

# Guided Study Program in System Dynamics

System Dynamics in Education Project

System Dynamics Group

MIT Sloan School of Management<sup>1</sup>

Solutions to Assignment #9

November 24, 1998

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## ***Reading Assignment:***

*Please refer to Road Maps 4: A Guide to Learning System Dynamics (D-4504-4) and read the following papers from Road Maps 4:*

- *Beginner Modeling Exercises Section 4, Mental Simulation: Adding Constant Flows, by Alan Coronado (D-4546)*
- *Problems with Causal Loop Diagrams, by George Richardson (D-3312-1)*

*Please refer to Road Maps 5: A Guide to Learning System Dynamics (D-4505-4) and read the following paper from Road Maps 5:*

- *Graphical Integration Exercises Part 3: Combining Flows, by Kevin Agatstein and Lucia Breierova (D-4596)*

## ***Exercises:***

### 1. Beginner Modeling Exercises: Adding Constant Flows

*After reading this paper and doing all the included exercises, please answer the following questions by using mental simulation:*

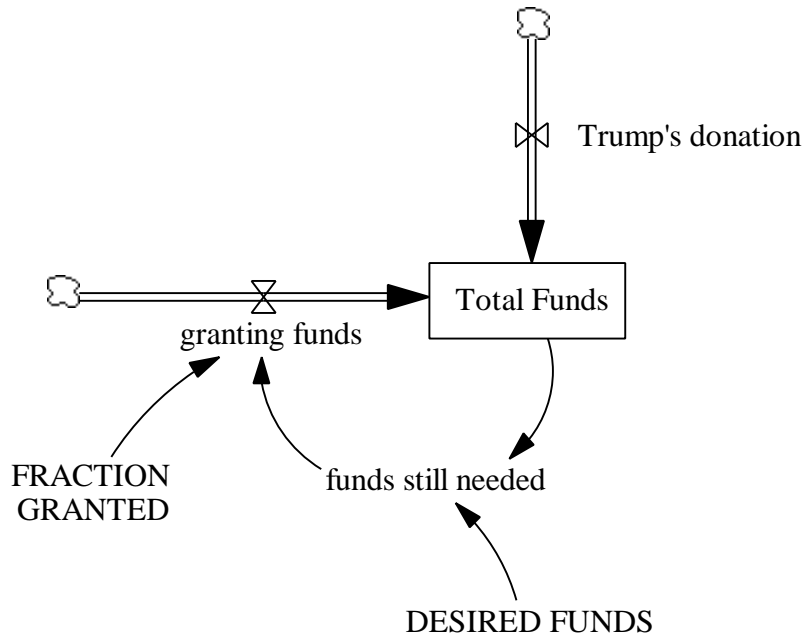
A. *Refer back to the scenario in exercise 2 of assignment 8. Imagine that Donald Trump vows to make a quarterly donation of \$5,000 (he owns a building on the southwest corner of the park, and hopes that a new botanical garden will attract residents to his building) until the park raises enough money. How does this constant inflow of funds affect the equilibrium time and value of the system?*

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Trump's quarterly donation adds a constant inflow of \$5,000 per quarter to the system. Modify the goal-gap inflow model from exercise 2 of assignment 8 by adding a constant inflow. Do not add \$5000 to the "granting funds" inflow. "Granting funds" and "Trump's donation" represent two separate, dynamic actions and hence should be represented by separate flows.

Model diagram:



Model equations:

$$\text{DESIRED FUNDS} = 1e+006$$

Units: dollar

The total amount of money needed for the project.

$$\text{FRACTION GRANTED} = 0.5$$

Units: 1/Quarter

The fraction of the needed funds that is granted by the Office of Management and Budget per quarter.

$$\text{funds still needed} = \text{DESIRED FUNDS} - \text{Total Funds}$$

Units: dollar

The difference between desired and current funds for the project is the amount that still remains to be collected.

$$\text{granting funds} = \text{funds still needed} * \text{FRACTION GRANTED}$$

Units: dollar/Quarter

The amount of money the Office of Management and Budget grants per quarter.

Total Funds = INTEG (granting funds + Trump's donation, 0)

Units: dollar

The amount of money that has been granted for the project at any given time.

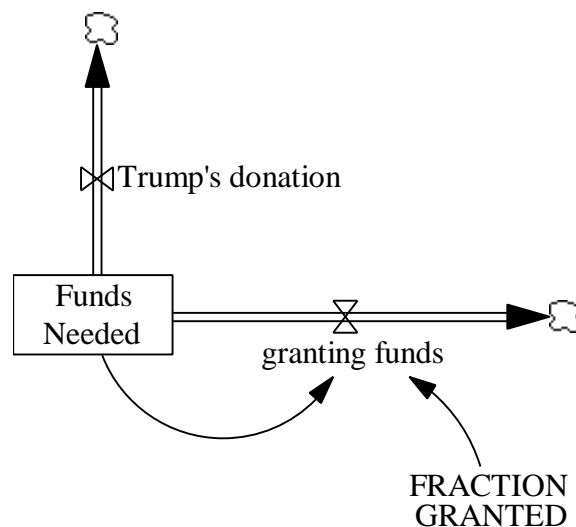
Trump's donation = 5000

Units: dollar/Quarter

Trump's quarterly donation to the total funds.

Alternatively, you can add a constant outflow to the outflow decay structure from exercise 2 of assignment 8:

Model diagram:



Model equations:

FRACTION GRANTED = 0.5

Units: 1/Quarter

The fraction of the needed funds that is granted by the Office of Management and Budget per quarter.

Funds Needed = INTEG (-granting funds - Trump's donation, 1e+006)

Units: dollar

The amount of funds that still needs to be collected.

granting funds = Funds Needed \* FRACTION GRANTED

Units: dollar/Quarter

The amount of money the Office of Management and Budget grants per quarter.

