

Guided Study Program in System Dynamics

System Dynamics in Education Project

System Dynamics Group

MIT Sloan School of Management

Solutions to Assignment #3
Wednesday, October 14, 1998

Reading Assignment:

Read the following sections from “Vensim PLE: User’s Guide, Version 3.0”:

- Section 2: *The Vensim PLE User Interface*
- Section 6: *Building a Simulation Model*

The Vensim PLE User’s Guide can be downloaded in pdf format from:

<http://www.vensim.com/download.html>

Please refer to Road Maps 2: A Guide to Learning System Dynamics (D-4502-4) and read the following papers from Road Maps 2:

- *The First Step (D-4694)*
- *Beginner Modeling Exercises (D-4347-6)*

Exercises:

In this assignment, you will build several models using Vensim PLE. The purpose of the assignment, however, is not only to increase your familiarity with Vensim PLE, but also to demonstrate some important ideas and principles of system dynamics.

1. The First Step

A. The First Step (D-4694) was written using the STELLA II simulation software, so you will need to make the necessary adjustments for Vensim PLE. Because all papers in Road Maps use STELLA II, you should make sure you know how to make the conversion between STELLA II and Vensim PLE models and equations. We recommend that you take the time now to read section 2 of the Vensim PLE User’s Guide: The Vensim PLE User Interface (pp. 7-14). The section reviews the main features of the software.

B. Then turn to section 6 of the *Vensim PLE User's Guide: Building a Simulation Model* (pp. 53-64). Section 6 builds a population model similar to that presented in *The First Step*, using *Vensim PLE*. Browse through *Building a Simulation Model*. Refer back to the chapter if you have any problems making the models described in *The First Step*. If you still are having difficulties, please contact us as soon as possible.

C. Please read *The First Step*, building all of the models and running all of the simulations as you go along. In your assignment solutions document, please include the following:

- the model diagram as in Figure 17 and the model equations
- the graph of the model behavior as in Figures 21, 23, 26, and 27
- the model diagram as in Figure 32 and the model equations
- the graph of the model behavior as in Figures 33 and 35

The solutions to this exercise are included in *The First Step*.

2. Beginner Modeling Exercises

A. Please read *Beginner Modeling Exercises (D-4347-6)*, answer all the questions and build all the models in *Vensim PLE*.

In your assignment solutions document, please include the following:

- for either the skunk population model or the landfills model, include the model diagram, the model equations, and a graph of the model behavior
- choose two of the remaining 12 models and include the model diagram, the model equations, and a graph of the model behavior for each of the two models

The solutions to this part are included in *Beginner Modeling Exercises*.

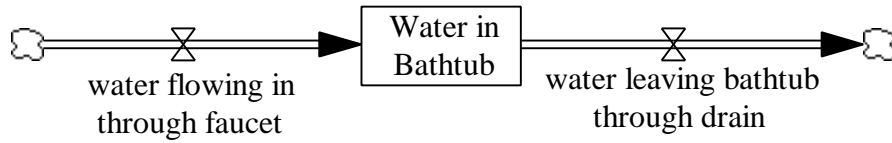
B. Think of two other simple systems that one could model as a stock with an inflow and/or outflow. For each of them, please include the following:

- short description of the system
- list of variables and their identification as stocks or flows
- model diagram
- model equations, including documentation and units
- graph of model behavior

The first system I've decided to model is a bathtub. The system includes water flowing into the tub from a faucet and water leaving the tub through a drain. We assume the tub is initially empty, that water flows in through the faucet at the rate of three gallons per minute, and that water leaves the tub at the rate of two gallons per minute. How much water will be in the tub after one hour? The variables for this system include:

water flowing in through faucet (flow), water in bathtub (stock), and water leaving bathtub through drain (flow).

Model diagram:



Model equations:

water flowing in through faucet = 3

Units: gallon/minute

The rate at which water flows into the bathtub through the faucet.

Water in Bathtub = INTEG (water flowing in through faucet - water leaving bathtub through drain, 0)

Units: gallon

The initial amount of water in the bathtub.

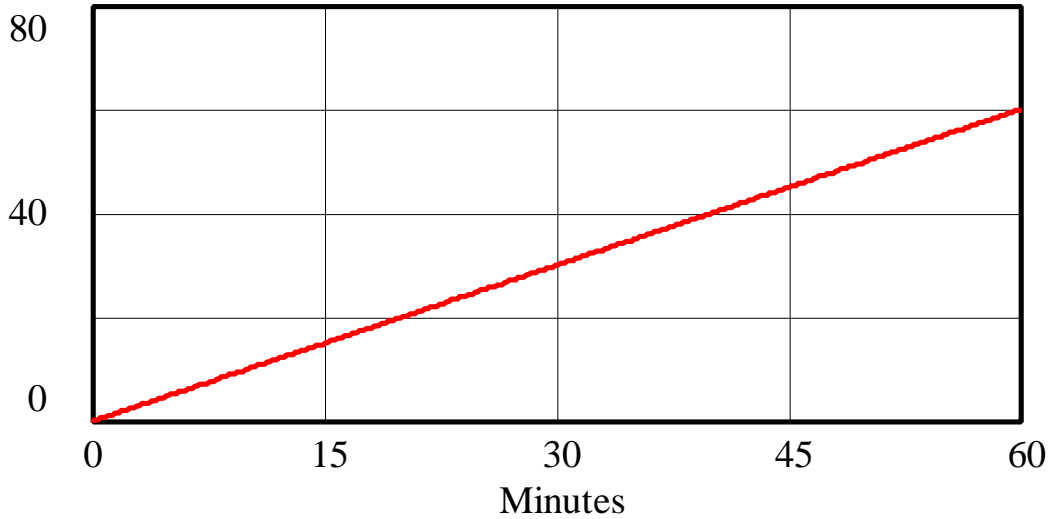
water leaving bathtub through drain = 2

Units: gallon/minute

The rate at which water leaves the bathtub through the drain.

Model behavior:

Water in Bathtub



Water in Bathtub : bathtub ————— gallon

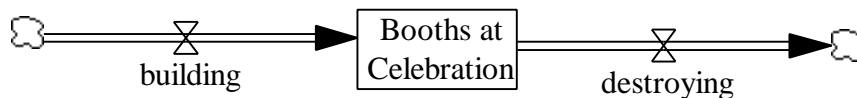
After one hour, there will be 60 gallons of water in the bathtub.

At the Greenwood County annual Wellness Celebration, 235 booths are needed on the downtown streets to house all the groups participating. 120 people begin an around the clock assembly of these booths at 10PM on the evening before the festival. They can assemble the booths at the rate of 75 booths in 2 hours. Unfortunately, a windstorm comes through and destroys the booths at a rate of 21 per hour. Will the booths be ready for the festival to begin at 9AM?

List of variables and their identification as stocks and flows:

Booths	Stock
building	Flow
destroying	Flow

Model diagram:



Model equations:

Booths at Celebration = INTEG (building - destroying, 0)

Units: booth

The number of booths that were up when people arrived to begin building.

building = 37.5

Units: booth/Hour

The rate at which booths can be assembled.

