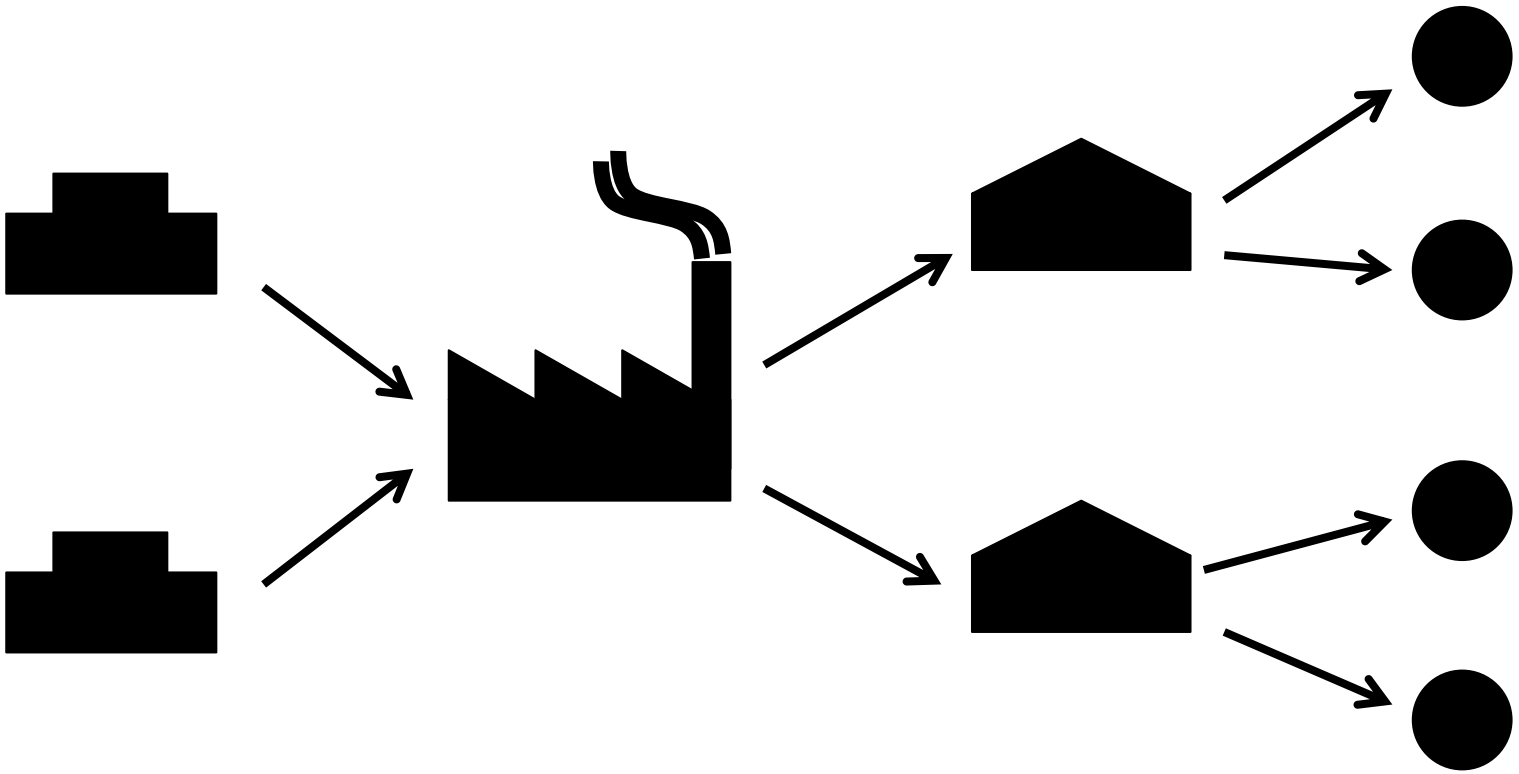
D Lab: Supply Chains

Inventory Management

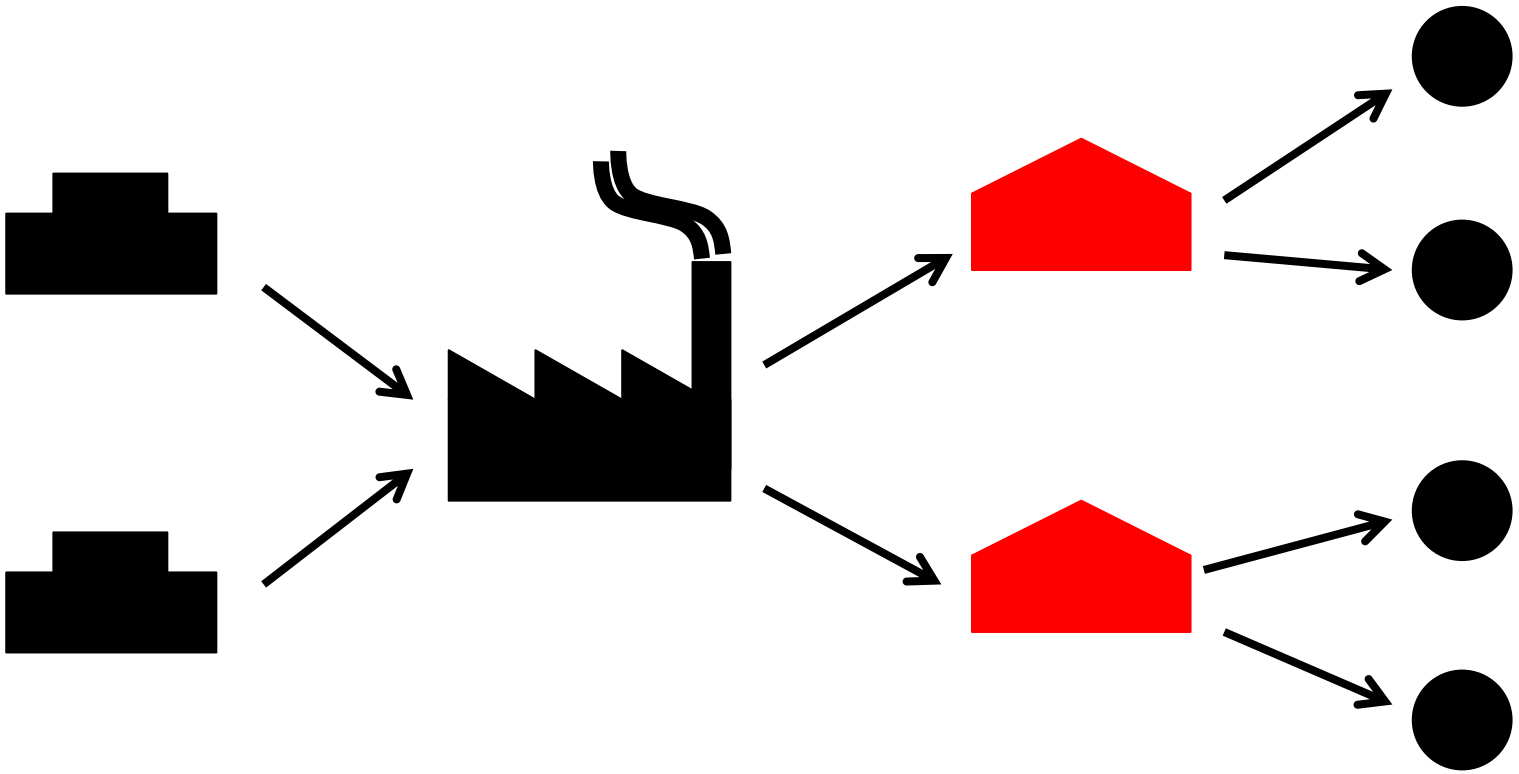
Class outline:

- Roles of inventory
- Inventory related costs
- Types of inventory models
- Focus on EOQ model today (Newsvender model next class)

Inventory Management



Inventory Management



Inventory

- Inventory: goods that a business holds for resale and/or redistribution

- Why should a company hold inventory?