Trump's donation = 5000

Units: dollar/Quarter

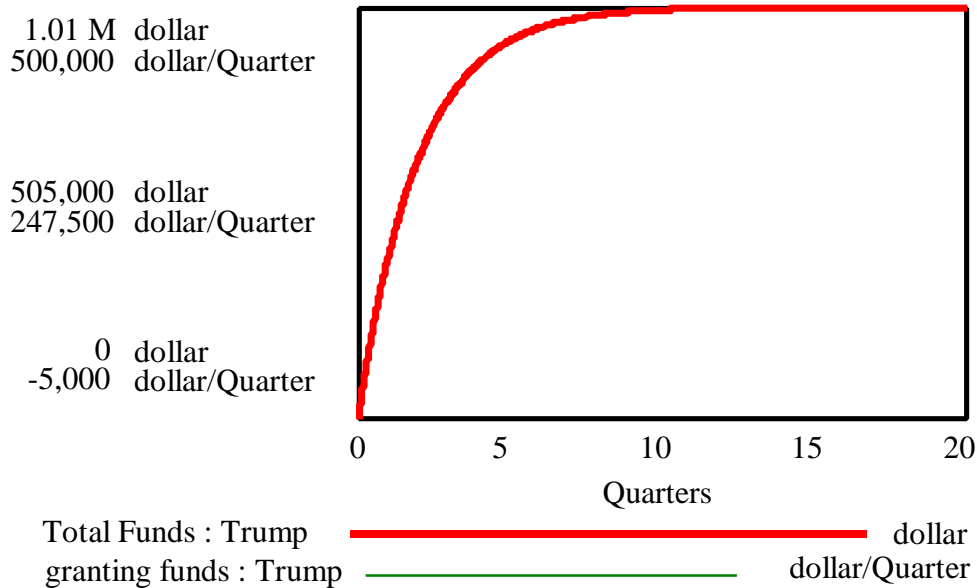
Trump's quarterly donation to the funds.

To mentally simulate the system, begin by calculating the equilibrium value. We will use the first formulation of the system (the second formulation yields identical results). For the stock “Total Funds” to be at equilibrium, its net flow, the sum of “Trump’s donation” and “granting funds” must be zero. Therefore, “Trump’s donation” must equal the negative of “granting funds.” “Trump’s donation” is constant at \$5,000 per quarter. The inflow “granting funds” equals one half of the “funds needed,” which equals the “DESIRED FUNDS” of \$1 million minus the current amount of “Total Funds.” Hence, “Total Funds” is at equilibrium when:

$$\begin{aligned} \text{Trump's donation} + \text{granting funds} &= 0 \\ \text{Trump's donation} &= - \text{granting funds} \\ \text{Trump's donation} &= - \text{funds needed} * \text{FRACTION GRANTED} \\ \text{Trump's donation} &= - (\text{DESIRED FUNDS} - \text{Total Funds}) * \text{FRACTION GRANTED} \\ \$5000 / \text{quarter} &= - (\$1,000,000 - \text{Total Funds}) * 0.5 / \text{quarter} \\ \$5000 &= - \$500,000 + (\text{Total Funds} * 0.5) \\ \text{Total Funds} &= \$505,000 * 2 \\ \text{Total Funds} &= \$1,010,000 \end{aligned}$$

Hence, the equilibrium value of “Total Funds” is \$1.01 million. Furthermore, at equilibrium, “granting funds” equals –\$5,000 per quarter. Something must be wrong. Indeed, simulating the model generates the following behavior:

### Total Funds with Trump's donation

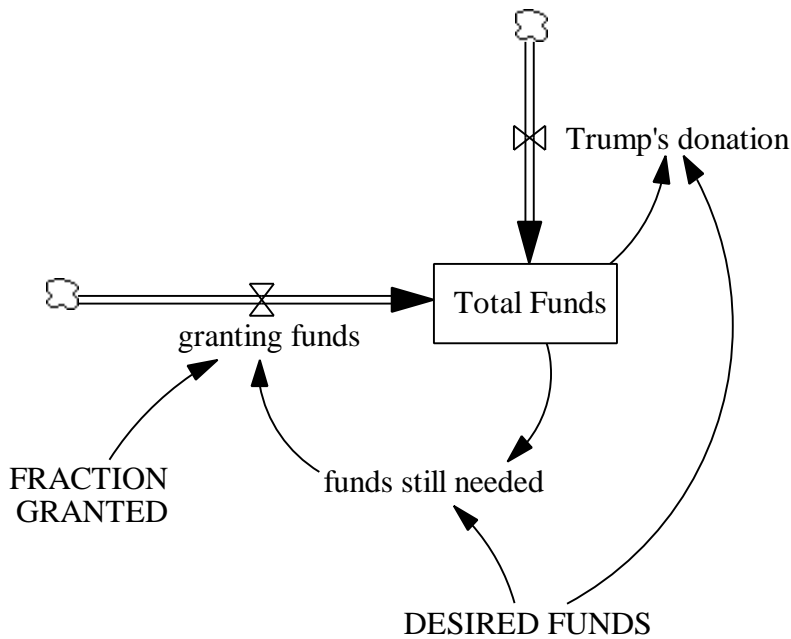


The results of the simulation make no sense when translated back to the system being modeled. Why would the city raise more money than it needs? The problem is that, with the present formulation of the model, even after the goal of “DESIRED FUNDS” is met,

“Trump’s donation” keeps coming. When “Total Funds” equals “DESIRED FUNDS,” the amount of “funds still needed” is zero, and the inflow “granting funds” is zero. The inflow “Trump’s donation,” however, continues to add \$5000 to the system every quarter. “Total Funds” thus becomes larger than “DESIRED FUNDS,” and the goal-gap process in the model begins to work backwards (negatively) as funds are removed from the stock of “Total Funds” in order to close the gap. This model is not a good representation of what would happen in the real-world system. To make the model more realistic, the flow “Trump’s donation” should be reformulated so that the donation stops once the goal has been reached.

