INITIAL TOTAL # OF APARTMENTS)

Units: apartments

Graph Showing Oscillatory Behavior

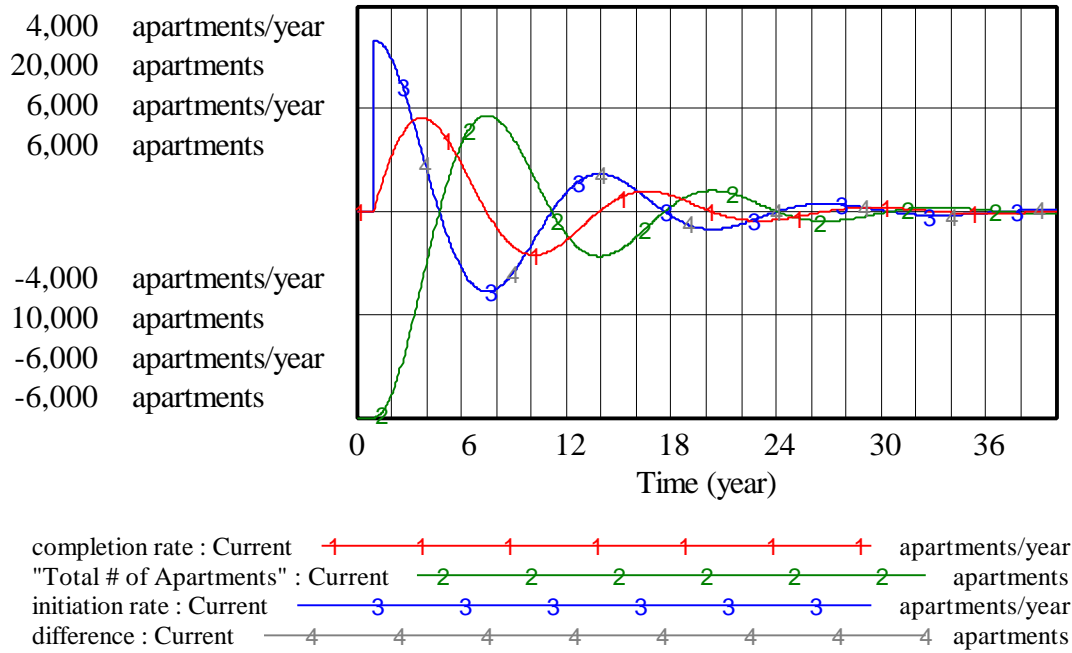


Figure 24: Vensim Equivalent of Simulation showing oscillatory behavior