Roles of Inventory

- Anticipation Stock
- Cycle Stock
- Safety Stock
- Pipeline Stock
- Decoupling Stock

Anticipation Stock

- Stock built in **anticipation** of demand or price change
- Production capacity cannot meet demand at the time that occurs

Seasonal demand

Capacity Limits

Cycle Stock

- Stock created due to cyclic nature of replenishment
- Exists due to economies of scale in replenishment (eg, a fixed cost in placing an order)
- When we reduce time between orders, cycle stock goes down
- Milk at home or in a grocery store

Safety Stock

- Exists to protect against uncertainty in demand, in lead times, and in yields
- Larger uncertainty -> larger safety stock
- Serves as a counter measure to uncertainty and disruption in the supply chain

Pipeline Stock

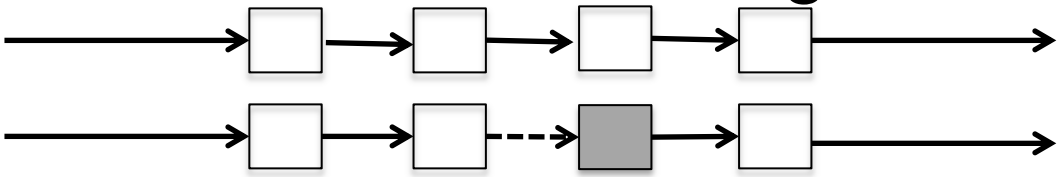
- Stock that has been ordered but has not arrived (in-transit stock)

- Caused by unavoidable time lags and delays

Decoupling Stock

- Stock that is positioned within a manufacturing setting or supply chain, whose purpose is to decouple the system

- Allows the upstream segment to operate independently of the downstream segment



- Can often be viewed analogous to a safety stock

Roles of Inventory

- Anticipation Stock
- Cycle Stock
- Safety Stock
- Pipeline Stock
- Decoupling Stock

Types of Inventory

- Raw materials

- Work-in-process

- Finished goods

Costs of holding inventory

What are the costs of holding inventory?

- Cost of capital
- Cost of storage space and handling
- Inventory risk costs: obsolescence, damage, theft
- Inventory service costs: taxes, insurance

Inventory related costs

- Ordering Costs
  - Fixed and Variable
- Shortage Costs
  - Expedited Shipping, Reimbursement to Customers, Loss of Customer Goodwill
  - These costs are hard to estimate…

Fundamental Questions

- What items should be stocked?
- Where should items be stocked?
- How much should be ordered?
- When should an order be placed?

Types of inventory models

- **Demand:** constant, deterministic, stochastic
- **Lead times**: “0”, “>0”, stochastic
- **Horizon**: single period, finite, infinite
- **Products**: one product, multiple products
- **Capacity**: order/inventory limits, no limits
- **Service**: meet all demand, shortages allowed

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Economic Ordering Quantity (EOQ) Model

EOQ: Motivating Example

- You are managing diesel inventory of an outpost for a humanitarian organization in Uganda
- Your task is to determine when to order diesel fuel and how much to order
- Initiating an order costs $250 (transportation, road security)
- The fuel costs $1/liter
- To hold the fuel, it costs $0.50/year/liter (cost of capital – e.g. redirected from medicine)
- Your demand is constant and stable at 4,000 liters/year

EOQ: Motivating Example

- What is the key tradeoff?
  - Batch size too large (too much average inventory) versus
  - Batch size too small (too much ordering cost)

- For a fixed demand rate, the larger the order, the larger the holding cost

- The smaller the order, the larger the fixed ordering cost per unit

Other Examples

- Can you think about examples from your projects that are similar to the previous example?

- What's the order costs? What's the holding costs?