The model can be changed as follows:

Model diagram:

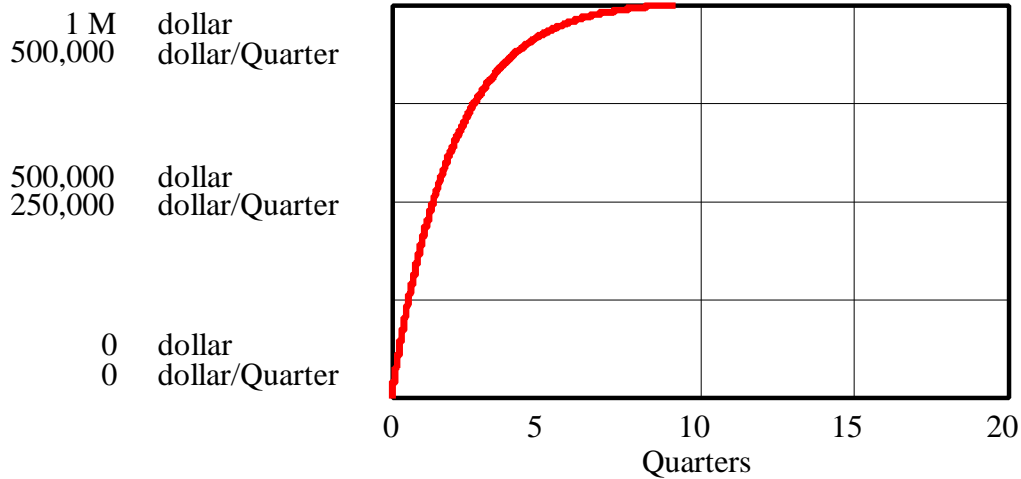


Modified model equations:

Trump's donation = IF THEN ELSE(Total Funds < DESIRED FUNDS, 5000, 0)  
 Units: dollar/Quarter  
 Trump’s quarterly donation to the total funds.

Model behavior:

### Trump's donation reformulated



Total Funds : Trump reformulated      —————      dollar  
 granting funds : Trump reformulated      —————      dollar/Quarter

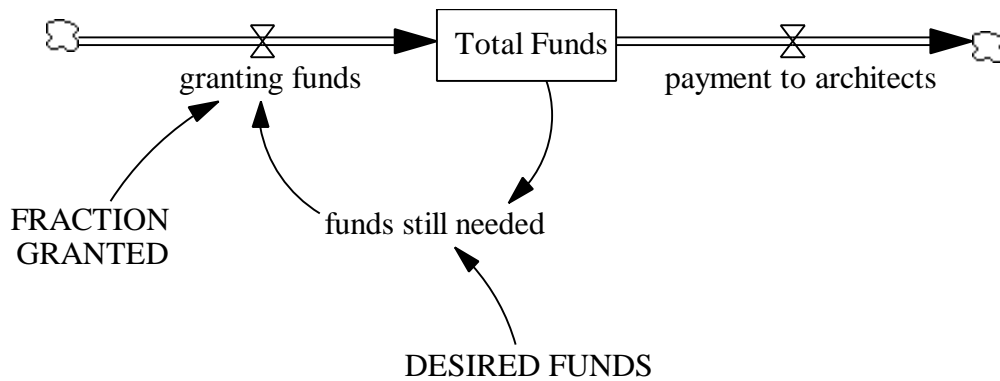
Note that because of the IF THEN ELSE formulation, the equilibrium values of the level of total funds and funds needed may be distorted by the time step. Reduce the time step to reduce the distortion. Also, note that we discourage the use of IF THEN ELSE and other such special function statements unless they are absolutely necessary and realistic. Often, table functions are a better way to model many situations, even though an IF THEN ELSE statement may seem okay. It is not often that discontinuities as implied by this statement are seen in the real world. Hence, be very aware of the implications of such statements.

Note that now the equilibrium value of the system is again the amount of desired funds. Note also that the curves approach the equilibrium slightly faster (it takes about 5.6 quarters instead of 6 quarters for “Total Funds” to reach 95% of their equilibrium value) than in the original model because of the additional inflow of “Trump’s donation.”

*B. Imagine that, despite Trump’s donation, you must pay the architecture firm \$10,000 per quarter to retain its services. Such an expense would make sure that the firm does not take on any other major projects that would interfere with the botanical garden, so that construction can commence as soon as you gather all necessary funding. How does this constant outflow of funds affect the equilibrium time and value of the system? Explain the behavior that you observe.*

For the sake of simplicity, consider the system in which Trump is broke. To take into account the quarterly payments to the architects, add a constant outflow of \$10,000 per quarter to the goal-gap inflow model from exercise 2 of assignment 8.

Model diagram:



Modified model equations:

payment to architects = 10000

Units: dollar/Quarter

The amount of money paid to architects every quarter to retain their services on the project.

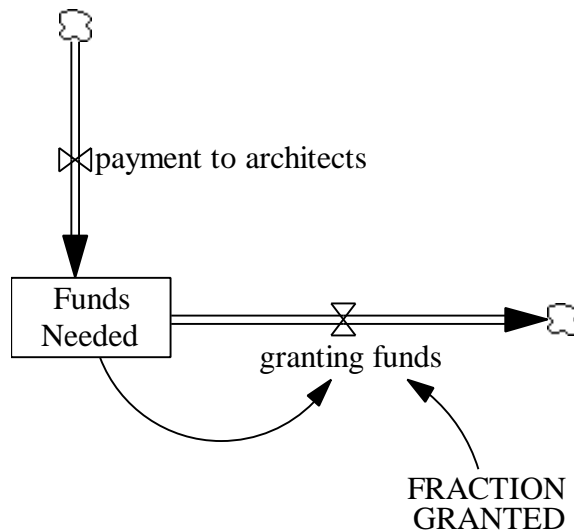
Total Funds = INTEG (granting funds - payment to architects, 0)

Units: dollar

The amount of money that has been granted for the project at any given time.

Alternatively, you can add a constant outflow of \$10,000 per quarter to the outflow decay structure from exercise 2 of assignment 8:

Model diagram:



Modified model equations:

Funds Needed = INTEG (-granting funds + payment to architects, 1e+006)

Units: dollar

The amount of funds that still needs to be collected.

payment to architects = 10000

Units: dollar/Quarter

The amount of money paid to architects per quarter to retain their services on the project.