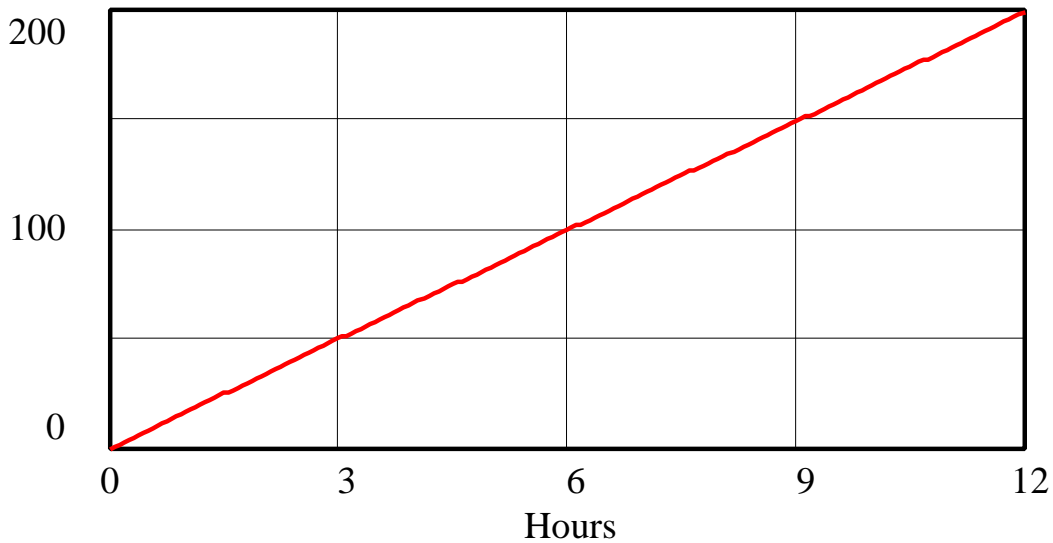
destroying = 21

Units: booth/Hour

The rate at which booths were destroyed by the wind.

Model behavior:

Booths at Celebration



Booths at Celebration : booths ————— booth

Clearly, at 9AM (that is, 11 hours after the start of the simulation), there will be fewer than 200 booths assembled, so not all 235 booths needed will be ready by the time the festival begins.

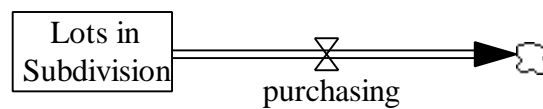
There were 423 lots to be sold in our subdivision when it opened in 1990. Between 1990 and 1992, a total of 53 lots were sold. Since that time, 17 lots are being purchased each year for home construction. If this rate continues, how long will it be before the subdivision is completely filled?

List of variables:

Lots in subdivision
purchasing

Stock
Flow

Model diagram:



Model equations:

Lots in Subdivision = INTEG (-purchasing, 370)

Units: lot

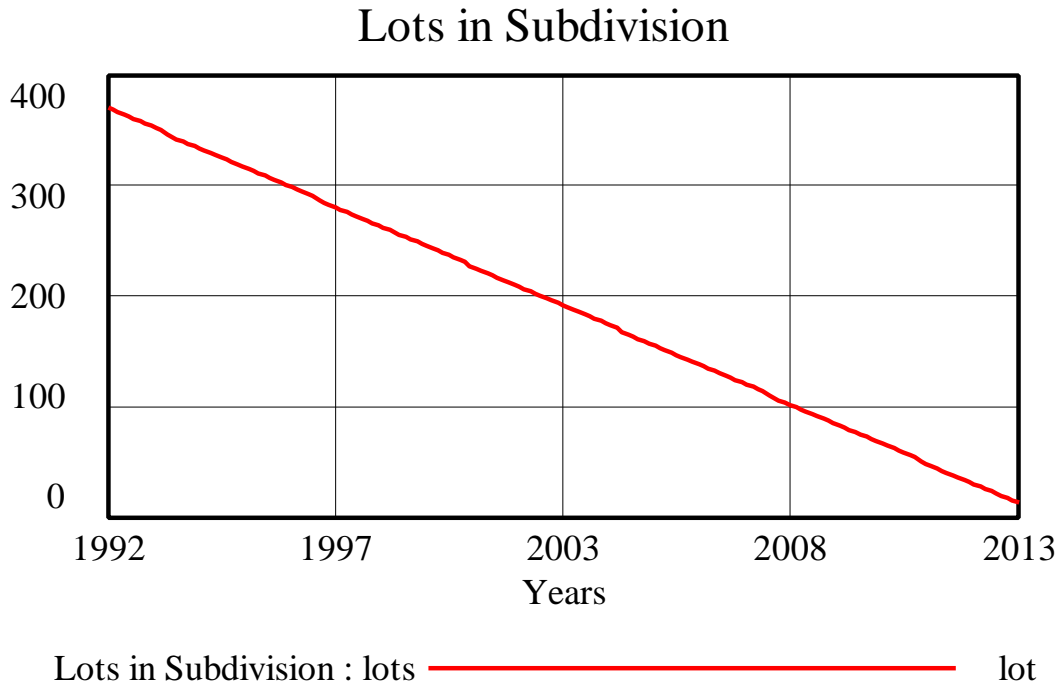
The number of lots available for purchase in 1992.

purchasing = 17

Units: lot/Year

The number of lots purchased for homes every year.

Model behavior:



The subdivision will be almost completely filled by the end of year 2013.

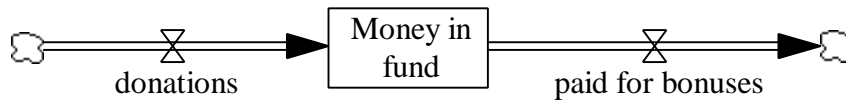
Money for Corruption

Money is placed into a special account to build up a fund to help poor people. Money from donors is added at the rate of \$5,000 per month. Corrupt officials, however, use some of the money to pay themselves “bonuses.” They take this money at the rate of \$1,800 per month. At the end of five years, how much money will be available for poor people? How much would there have been if the illegal bonuses were not paid?

Model Variables:

Money in fund	stock
donations	flow
paid for bonuses	flow

Model diagram:



Model equations:

donations = 5000

Units: dollar/Month

The amount of money received each month in donations.

Money in fund = INTEG (donations - paid for bonuses, 0)

Units: dollar

The amount of money that the fund starts with.

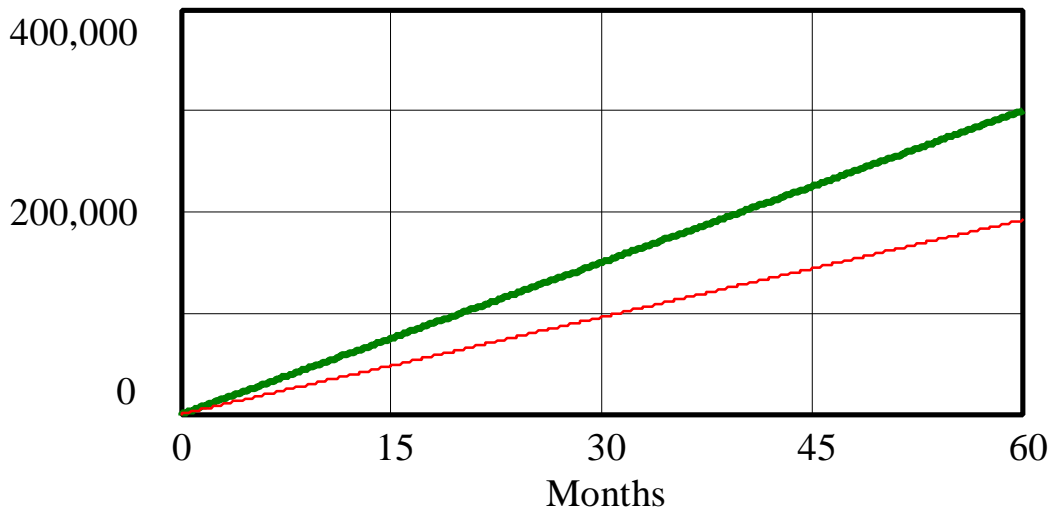
paid for bonuses = 1800

Units: dollar/Month

The amount of money that is stolen (as bonuses) from the account each month.