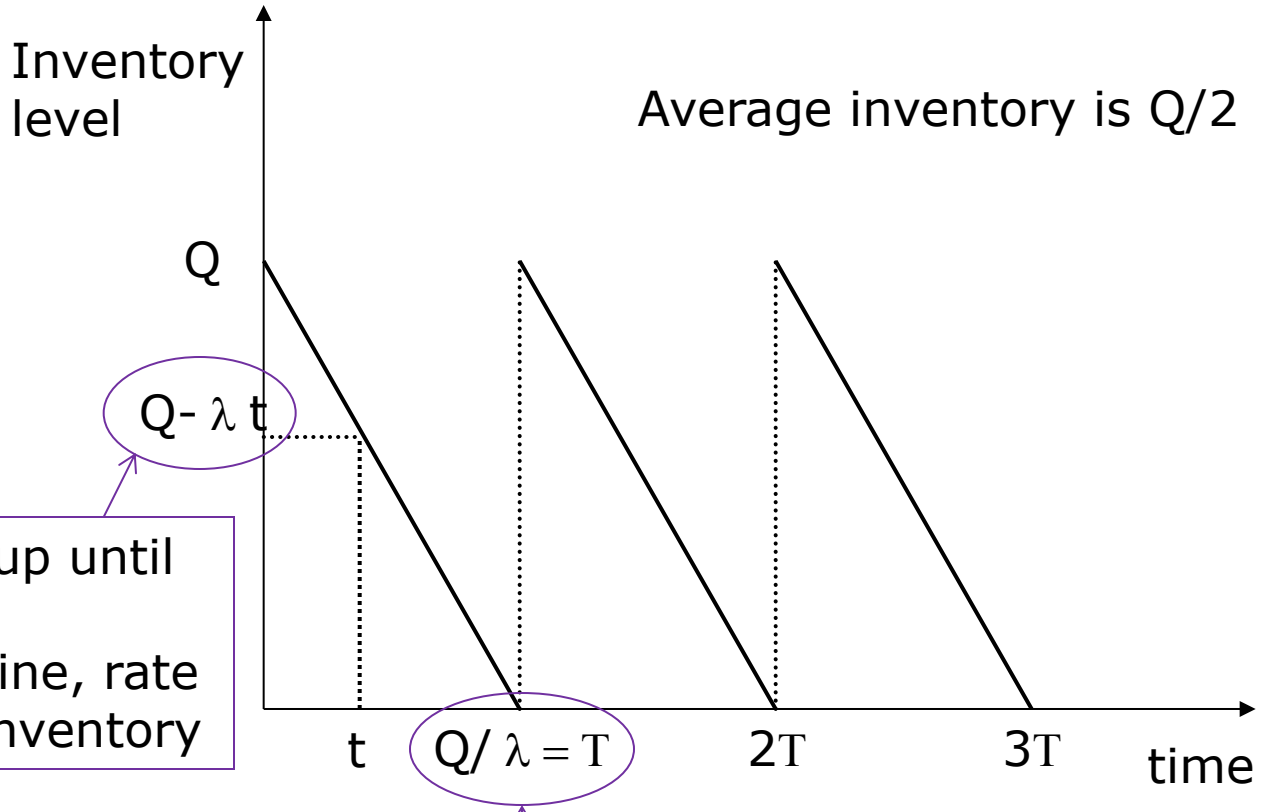
EOQ: Notation

- Decision Variables
  - Fixed order quantity: Q
  - Time between orders: T
- Inventory Costs
  - Fixed order cost: K
  - Variable cost/unit (purchase price): c
  - Inventory holding cost/unit/time: h
- Assume constant demand rate of λ units/time; must meet all demand

EOQ: Key Observations

- It's best to order only when inventory is zero. Why?
  - Zero lead time → Instantaneous replenishment
  - Therefore, no need to order inventory if can fill orders from existing stock
- The optimal ordering quantity will be constant.
  - Safety stock is no longer beneficial (constant demand, 0 lead time) and only incurs holding cost

EOQ: Graphical Representation



λt = demand up until time t
- λ= slope of line, rate of change in inventory

Given our order quantity, we know when to place the order, and vice versa.