First, calculate the equilibrium of the system. Take as an example the first formulation of the system. For the stock “Total Funds” to be at equilibrium, its net flow must be zero, so “granting funds” must equal “payment to architects.” The “payment to architects” outflow is constant at \$10,000 per quarter. The “granting funds” inflow equals one half of the “funds still needed,” which equals the difference between the “DESIRED FUNDS” of \$1 million and the current amount of “Total Funds.” Hence, “Total Funds” is at equilibrium when:

$$\text{payment to architects} - \text{granting funds} = 0$$

$$\text{payment to architects} = \text{granting funds}$$

$$\text{payment to architects} = \text{funds still needed} * \text{FRACTION GRANTED}$$

$$\text{payment to architects} = (\text{DESIRED FUNDS} - \text{Total Funds}) * \text{FRACTION GRANTED}$$

$$\$10,000/\text{quarter} = (\$1,000,000 - \text{Total Funds}) * 0.5 / \text{quarter}$$

$$\$10,000 = \$500,000 - \text{Total Funds} * 0.5$$

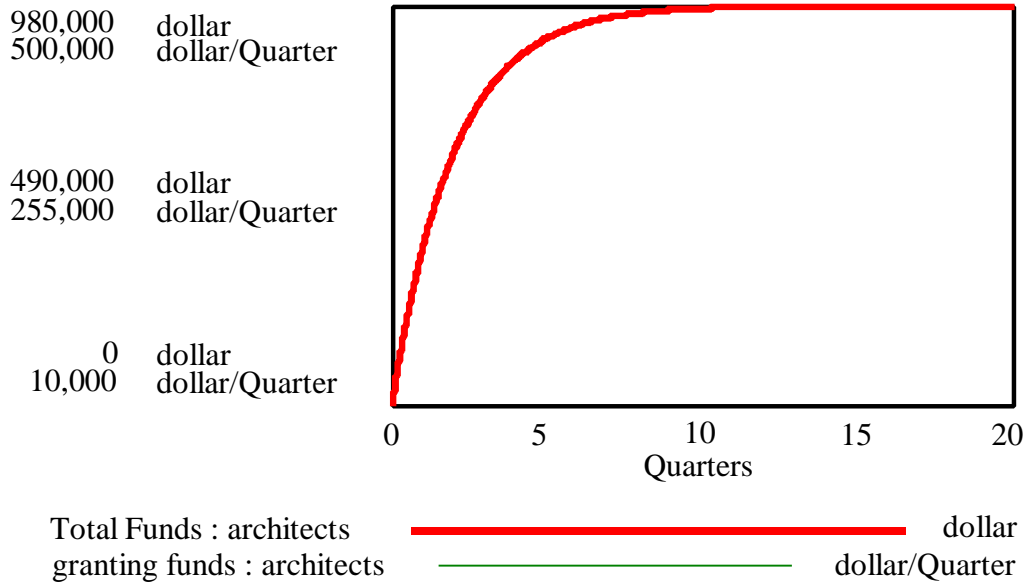
$$\text{Total Funds} = \$490,000 * 2$$

$$\text{Total Funds} = \$980,000$$

Hence the equilibrium value of “Total Funds” is \$980,000. Indeed, simulating the model generates the following behavior:



### Payments to architects



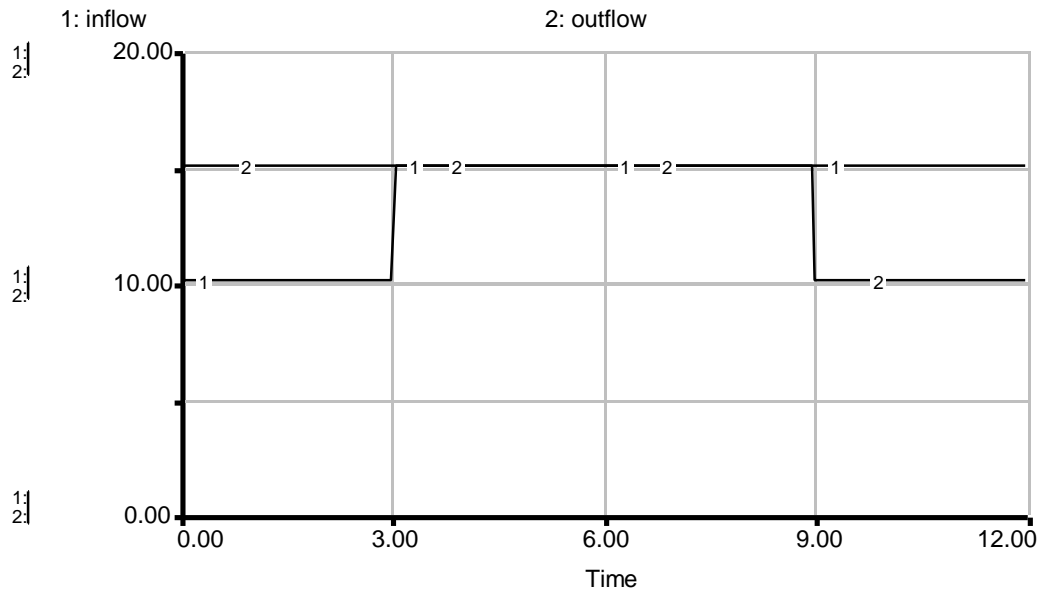
With the present formulation of the model, the project never receives enough funding. When the project needs only the final \$20,000 to be complete, half of that amount, \$10,000, is granted. All of the granted money then has to go to paying the architects their quarterly fee of \$10,000. In the next quarter, the “Total Funds” has not changed, so the process repeats itself. The project continues to raise funds that it hands over to the architects. The project misses its goal by \$20,000. Is this behavior realistic? In the real world, many projects do eventually reach completion, regardless of whether they require the service of architects. A more realistic model would contain a more complicated structure determining the payments due to the architecture firm. For example, you could calculate in advance the amount you expect to pay the architects, and incorporate the results into the desired funds amount.

#### 2. Graphical Integration Exercises Part Three: Combining Flows

Using the skills you acquired in “Graphical Integration Exercises Part Three,” complete the following exercises. For each exercise, first calculate the net flow, and then integrate the net flow. Use a graphics application to create the graphs of the net flow and stock behaviors, and then paste the graphs into your assignment solutions document.

