Determination of Expected Profit for Newsboy for Uniform Demand

Assume that demand is from a normal distribution with mean and standard deviation given by μ,σ . Let the sales price be *p*, the salvage price be *s*, and the item cost be *c*.

$\Pi(Q)$ is the expected profit for the newsboy from ordering Q units.

$\phi(x|\mu,\sigma)$ denotes the probability density function for a normal distribution with parameters μ,σ.

$$\Pi(Q)=\int_{x=-\infty}^{x=Q}\left(px+s(Q-x)\right)\phi\left(x|\mu,\sigma\right)dx+\int_{x=Q}^{x=\infty}(pQ)\phi\left(x|\mu,\sigma\right)dx-cQ$$

Explanation:

- The first integral is over the demand realizations that are less than the order quantity Q; if demand equals x and if x < Q, then the newsboy will sell x units at price *p* and salvage (Q-x) units at *s*.
- The second integral is over the demand realizations that are more than the order quantity Q; in these cases, the newsboy can only sell Q units at price *p*.
- The last term is what the newsboy pays for ordering Q units.

To evaluate this expression, we re-write as follows:

$$\Pi(Q)=\int_{x=-\infty}^{x=Q}\left(px+s(Q-x)\right)\phi\left(x|\mu,\sigma\right)dx+\int_{x=Q}^{x=\infty}(pQ)\phi\left(x|\mu,\sigma\right)dx-cQ$$

$$=\int_{x=-\infty}^{x=Q}\left(px+s(Q-x)\right)\phi\left(x|\mu,\sigma\right)dx+\int_{x=Q}^{x=\infty}\left(px+s(Q-x)\right)\phi\left(x|\mu,\sigma\right)dx-\int_{x=Q}^{x=\infty}\left(px+s(Q-x)\right)\phi\left(x|\mu,\sigma\right)dx$$

$$+\int_{x=Q}^{x=\infty}(pQ)\phi\left(x|\mu,\sigma\right)dx-cQ$$

$$=\int_{x=-\infty}^{x=\infty}\left(px+s(Q-x)\right)\phi\left(x|\mu,\sigma\right)dx-\int_{x=Q}^{x=\infty}(p-s)(x-Q)\phi\left(x|\mu,\sigma\right)dx-cQ$$

We can then simplify this as follows:

$$\Pi(Q)=p\mu+s(Q-\mu)-(p-s)\int_{x=Q}^{x=\infty}(x-Q)\phi\left(x|\mu,\sigma\right)dx-cQ$$

Now the crux of the evaluation is to evaluate the third integral; for normal distribution the following can be shown, directly from algebraic transformations:

$$\int_{x=Q}^{x=\infty} (x-Q)\phi(x|\mu,\sigma)\,dx = \sigma \int_{x=z}^{x=\infty} (x-z)\phi(x|0,1)\,dx$$

$$where\ z = \frac{Q-\mu}{\sigma}$$

Note that $\phi(x|0,1)$ is the probability density function for the standard normal distributed random variable with mean of 0 and standard deviation of 1.

The above expression is known as the partial loss function and can be calculated as follows:

$$PartialLossFunction(z) = \int_{x=z}^{x=\infty} (x-z)\phi(x|0,1)\,dx = \phi(z|0,1) - z \times (1-\Phi(z))$$

$$= NORMDIST(z,0,1,FALSE) - z \times (1 - NORMDIST(z,0,1,TRUE))$$

Where $\Phi(z)$ is the cumulative distribution function for the standard normal. The spreadsheet commands are shown for calculation.

Thus the newsboy profit calculation is

$$\Pi(Q) = p\mu + s(Q-\mu) - (p-s)\,PartialLossFunction\left(z = \frac{Q-\mu}{\sigma}\right) - cQ$$