Model behavior:

Money in fund



Money in fund : money ————— dollar
 Money in fund : money - no theft ————— dollar

At the end of five years, there will be almost \$200,000 available for poor people. If the illegal bonuses were not paid, there would have been \$300,000 available.

3. Independent modeling exercise

The independent modeling exercise was the first in a series of exercises designed to give you practice in the conceptualization, formulation, testing, and evaluation of a system dynamics model.

A. Imagine a small forest of fir trees, located deep in the Pacific Northwest. The forest contains 1000 trees, half of which are still saplings. The other half have already grown to be tall, towering trees. Over the course of each year, the locals chop down, on average, 10 of the tall trees. Every year the third graders in Miss Pringle's class plant 10 young saplings in the forest and carefully watch them grow. Miss Pringle tells her class that it will take their saplings 50 years to grow to resemble the older trees.

This exercise will guide you through the steps to make a simple model of the forest, in order to understand how the distribution of trees in the forest changes over time. Our model will contain the elements listed below; identify each element as either stock, flow, or constant, and label its units. Then take each stock and determine its inflows and outflows.

- *mature trees*
- *planting*
- *time for saplings to mature*
- *saplings*
- *harvesting*
- *maturing*

Start the modeling process by focusing on a problem: understanding how the distribution of trees in a forest changes over time due to deforestation. Given that problem, make a list of important elements. Then identify those elements as being stocks, flows, or constants, and determine the inflows and outflows of each stock, as in the table below:

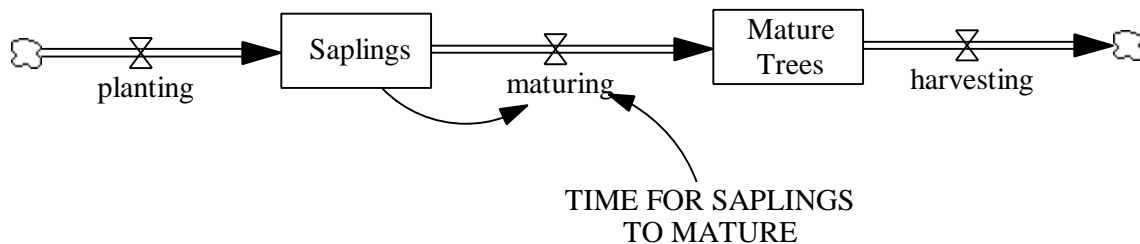
	Units	Type
mature trees	trees	stock
planting	trees/year	flow (into saplings)
time for saplings to mature	years	constant
saplings	trees	stock
harvesting	trees/year	flow (out of mature trees)
maturing	trees/year	flow (out of saplings into mature trees)

The inflow into the stock “Saplings” is “planting.” The outflow from “Saplings” is “maturing.”

The inflow into the stock “Mature Trees” is “maturing.” The outflow from “Mature Trees” is “harvesting.”

B. Using Vensim PLE, combine the elements to represent the structure of the system.

A model diagram of the system looks as follows:



C. Using the description of the system provided in part A, define the equations for each flow. Then supply the constants and initial values. Remember to document all equations. In your assignment solutions document, include the model diagram and documented equations. (Hint: If there are N saplings in a forest and saplings need M years to mature, then N/M of the saplings mature each year.)

Note that the equations for stocks are already set by the structure itself: the stocks are the integrals of their inflows minus their outflows. The initial values of the stocks can be obtained from the system description: “the forest has 1000 trees, half of which are still saplings.” Therefore, the initial values for both “Saplings” and “Mature Trees” are 500 trees each. The system description says that the third-graders plant ten saplings every year, so “planting” is a constant flow of 10 trees/year. The description also states that the locals chop down, on average, ten mature trees each year. So “harvesting” is a constant flow of 10 trees/year. The saplings take 50 years each to mature. So the “maturing” rate is the current number of “Saplings” divided by the 50 years that it takes for each sapling to mature. In other words, every year $1/50^{\text{th}}$ of the current number of “Saplings” mature.

The model equations are:

$$\text{harvesting} = 10$$

Units: trees/Year

The number of mature trees that the locals chop down each year.

$$\text{Mature Trees} = \text{INTEG}(\text{maturing} - \text{harvesting}, 500)$$

Units: trees

The number of mature trees in the forest.

$$\text{maturing} = \text{Saplings} / \text{TIME FOR SAPLINGS TO MATURE}$$

D-4715-1

Units: trees/Year

The average maturing rate is the number of saplings that are ready to mature each year.

planting = 10

Units: trees/Year

The number of saplings that Miss Pringle and her class plant each year.

Saplings = INTEG (planting - maturing, 500)

Units: trees

The number of saplings in the forest.

TIME FOR SAPLINGS TO MATURE = 50

Units: Year

After 50 years, a sapling becomes a mature tree.

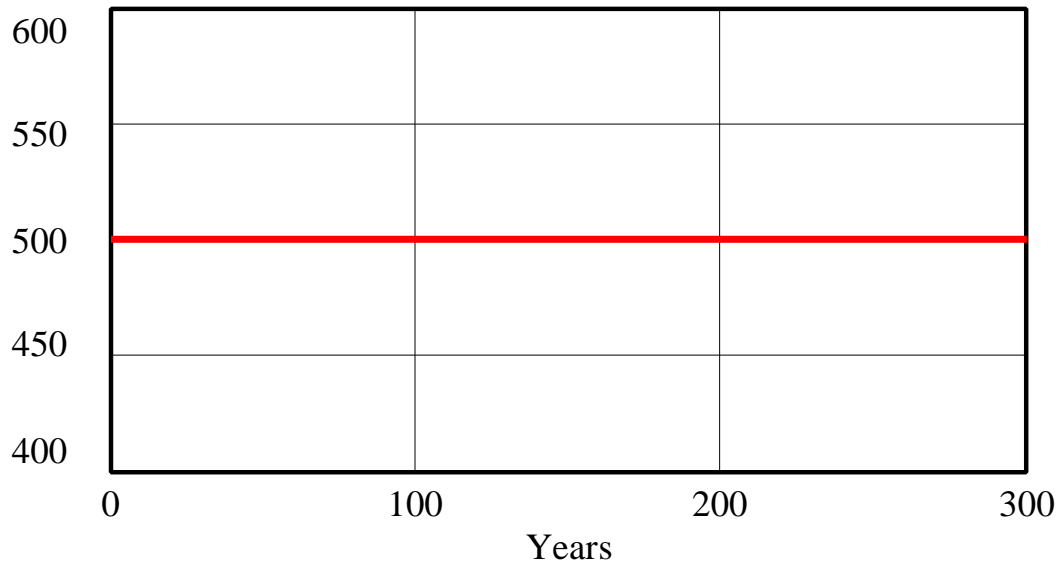
D. Before you simulate a model, it is often useful to first make a general sketch of the behavior that you expect to see. The best way to make the sketch is to draw a horizontal time axis and a vertical axis for the key variables (the stocks in the model, for example). Then trace what you expect would be the general behavior of each variable (growth, growth followed by a collapse, decay, oscillations, etc.). Give a general idea of the time horizon (will your model be valid over a period of hours, months, years, or centuries?). Identify any critical points (the peak before a collapse, or an inflection point between exponential growth and asymptotic growth, for instance), and try to approximate at which time they would occur. Such sketches are called “reference modes.” After you have drawn some reference modes, you can simulate the model. If the behavior that the model generates differs from the behavior you predicted, then you know that there must be an inconsistency in either the model you formulated or your mental model.