A. Assume that the initial value of the stock is 0.

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*From time = 0 to time = 3, the inflow is constant at 10. At time = 3, the inflow steps up to 15 and remains at 15 until time = 12.*

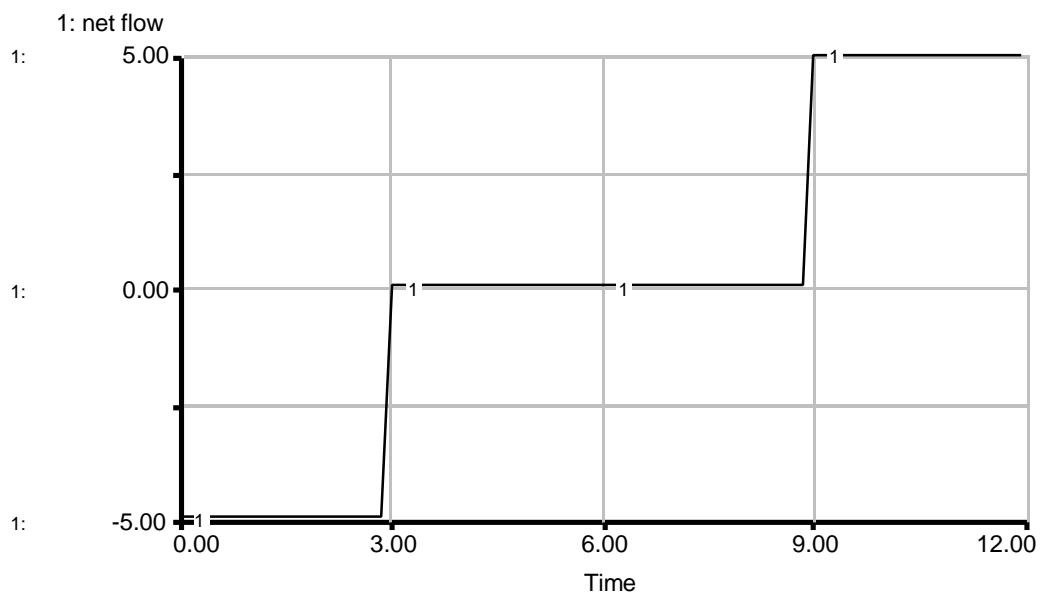
*The outflow is constant at 15 from time = 0 to time = 9. At time = 9, the outflow steps down to 10 and remains at 10 until time = 12.*

First calculate the net flow:

From time = 0 to time = 3, net flow = inflow – outflow = 10 – 15 = –5.

From time = 3 to time = 9, net flow = inflow – outflow = 15 – 15 = 0.

From time = 9 to time = 12, net flow = inflow – outflow = 15 – 10 = 5.

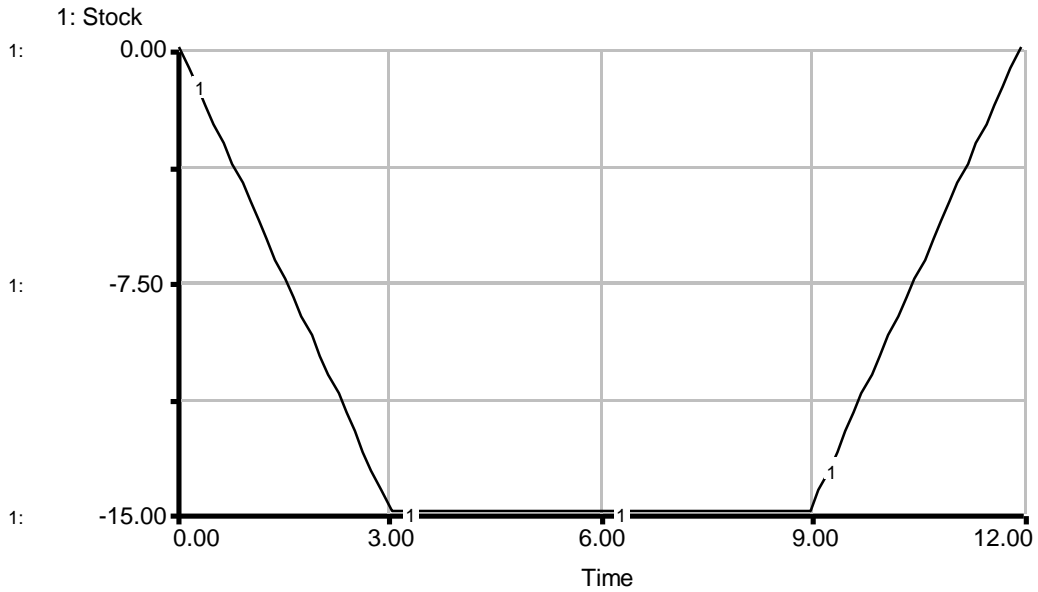


Now graphically integrate the net flow:

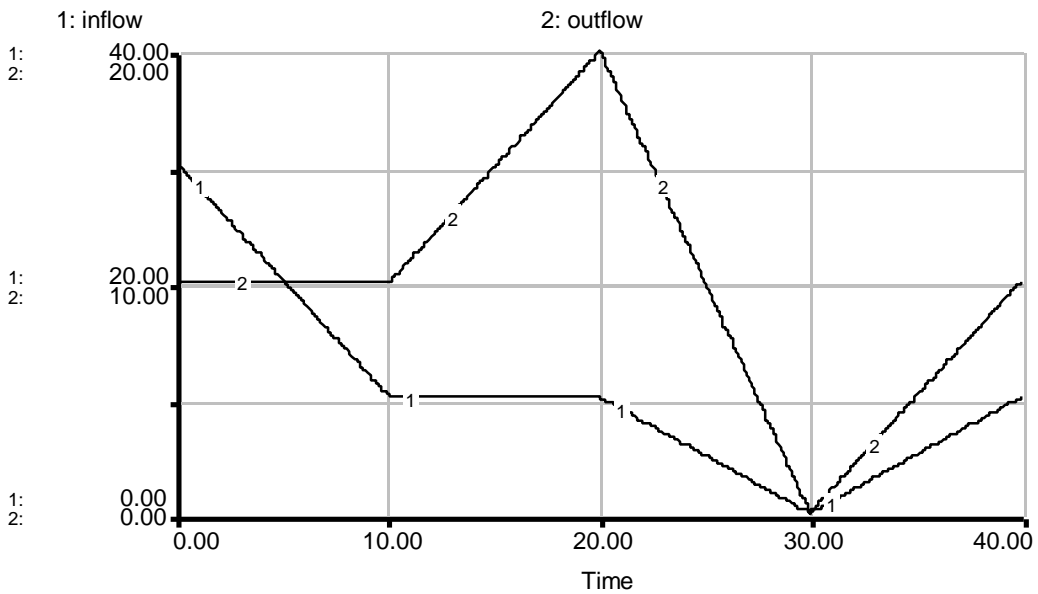
From time = 0 to time = 3, the area between the graph of the net flow and the zero flow line is  $(3 - 0) * -5 = -15$ , so the value of the stock decreases linearly with slope of  $-5$  by 15 units, to  $-15$  units.