EOQ: Objective

- Total average cost = average holding cost + average order cost
  - Average holding cost =(holding $/unit/time)*(avg. inventory)
    $= hQ/2$
  - Average order cost = $\frac{\text{order cost per replenishment cycle}}{\text{length of replenishment cycle}}$
    $=(K + cQ)/T$
    $= K\lambda/Q + c\lambda$
- Meet all demand while minimizing the total average cost ($/time)

EOQ: Optimization

- min(Total average cost) = min f(Q)

  $=\min(hQ/2 + K\lambda/Q + c\lambda)$
- 1st Order Condition: $f'(Q^*)=0$

  $f'(Q)=h/2 - K\lambda/Q^2$
- 2nd Order Condition: $f''(Q^*)\geq 0$

  $f''(Q)=2K\lambda/Q^3$ (≥ 0 for any quantity)

$$Q^* = \sqrt{\frac{2K\lambda}{h}}$$

$$T^* = \sqrt{\frac{2K}{h\lambda}}$$

EOQ: Optimization Results

- Order Quantity/Order Time Decisions

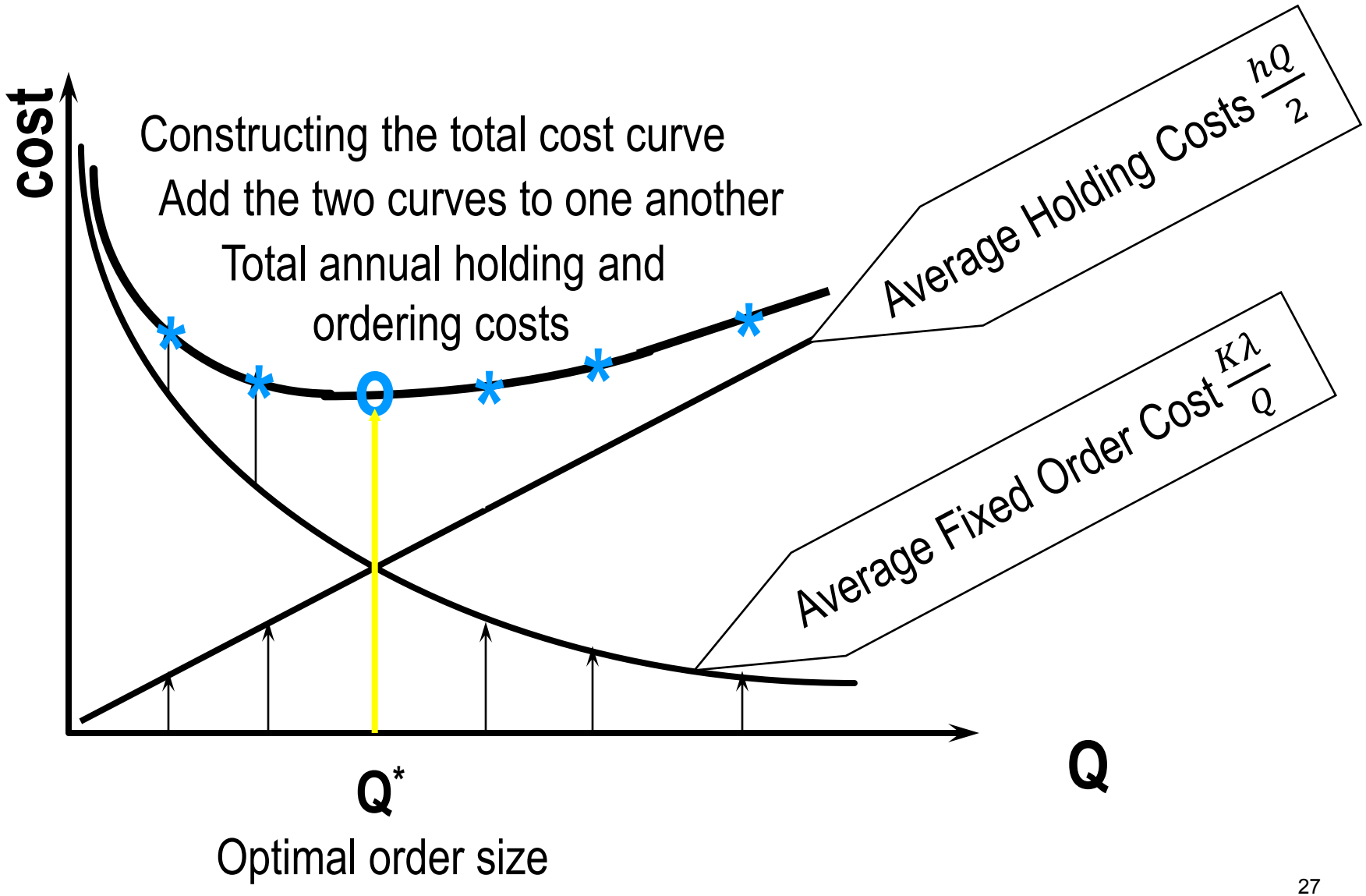
$$Q^* = \sqrt{\frac{2K\lambda}{h}} \qquad T^* = \sqrt{\frac{2K}{h\lambda}}$$