Draw reference modes for the stocks in the forest model. You do not need to include these in your assignment solutions document, but you do need to draw them in order to be able to answer the next question.

E. Simulate the model over 300 years. Include graphs of the behavior of all stocks in your assignment solutions document. Did the model generate the behavior you predicted? Why or why not?

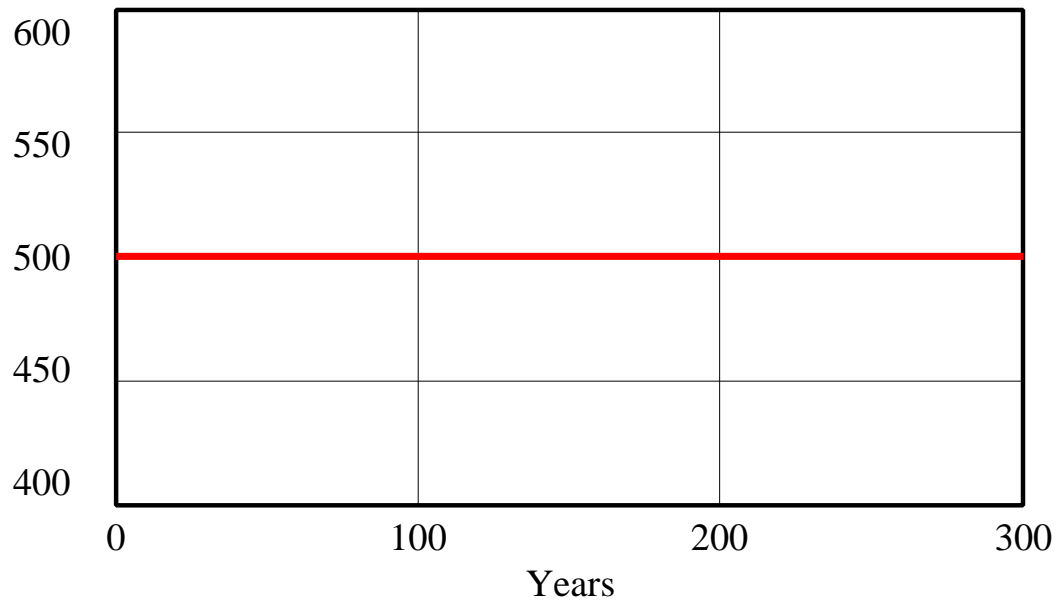
The model generated the following behavior:

Saplings - part E.



Saplings : trees - part E  trees

Mature Trees - part E.



Mature Trees : trees - part E  trees

Both stocks remain in equilibrium at their initial values during the 300 years of simulation. Such behavior should be expected in the reference modes because the inflow into each stock equals the outflow from the stock.

F. As more and more people move to the Pacific Northwest to be closer to nature, the demand for firewood increases. Now the locals chop down five more trees each year. Miss Pringle, the third grade teacher, decides to compensate by planting five more saplings each year.

Make the necessary changes in the forest model to reflect the new scenario. In your assignment solutions document, include the documented equations that you changed.

Before running a simulation of the new scenario, draw reference modes for each stock in the model. Then simulate the model. In your assignment solutions document, include graphs of the behavior of all stocks.

Did the model generate the behavior that you predicted? In a paragraph, explain which aspects of the forest system, reflected in the model, combined to produce the behavior observed in the new scenario. What lessons can you draw from these observations?

The new model equations are as follows:

harvesting = 15

Units: trees/Year

The number of mature trees that the locals chop down each year.

Mature Trees = INTEG (maturing - harvesting, 500)

Units: trees

The number of mature trees in the forest.

maturing = Saplings/TIME FOR SAPLINGS TO MATURE

Units: trees/Year

The average maturing rate is the number of saplings that are ready to mature each year.

planting = 15

Units: trees/Year

The number of saplings that Miss Pringle and her class plant each year.

Saplings = INTEG (planting - maturing, 500)

Units: trees

The number of saplings in the forest.

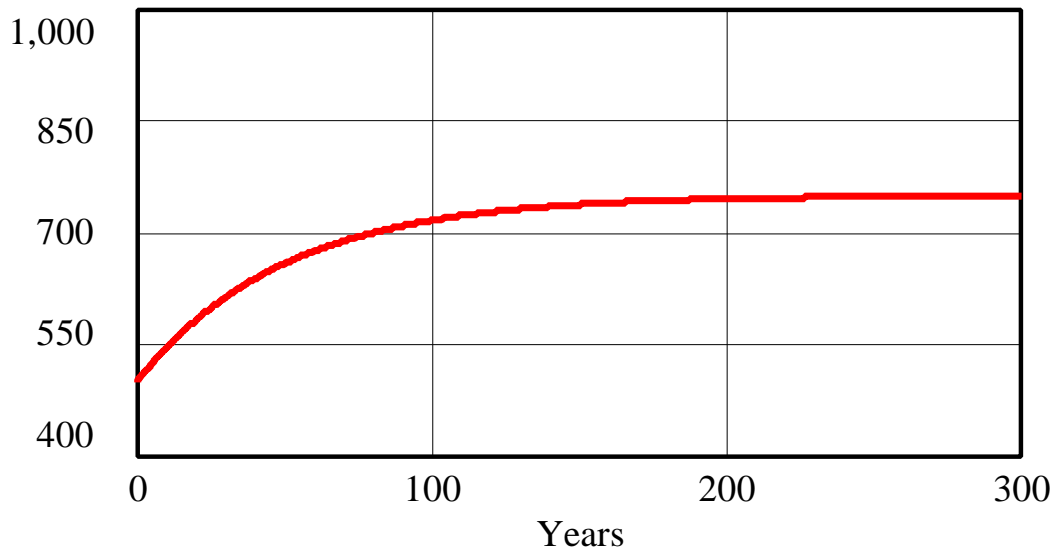
TIME FOR SAPLINGS TO MATURE = 50

Units: Year

After 50 years, a sapling becomes a mature tree.

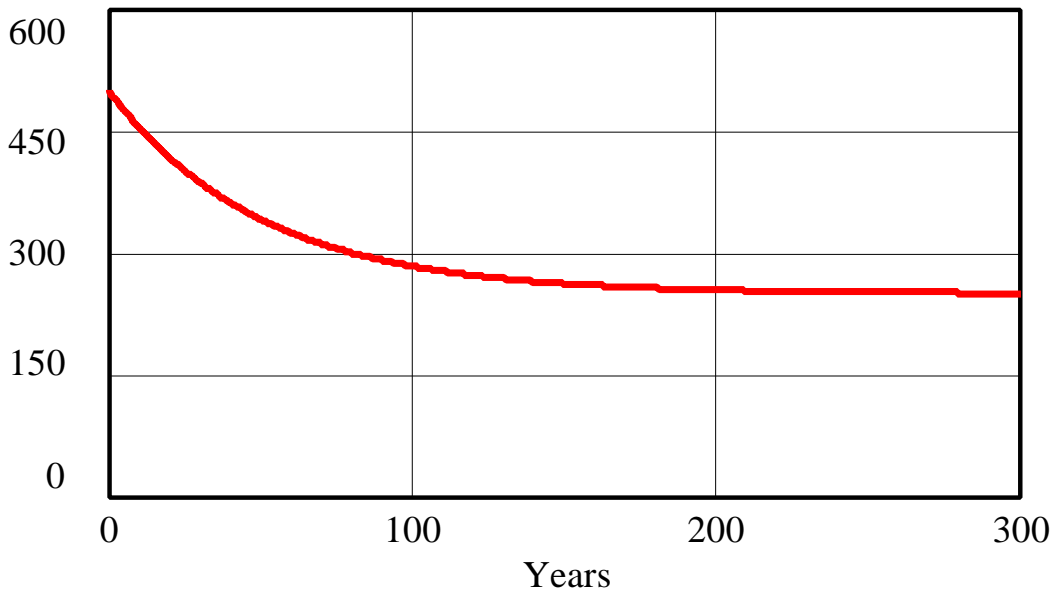
The new model generated the following behavior:

Saplings - part F.