From time = 3 to time = 9, the net flow is at 0, so the stock is not changing.

From time = 9 to time = 12, the area between the net flow graph and the zero flow line is  $(12 - 9) * 5 = 3 * 5 = 15$ . The value of the stock increases linearly with slope of 5 by 15 units, from  $-15$  units back to 0 units.



*B. Again assume that the initial value of the stock is 0.*



The inflow starts at 30. From time = 0 to time = 10, the inflow decreases linearly with slope of  $-2$ . From time = 10 to time = 20, the inflow remains constant at 10. From time = 20 to time = 30, the inflow decreases linearly with slope of  $-1$ . From time = 30 to time = 40, the inflow increases linearly with slope of  $+1$ .

From time = 0 to time = 10, the outflow is constant at 10. From time = 10 to time = 20, the outflow increases linearly with slope of  $+1$ . From time = 20 to time = 30, the outflow decreases linearly with slope of  $-2$ . From time = 30 to time = 40, the outflow increases linearly with slope of  $+1$ .

First calculate the net flow:

At time = 0, net flow = inflow – outflow =  $30 - 10 = 20$ .

At time = 10, net flow = inflow – outflow =  $10 - 20 = 0$ .

So from time = 0 to time = 10, the net flow is positive but decreases linearly with slope of  $-2$ .

At time = 20, net flow = inflow – outflow =  $10 - 20 = -10$ .

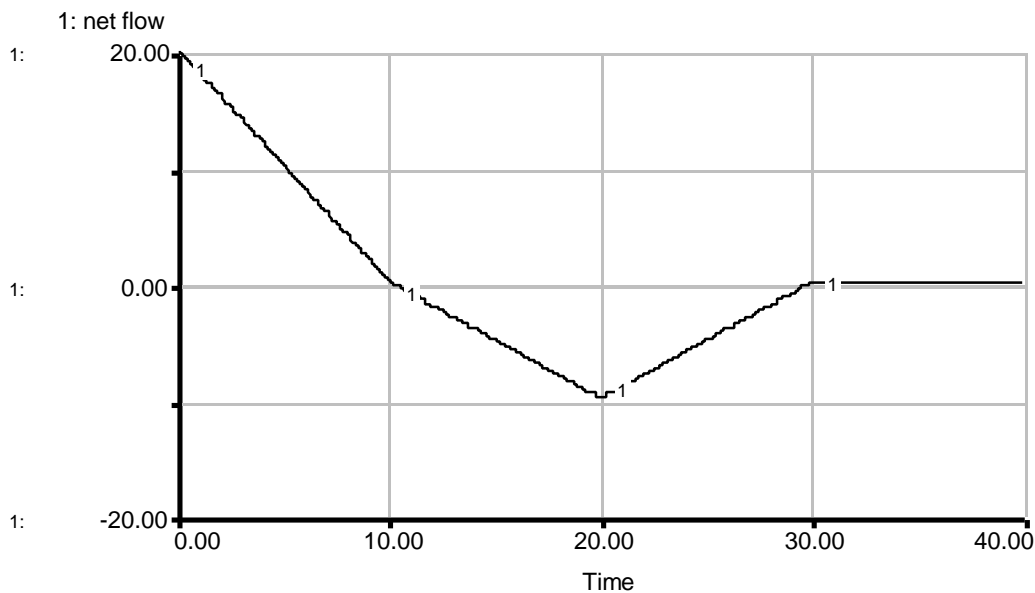
So from time = 10 to time = 20, the net flow is negative and decreases linearly with slope of  $-1$ .

At time = 30, net flow = inflow – outflow =  $0 - 0 = 0$ .

So from time = 20 to time = 30, the net flow is still negative but increases linearly with slope of  $+1$ .

At time = 40, net flow = inflow – outflow =  $10 - 10 = 0$ .

So from time = 30 to time = 40, the net flow remains constant at 0.



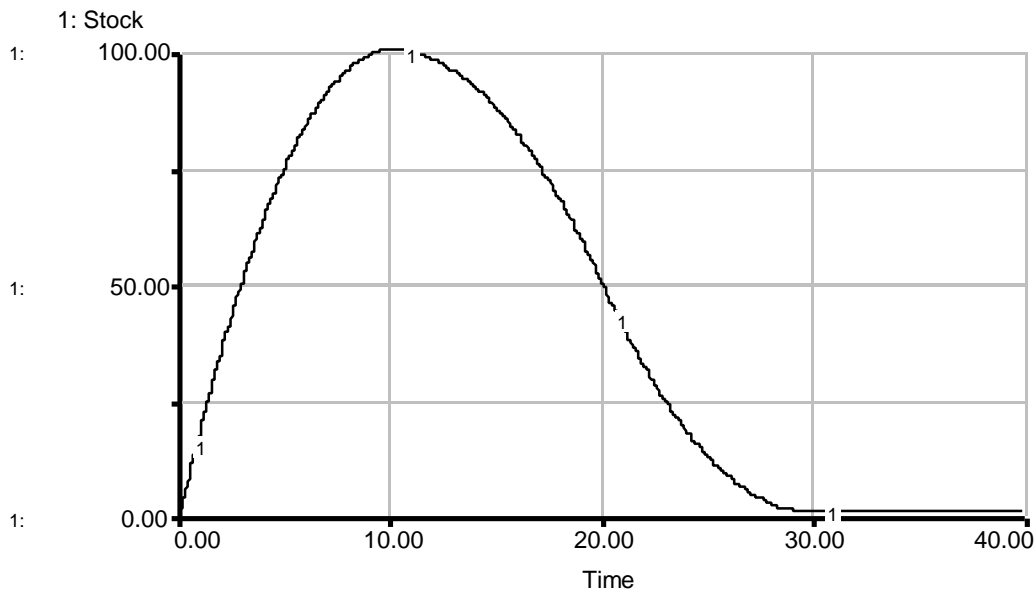
Now graphically integrate the net flow:

From time = 0 to time = 10, the area under the net flow graph is  $(10 - 0) * 20 / 2 = 100$ . The change in the value of the stock is thus 100 units, and the stock increases to 100 units. Because the value of the net flow decreases during this time period, the slope of the stock graph also decreases, and the stock exhibits “decreasing” parabolic growth.