- Optimal Holding Cost $\frac{hQ^*}{2} = \sqrt{\frac{K\lambda h}{2}}$

- Optimal Fixed Order Cost $\frac{K\lambda}{Q^*} = \sqrt{\frac{K\lambda h}{2}}$

- Total Cost $c\lambda + \sqrt{2K\lambda h}$

EOQ: Graphical View



EOQ: Example Solution

- Optimal order quantity in liters =

$$Q^* = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{(2)(250)(4000)}{0.5}} = 2000$$

- Cycle time in year

$$T^* = Q^*/\lambda = .5$$

- Annual holding cost in dollars

$$\frac{hQ^*}{2} = \sqrt{\frac{K\lambda h}{2}} = \sqrt{\frac{(250)(4000)(0.5)}{2}} = 500$$

- Annual order cost in dollars

$$\frac{K + cQ^*}{T^*} = \sqrt{\frac{K\lambda h}{2}} + c\lambda = \sqrt{\frac{(250)(4000)(0.5)}{2}} + 1 \times (4000) = 4500$$

- Total annual cost in dollars

$$500 + 4500 = 5000$$

EOQ: Sensitivity Analysis

- What if we don't order **exactly** Q*?
  - Fixed order cost and holding cost are affected
  - Variable order cost is independent of Q*
  - Say we order γQ* instead…how is fixed order cost + holding cost affected?

EOQ: Sensitivity Analysis

$$
\begin{aligned}
C(Q^*) &= \sqrt{2K\lambda h} \\
C(\gamma Q^*) &= \frac{1}{\gamma}\sqrt{\frac{1}{2}K\lambda h} + \gamma\sqrt{\frac{1}{2}K\lambda h} \\
&= \sqrt{2K\lambda h}(\gamma + \tfrac{1}{\gamma})/2 \\
\frac{C(\gamma Q^*)}{C(Q^*)} &= (\gamma + \tfrac{1}{\gamma})/2
\end{aligned}
$$

γ	0.5	0.8	0.9	1	1.2	1.5	2
$\frac{C(\gamma Q^*)}{C(Q^*)}$	1.25	1.025	1.006	1	1.017	1.083	1.25

EOQ: Key Observations

- Optimal decision (order quantity Q*) independent of variable ordering cost c
- Rather insensitive to order quantities that are close to optimal, but *not* optimal
- Optimal fixed order cost equals optimal inventory holding cost

Extensions

- Lead-time L
  - Same ordering quantity
  - Order L periods in advance, when stock reaches L/D
- Finite production rates / Capacity restriction on orders
- Quantity discounts
- Allow shortage

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Economic Ordering Quantity (EOQ) Model

Next Class

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Newsvendor Model

15.772J / EC.733J D-Lab: Supply Chains
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