Saplings : trees - part F ————— trees

Mature Trees - part F.

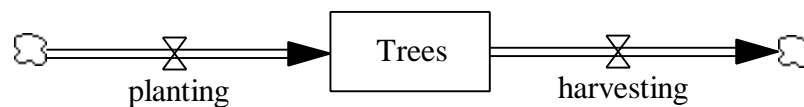


Mature Trees : trees - part F ————— trees

In this scenario, the system is no longer in equilibrium: the number of “Saplings” grows asymptotically, and the number of “Mature Trees” declines asymptotically. The asymptotic growth of “Saplings” occurs because initially, the “planting” rate is higher than the “maturing” rate. On the other hand, the number of “Mature Trees” declines because initially the rate of “maturing” is lower than the “harvesting” rate. The delay in the system, the average of 50 years that it takes for a sapling to become a mature tree ready to be harvested, causes the locals to harvest trees faster than the saplings can mature, so the forest ends with “too many” saplings and “too few” mature trees.

We are now going to look at the deforestation model in more detail. Do not feel intimidated if you do not understand everything that you read; the concepts will be revisited again and again in Road Maps and in later assignments.

A stock is at equilibrium when its inflows equal its outflows. A net flow into or out of a stock changes the value of the stock. The net flow is defined as the sum of all inflows minus the sum of all outflows. When the sum of all inflows equals the sum of all outflows, the net flow is zero, and the stock does not change. Take the deforestation model in an aggregate form. The stock of trees includes both saplings and mature trees. The total number of trees is increased by a constant planting rate and decreased by a constant harvesting rate.

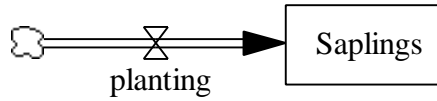


Then whenever the “planting” rate equals the “harvesting” rate, the number of trees taken out of the forest by the locals is put back in by Miss Pringle’s class, and the total number of “Trees” does not change. In both scenarios examined in this exercise, “planting” equals “harvesting,” so the total number of “Trees” does not change.

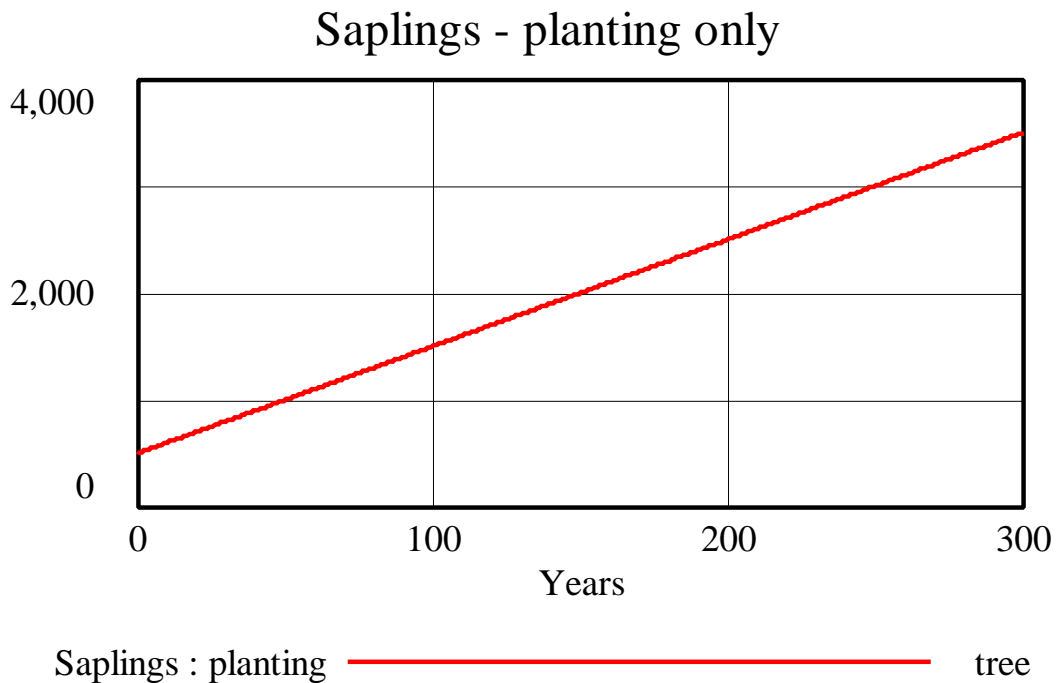
The above model is incomplete, however, because although the total number of “Trees” does not change when the “planting” rate equals the “harvesting” rate, the distribution of trees between “Saplings” and “Mature Trees” will change given different “planting” and “harvesting” rates. Imagine a scenario where the locals chop down all 500 mature trees in one year, and a very upset Miss Pringle spends the entire year planting 500 new saplings. Then the forest would still contain 1000 trees but, after the first year, they would all be saplings.

A disaggregated model of the forest that distinguishes between saplings and mature trees reveals the delay in the system: the 50 years that it takes for saplings to mature.

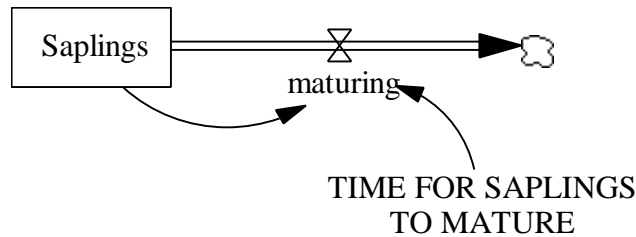
To understand why the first scenario produced an equilibrium, let’s break the model up into parts. First, take the constant “planting” inflow into the stock of “Saplings”:



Build the above structure in Vensim. Set the initial value of “Saplings” to be 500 trees and set the “planting” rate to be 10 trees per year. Simulate the model and graph the number of “Saplings” over a period of 300 years. The model should generate the following behavior, with the number of “Saplings” increasing by 10 every year:

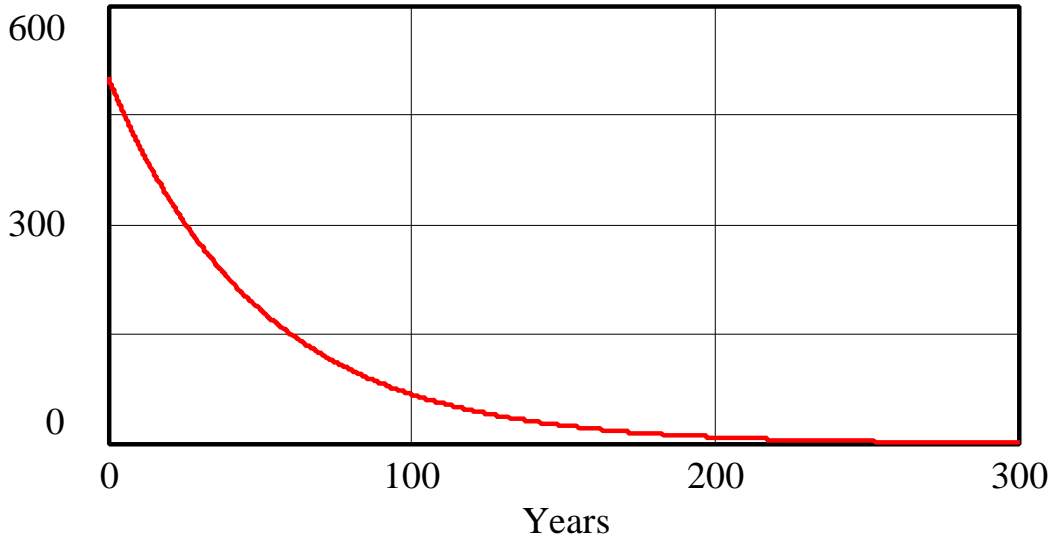


Now take the outflow in isolation. Delete the inflow and build up the “maturing” rate. Set the “maturing” rate to be the number of “Saplings” divided by the “TIME FOR SAPLINGS TO MATURE,” equal to 50 years.



Can you estimate how the number of “Saplings” will change over time? Simulate the model over a period of 300 years. The model should generate the following behavior:

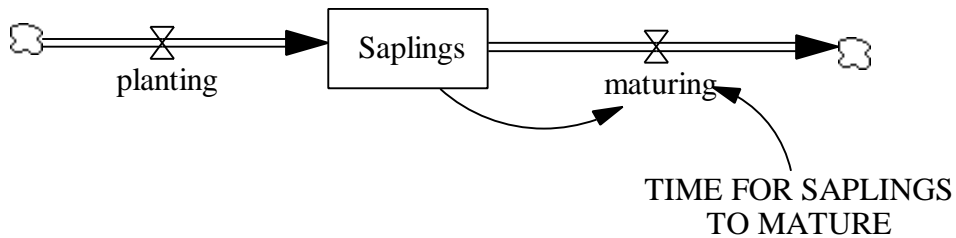
Saplings - maturing only



Saplings : maturing ————— tree

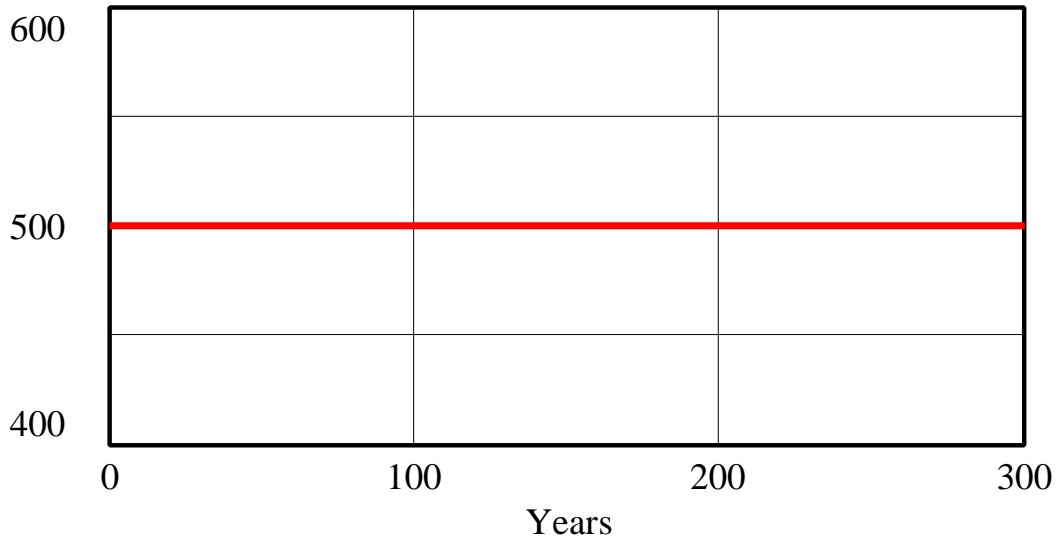
The process of “Saplings” becoming mature is simple negative feedback that generates asymptotic decline. As time goes on, the number of “Saplings” decreases, so the rate of “maturing” slows down, extracting less and less from the stock of “Saplings” until no “Saplings” remain—they have all become mature.

To analyze what happens when the model contains both the inflow and the outflow to the stock of “Saplings,” add the inflow “planting” back to the model and set it equal to 10 trees per year again:



Simulate the model over 300 years. The model should generate the following behavior:

Saplings - both flows



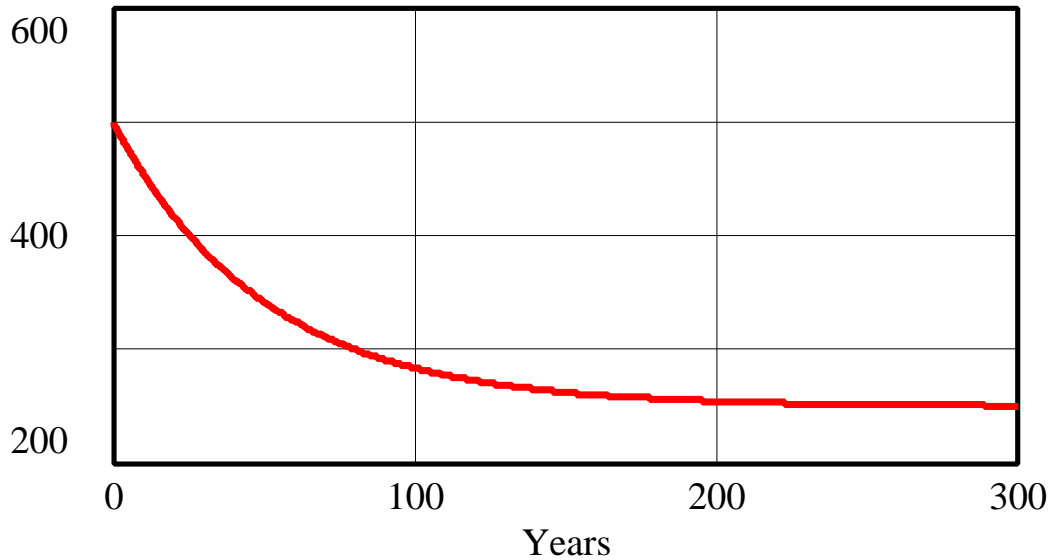
Saplings : both flows  tree

The behavior of the model seems counter-intuitive because when you look at each part of the model separately, as time goes on “maturing” decreases, extracting less and less from the stock, while “planting” always increases the stock by the same amount.

To understand why the model remains in equilibrium, simulate the model in your mind. In the first time period, ten trees are planted. In the same time period, the “maturing” rate is 500 trees divided by 50 years, or 10 trees per year. So the number of trees taken out of the stock is put back in, and the number of “Saplings” does not change. During the next time period, there are still 500 “Saplings,” so again ten trees mature, balancing the ten trees planted, and again the number of “Saplings” stays constant. The behavior during the third time period will therefore be the same again, and the number of “Saplings” remains at equilibrium.