From time = 10 to time = 20, the area between the net flow graph and the zero flow line is  $(20 - 10) * (-10) / 2 = -50$ . The value of the stock therefore decreases by 50 units, to 50 units. Because the absolute value of the net flow increases during this time period, the absolute value of the slope of the stock graph also increases, and the stock exhibits “decreasing” parabolic behavior.

From time = 20 to time = 30, the area between the net flow graph and the zero flow line is  $(30 - 20) * (-10) / 2 = -50$ , so the value of the stock decreases by 50 units, to 0 units. Because the absolute value of the net flow decreases during this time period, the absolute value of the slope of the stock graph also decreases, and the stock exhibits parabolic behavior.

From time = 30 to time = 40, the net flow is at 0, so the stock is not changing.



### 3. Problems with Causal Loop Diagrams

*Please read this paper carefully. You do not need to answer any questions about the paper, but if you can think of an instance when you had similar problems with causal-loop diagrams, feel free to share the experience with us.*

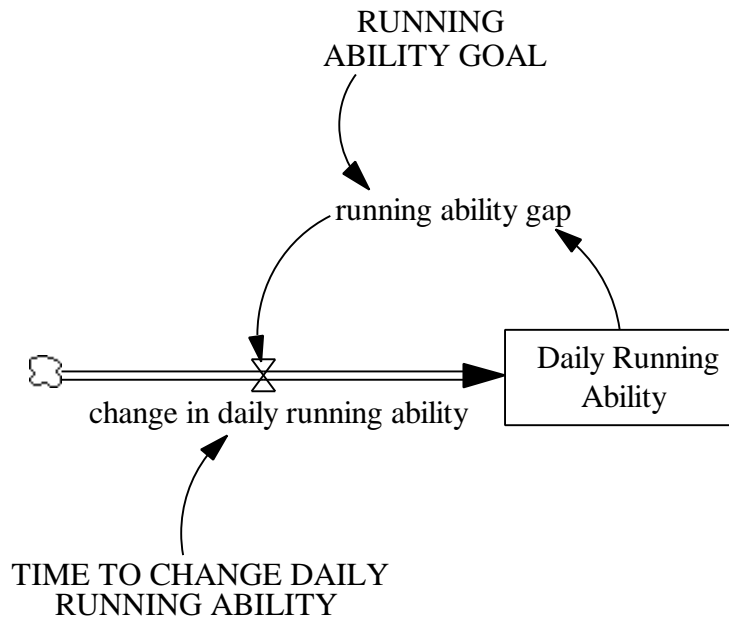
4. Independent Modeling Exercise: Eroding Goals

In assignment 7, you were asked to give examples of systems in which goals are constantly readjusted to match actual performance, creating behavior known as “eroding goals.” This exercise will develop a model of such systems.

A. I want to improve my fitness by running every day. I would like to be able to run three miles every day, but right now, I can only run one mile. Therefore, I decided that over a period of two weeks, I should increase the number of miles I can run to make my current running ability equal to my running goal.

Start building the model with a simple goal-gap structure. In your assignment solutions document, include the model diagram, documented equations, and a graph of model behavior. Explain the behavior that you observe.

Model diagram:



Model equations:

change in daily running ability = running ability gap / TIME TO CHANGE DAILY RUNNING ABILITY  
 Units: mile/day/Week  
 The weekly change in the number of miles that I can run every day.

Daily Running Ability = INTEG (change in daily running ability, 1)  
 Units: mile/day  
 The actual number of miles that I am able to run every day.

running ability gap = RUNNING ABILITY GOAL - Daily Running Ability  
 Units: mile/day

The gap between my desired running ability and my actual running ability.

RUNNING ABILITY GOAL = 3

Units: mile/day

The number of miles that I would like to be able to run every day.

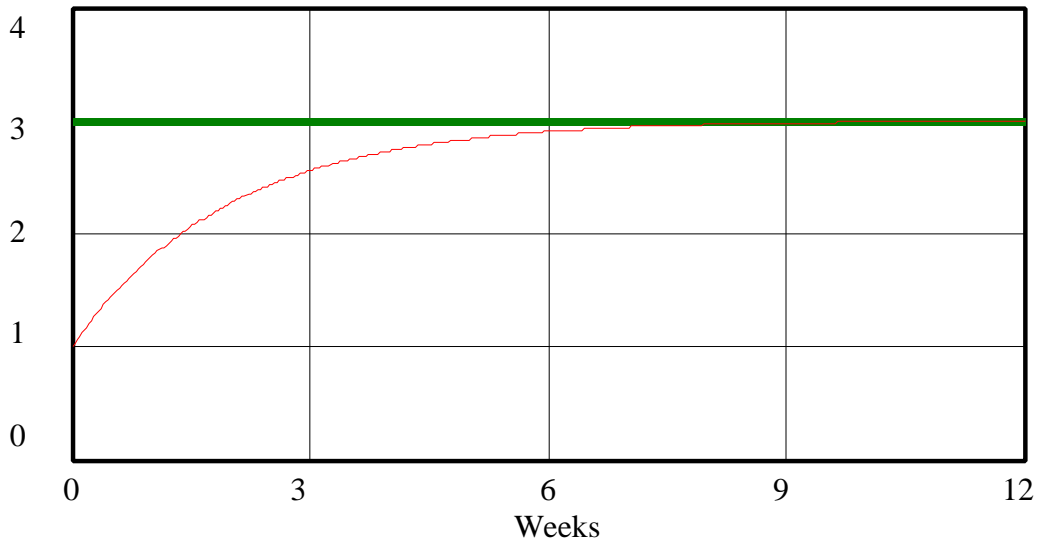
TIME TO CHANGE DAILY RUNNING ABILITY = 2

Units: Week

The time it takes me to change my running ability.

