15.433 INVESTMENTS

Class 23: Commodities

Spring 2003

Introduction

The following table taken from the annual report of the Bank for International Settlements (BIS) reports the spectacular growth in exchange-traded derivatives, such as futures and options.

The figures indicate the amount of the underlying controlled by the derivative in billions of US dollars at year end:

Type of Exchange Traded Instrument	1986	1987	1988	1989	1990	1995	1998	06/1999
Interest Rate Options and Futures	516	609	1'174	1'588	2'054	15'669	50'015	54'072
Currency Options and Futures	49	74	60	66	72	120	18'011	14'899
Stock Index Options and Futures	18	41	66	108	158	442	1'488	1'511
Commodities						142	415	441
Total	583	724	1'300	1'762	2'284	16'373	69'929	70'923

Table 1: Derivatives volumes, Source: Bank for International Settlements. Visit their Web Page http://www.bis.org/ for the latest statistics.

Constant Proportional Storage Costs (Known Storage Costs)

- Recall, financial assets may have cash payouts which have to be factored in the cost of carry relation. In the case of stocks, the payouts are dividends, while for Treasury bonds, they are coupons.

- Non-financial assets, such as gold, silver, oil, corn, etc., have storage costs associated with the cash-and-carry strategy. The alteration to the cash-and-carry strategy of before is that we have to consider, both, the cost of financing (i.e., the standard shorting of K bonds) as well as the cost of storage when purchasing the asset spot.

- Notice that dividends over the life of the forward contract are a benefit accruing solely to the spot asset holder and not the forward contract party, while storage costs over the life of the forward contract are a disbenefit borne by the spot asset holder and not the forward contract party.

- Hence, storage costs can be viewed as a negative dividend payout. Depending on how storage costs are defined, all we

have to do is to replace the present value of dividends in the cost of carry relation with the present value of storage costs, with the appropriate adjustment in sign.

- Usually, commodities have no „dividends" or cash payouts associated with them, hence, we only have to consider storage costs when determining the cost of carry relation for them.

- Similarly, financial assets usually have dividends or cash payouts but do not have storage costs associated with them, hence, we only have to consider dividends when determining the cost of carry relation for them.

Notation: Let:

- I(K) be the initial investment required to buy a forward contract maturing at date T with delivery price K.
- B(t,T) be the current price of a unit bond paying 1 at maturity.
- $a = \sum_{k=t+1}^{T} B(t,k)$ be the current price of an annuity paying \$1 at the end of each period until the forward contract matures.

- D be the constant Dollar storage payout per period on the risky asset.

- D (k) be the time-varying, known Dollar storage payout per period on the risky asset.

- y be the gross, continuously compounded, proportional storage cost yield.

- S(t) be the current Spot price of the underlying asset.

- F(t,T) be the current forward price of the underlying asset.

The cost of carry relation between the forward price, F(t,T), and the spot price, S(t), depends on the storage cost structure:

- No Storage Costs: F(t,T) $= F(t,T) = \dfrac{S(t)}{B(t,T)}$

- Constant Dollar Storage Costs: $F(t,T) = \dfrac{S(t) + D \cdot a}{B(t,T)}$, where D

  is the size of each (constant) storage cost payout and a is the current value of an annuity (a) paying \$1 at each ex-storage cost date.

- Time-varying, Known Dollar Storage Payouts:

$$F(t,T) = \frac{S(t) + \sum_{k=t+1}^{T} D(k) \cdot B(t,k)}{B(t,T)},$$ where

$\sum_{k=t+1}^{T} D(k) \cdot B(t,k)$ is the present value of the total storage costs associated with the underlying asset over the remaining life of the forward contract.

- Continuous Proportional Payouts: $F(t,T) = \dfrac{S(t)e^{y(T-t)}}{B(t,T)}$, where y is the continuously compounded, proportional storage cost measured as a percentage of the spot price.

Convenience Value of Commodities

- Convenience is the benefit from storage of the physical commodity, but not from holding the forward contract on the physical commodity.

- Storage may benefit from profit opportunities available when there are temporary shortages in the commodity, perhaps avoiding suspension of the production process.

- Frequently, due to a shortage, the asset underlying the forward contract cannot be borrowed in order to execute a short sale. The shortage arises mainly due to the convenience arising from owning the asset spot for the production process.

- When shortages occur, the spot price for the commodity is high, generating a convenience yield.

- When the convenience is high, the spot-forward curve goes into „backwardation" mode, i.e., (loosely speaking) the term structure of current forward prices is lower than the spot price. (Note: The spot price may also drop dramatically, due to a temporary glut or oversupply, causing the spot-forward curve

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to reverse into „contango", i.e., (loosely speaking) the term structure of forward prices rises above the spot price.)

- Some financial assets, like Treasury bonds and mortgage-backed securities, have convenience values due to their collateral value available in repo (repurchase) markets. In other words, there are times (i.e., during shortages) when these financial assets behave just like commodities.

Basis

In a well-functioning market, the forward price at delivery is equal to the prevailing spot price. Mathematically,

$$F(T,T) = S(T).$$

However, convergence is not always the norm. Frequently, the forward price does not converge to the spot price due to:

- the date of spot delivery T, does not match the date of hedge expiration T' (with $T \neq T'$),

- the assets underlying spot and forward contracts are not identical, and

- the date of spot delivery, T, is uncertain.

Example

Consider the following typical Short Hedge.

a)   A farmer has wheat to sell at harvest time, which we pretend to know to be at date T with certainty.

b)   At t = 0: farmer shorts a forward contract at F(0,T') for delivery at T' after (but close to) T. (Assume that's because there isn't a forward contract of identical maturity to the spot delivery date T.)

At t = T: farmer sells spot wheat at S(T) and „closes" out forward position at F(T,T').

c)   The farmer's payoff (Note: He has a short hedge):

$$\underbrace{\left[S(T) - F(T,T') \right]}_{basisrisk} + \underbrace{F(T,T')}_{locked-invalue}$$

The difference between these two prices, i.e., [S(T) - F(T,T')], is known as forward-spot basis. The risk the hedger (farmer) faces due to this difference is called basis risk.

(Important Note: Strictly speaking, basis risk only arises in futures contracts and is known as futures-spot basis. Basis is due to the

standardization of (exchange-traded) futures contracts. Forward contracts are usually tailor-made and hence do not carry basis risk. We speak in terms of forward contracts just to illustrate ideas and because we do not cover futures contracts until later. Hence, the term „futures contracts" is more correct usage instead of „forward contracts" in this sub-section. And ignore margins.)

- Example of 1: The forward (futures) contract expires on June 15 while spot delivery is set for May 15. The norm is to choose a forward (futures) delivery month T', which falls after the spot delivery date T. This is because the front-end forward (futures) price is usually very volatile when close to expiration. Picking a later-dated forward (futures) contract reduces this risk. The forward (futures) contract is usually unwound just prior to spot delivery.

- Example of 2: The underlying contract in the forward (futures) contract is corn while the spot delivery calls for wheat. Usually, the forward (futures) with an underlying which is most closely correlated in price to the spot contract is chosen.

- Example of 3: The wheat processor taking spot delivery from the farmer is not certain when she needs to take delivery. In other words, the timing of buyers' demand is not known with certainty.

Strengthening of the basis (i.e., [S(T) - F(T,T')]   ing) benefits the short hedger (farmer). Conversely, weakening of the basis benefits the long hedger.

Preparation for Next Class

Please read Read Statman (1999) and Nagy and Obenberger (1994).