If the rate of “planting” was smaller than the initial “maturing” rate, the number of “Saplings” would start decreasing, and so will the rate of “maturing.” The “maturing” rate will continue to decline until it equals the “planting” rate. Simulate the model with a “planting” rate of five trees per year. The model should exhibit the following behavior:

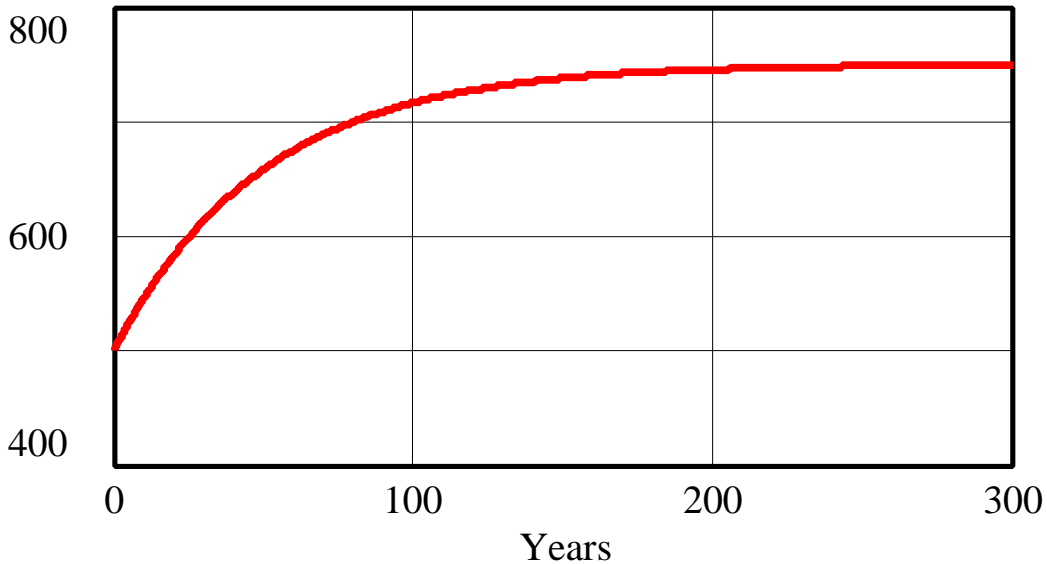
Saplings - planting = 5



Saplings : planting=5  tree

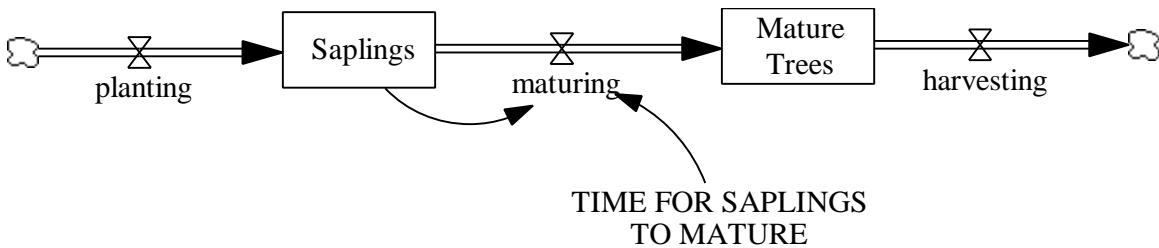
If the “planting” rate was larger than the initial “maturing” rate, the number of “Saplings” would start increasing, and so will the “maturing” rate. The number of “Saplings” would grow until the “maturing” rate equals the “planting” rate. Simulate the model with a “planting” rate of 15 trees per year. The model should generate the following behavior:

Saplings - planting = 15



Saplings : planting=15 ————— tree

Now look back at the model built in Exercise 3:

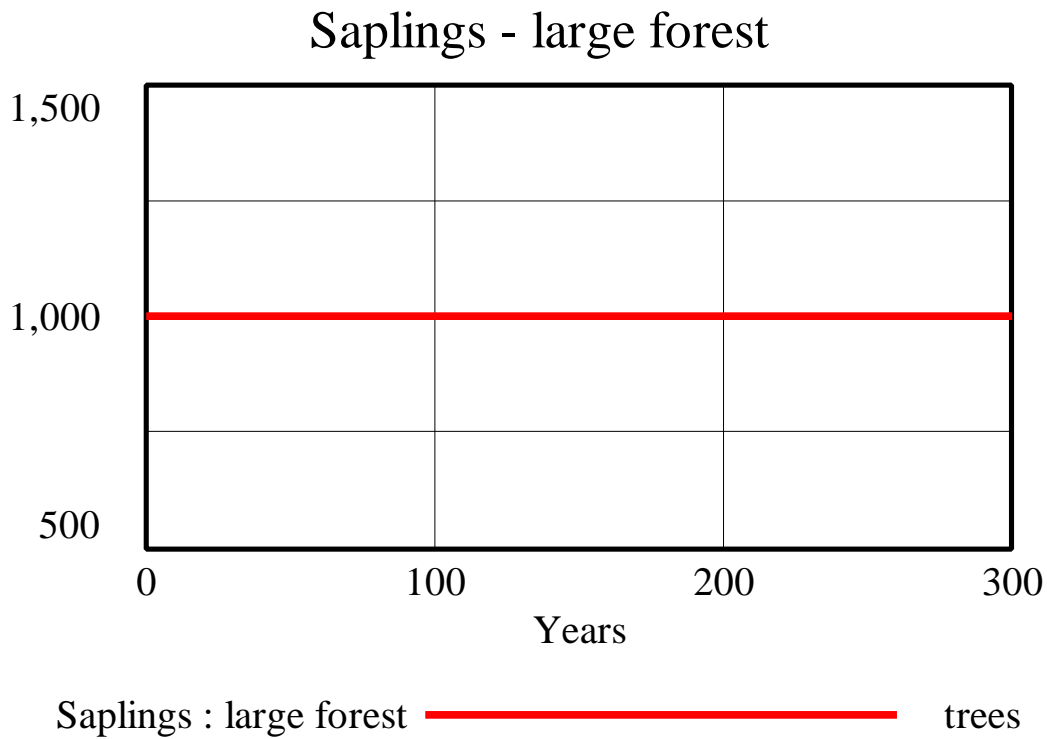


If the initial number of “Saplings” is 500 and the “TIME FOR SAPPLINGS TO MATURE” is 50 years, the initial “maturing” rate is 10 trees/year. If the “planting” and “harvesting” rates are also 10 trees/year, then the inflow into each stock equals the outflow from the stock, and the entire system is at equilibrium. In the scenario from part F, the “planting” rate is larger than the “maturing” rate, so the number of “Saplings” grows. The “harvesting” rate is also larger than the “maturing” rate, so the number of “Mature Trees” declines. The new equilibrium is found when the “maturing” rate is equal to the “planting” and “harvesting” rates: the “maturing” rate is 15 trees/year when the number of “Saplings” is 750. By that time, the number of “Mature Trees” has decreased to 250.