Model behavior:

### My running ability



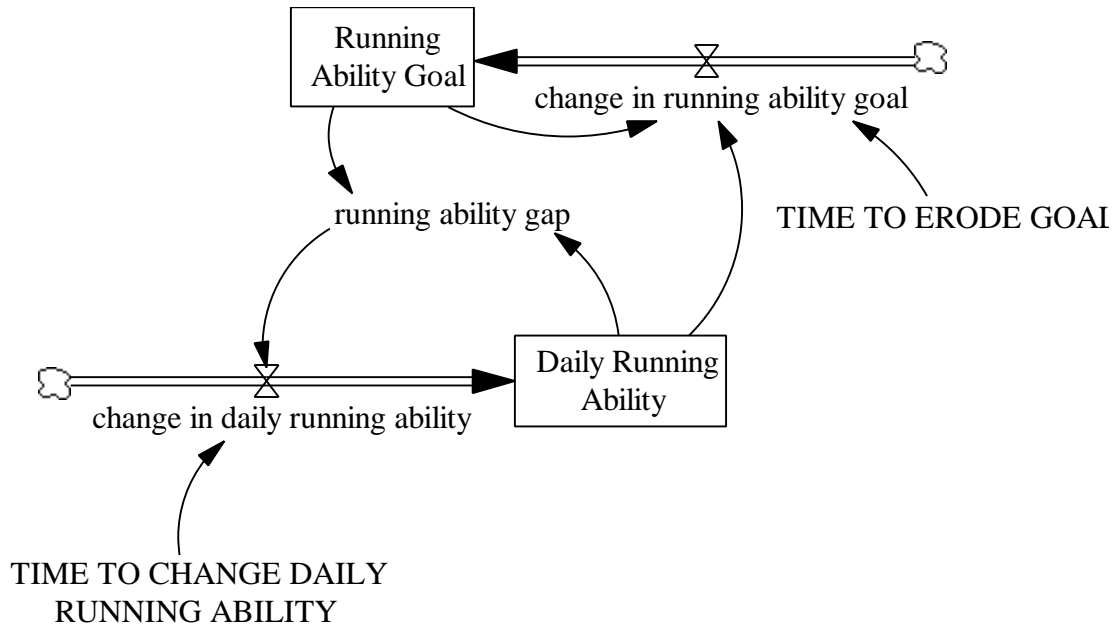
Daily Running Ability : running ———— mile/day  
 RUNNING ABILITY GOAL : running ————— mile/day

The system behaves as should be expected: because my “Daily Running Ability” is below my “RUNNING ABILITY GOAL,” I increase my daily running to close the “running ability gap.” My “Daily Running Ability” thus increases asymptotically towards the “RUNNING ABILITY GOAL.”

*B. Soon enough, however, I realize that I don’t like to lag behind my goal for so long. Hence, I change my goal for running, based on the difference between my current running ability and my current goal, over a period of four weeks.*

*Modify the model from part A to represent this “eroding goal,” and simulate the model. In your assignment solutions document, include the new model diagram, documented equations, and a graph of model behavior. Explain the behavior that you observe.*

*Hint: What kind of feedback is there between my current running ability and the goal?*

Model diagram:Model equations:

change in daily running ability = running ability gap / TIME TO CHANGE DAILY RUNNING ABILITY

Units: mile/day/Week

The weekly change in the number of miles that I can run every day.

change in running ability goal = (Daily Running Ability - Running Ability Goal) / TIME TO ERODE GOAL

Units: mile/(day\*Week)

The number of miles per day by which I change my running ability goal every week.

Daily Running Ability = INTEG (change in daily running ability, 1)

Units: mile/day

The actual number of miles that I am able to run every day.

running ability gap = Running Ability Goal - Daily Running Ability

Units: mile/day

The gap between my desired running ability and my actual running ability.

Running Ability Goal = INTEG (change in running ability goal, 3)

Units: mile/day

The number of miles that I would like to be able to run every day.

TIME TO CHANGE DAILY RUNNING ABILITY = 2

Units: Week



The time it takes me to change my running ability.

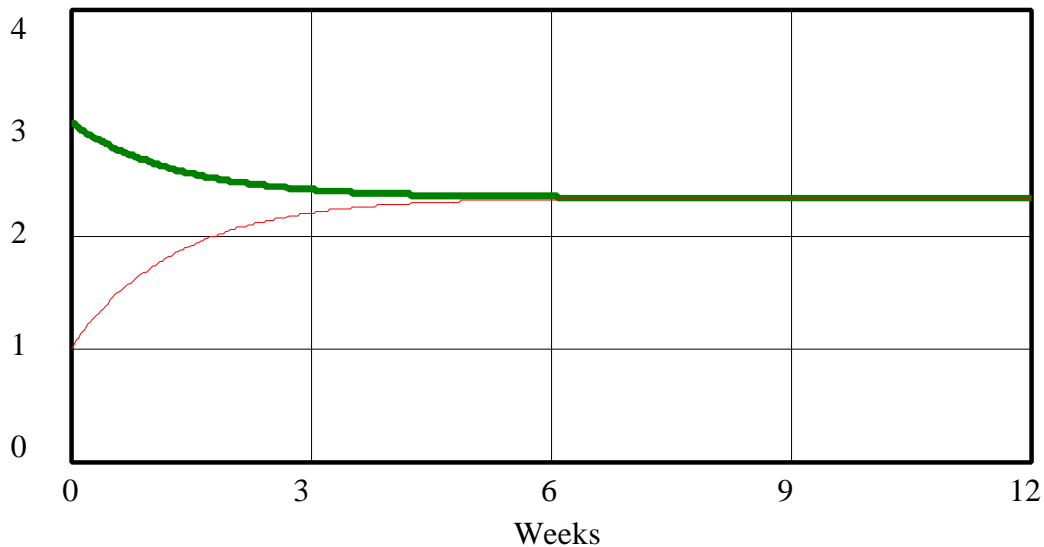
TIME TO ERODE GOAL = 4

Units: Week

The time it takes me to adjust my running ability goal towards my actual running ability.

Model behavior:

### My running ability



Daily Running Ability : eroding goal      ————      mile/day  
Running Ability Goal : eroding goal      —————      mile/day

As before, my “Daily Running Ability” is initially below my “Running Ability Goal,” so I start increasing my “Daily Running Ability.” At the same time, however, because I am not satisfied with being so far from my “Running Ability Goal,” I start adjusting the goal downwards based on the difference between my current “Daily Running Ability” and the “Running Ability Goal,” thus creating an additional negative-feedback loop. The closer my “Daily Running Ability” and my “Running Ability Goal” become, the slower I adjust them, until finally my “Daily Running Ability” reaches my “Running Ability Goal.” Notice that I reach my goal earlier than in the original simulation with a constant goal, but my “Daily Running Ability” is lower.