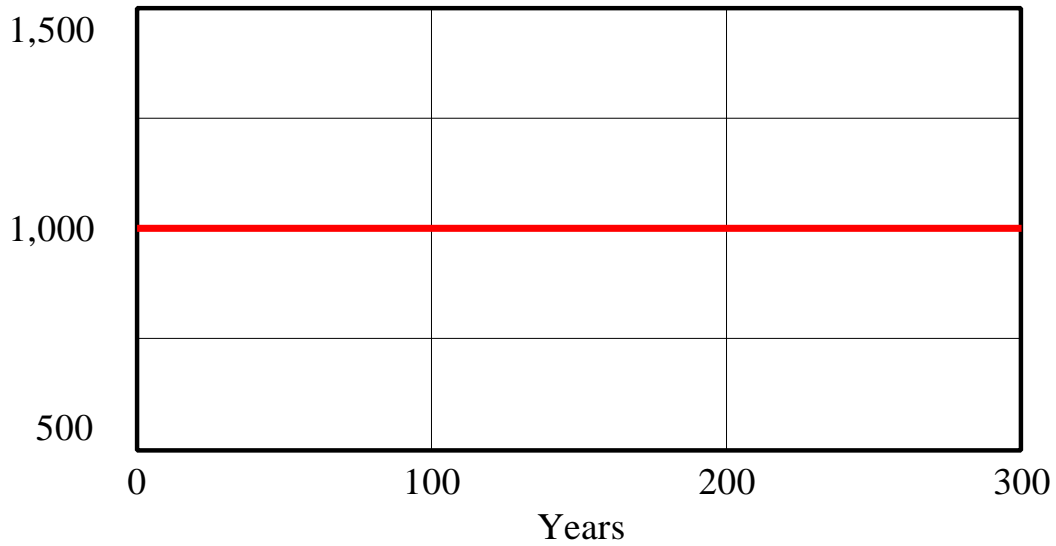
One comment about the continuous nature of this model: system dynamics models represent continuous, rather than discrete systems. When one draws an outflow of “harvesting” equal to 10 trees/year from the stock of “Mature Trees,” one represents a

system in which ten trees are harvested *over the course* of a year. The trees are not all harvested on New Year's Day. In fact, the locals do not harvest one tree every 36.5 days either; in the model, the locals are there every second, peeling away at the bark and lopping off tiny branches one at a time. The mathematical equations in behind the model represent a system in which infinitesimal fractions of each tree are being planted, matured, and harvested every infinitesimal period of time. Of course, this representation may not always make too much sense in the real world, but in general the approximation of continuous events for discrete events is appropriate and allows a modeler to understand more complex systems.

To make sure that you understand the dynamics underlying the model, imagine another scenario. A nearby forest has 2000 trees, half of which are saplings and half of which are mature trees. The locals in this forest usually chop down 20 mature trees and plant 20 young saplings over the course of a year. Modify the model to represent this scenario, and simulate the model over 300 years. The model should generate the following behavior:



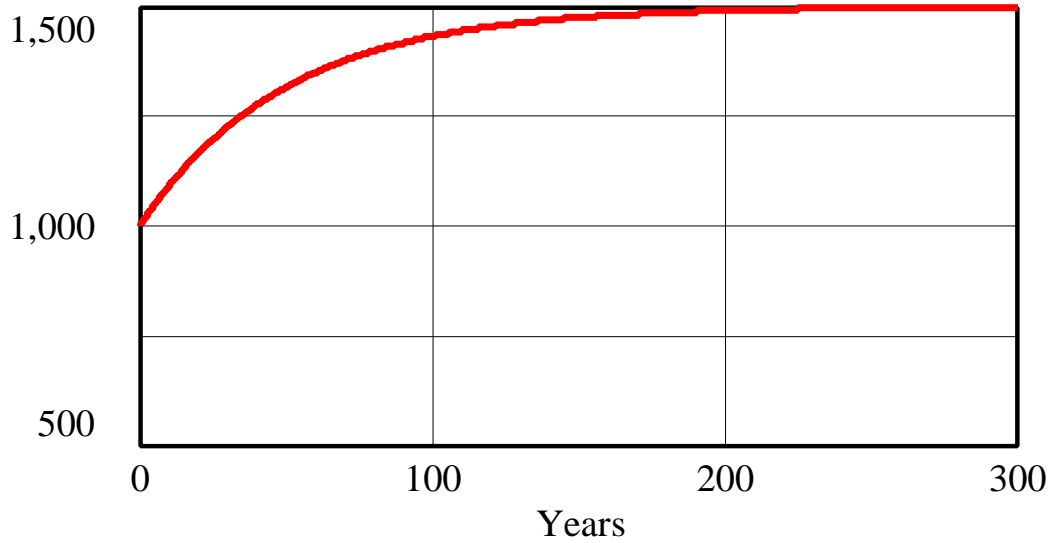
Mature Trees - large forest



Mature Trees : large forest ————— trees

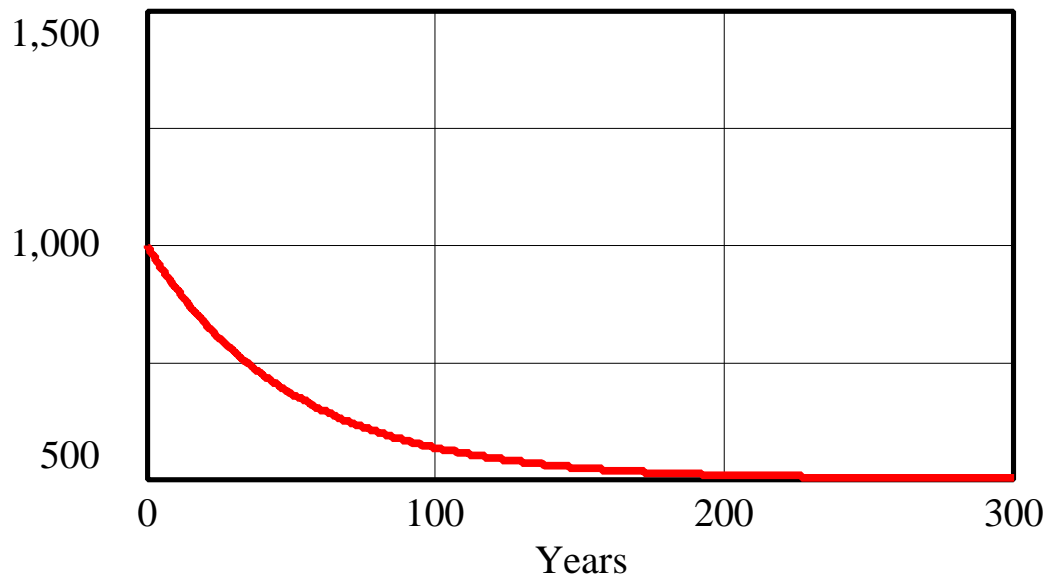
Predictably, the system remains at equilibrium. For the final scenario, imagine that the locals increase the rates at which they plant and harvest trees to 30 trees/year. Simulate the model under this scenario. The model should generate the following behavior:

Saplings - large forest, rates = 30



Saplings : large forest, rates=30 ————— trees

Mature Trees - large forest, rates = 30



Mature Trees : large forest, rates=30 ————— trees

The “planting” and “harvesting” rates of 30 trees per year are greater than the initial “maturing” rate of 20 trees per year, so the number of “Saplings” increases while the number of “Mature Trees” decreases. As the number of “Saplings” grows, the number of “Saplings” that mature each year also grows, until the rate of “maturing” reaches 30 trees/year, at which point the number of “Saplings” settles at equilibrium. To calculate the equilibrium number of “Saplings,” equate the equilibrium “maturing” rate to the number of “Saplings” divided by the “TIME FOR SAPLINGS TO MATURE”:

$$30 \text{ trees/year} = \text{Saplings}/50 \text{ years} = 1500 \text{ trees}/50 \text{ years}$$

So the equilibrium number of “Saplings” is 1500. The rest of the trees in the forest are “Mature Trees.” Hence, in this final scenario, as in the scenario from part F, the distribution of trees in the forest changes. The equilibrium values of “Saplings” and “Mature Trees” depend on the initial numbers of trees and the discrepancy between the initial “maturing” rate and the “planting” and “harvesting” rates.