15.401 Finance Theory

MIT Sloan MBA Program

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Lecture 12: Introduction to Risk and Return

Critical Concepts

- Motivation
- Statistical Background
- Empirical Properties of Stock Returns
- Anomalies

Readings

- Brealey, Myers, and Allen Chapters 7, 24.1, 24.4

Motivation

NPV and Other Valuation Techniques Need Cost of Capital

- Opportunity cost
- Required rate of return
- Risk-adjusted discount rate
- Determined by “the market”
- How???

Introduce Risk Into The Valuation Process

- How to measure risk
- How to estimate the required rate of return for a given level of risk
- Related questions:
  - How risky are stocks and what have their returns been historically?
  - Is the stock market “efficient”?
  - How can we gauge the performance of portfolio managers?

Statistical Background

Terminology

$$\text{Return } R_{it} \equiv \frac{D_{it} + P_{it} - P_{it-1}}{P_{it-1}} = \frac{D_{it} + P_{it}}{P_{it-1}} - 1$$

$$\text{Expected Return} \equiv \mathsf{E}[R_{it}]$$

$$\text{Excess Return} \equiv R_{it} - r_f$$

$$\text{Risk Premium} \equiv \mathsf{E}[R_{it}] - r_f$$

Statistical Background

Terminology

- **Mean, variance, standard deviation:**

$$\mu_i \equiv \mathsf{E}[R_{it}]$$

$$\sigma_i^2 \equiv \mathsf{E}[(R_{it}-\mu_i)^2]$$

$$\sigma_i = \sqrt{\sigma_i^2}$$

- **Sample estimators:**

$$\hat{\mu}_i \equiv \frac{1}{T}\sum_{t=1}^{T} R_{it}$$

$$\hat{\sigma}_i^2 \equiv \frac{1}{T-1}\sum_{t=1}^{T}(R_{it}-\hat{\mu}_i)^2$$

$$\hat{\sigma}_i = \sqrt{\hat{\sigma}_i^2}$$

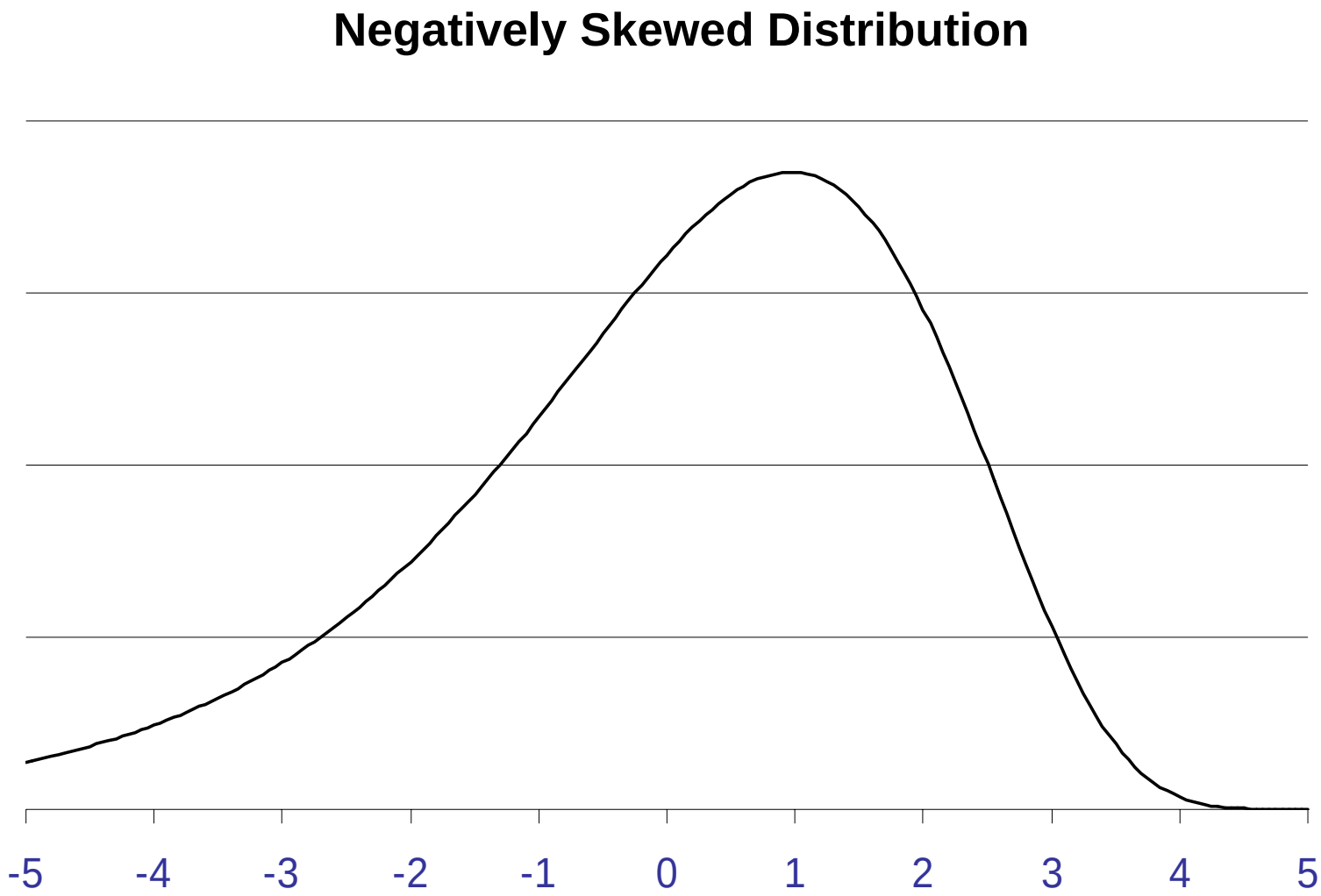
Other Statistics

- **Median**
  - 50th percentile (probability of 1/2 that R_t < median)
- **Skewness**
  - Is the distribution symmetric?
  - Negative: big losses are more likely than big gains
  - Positive: big gains are more likely than big losses
- **Correlation**
  - How closely do two variables move together?

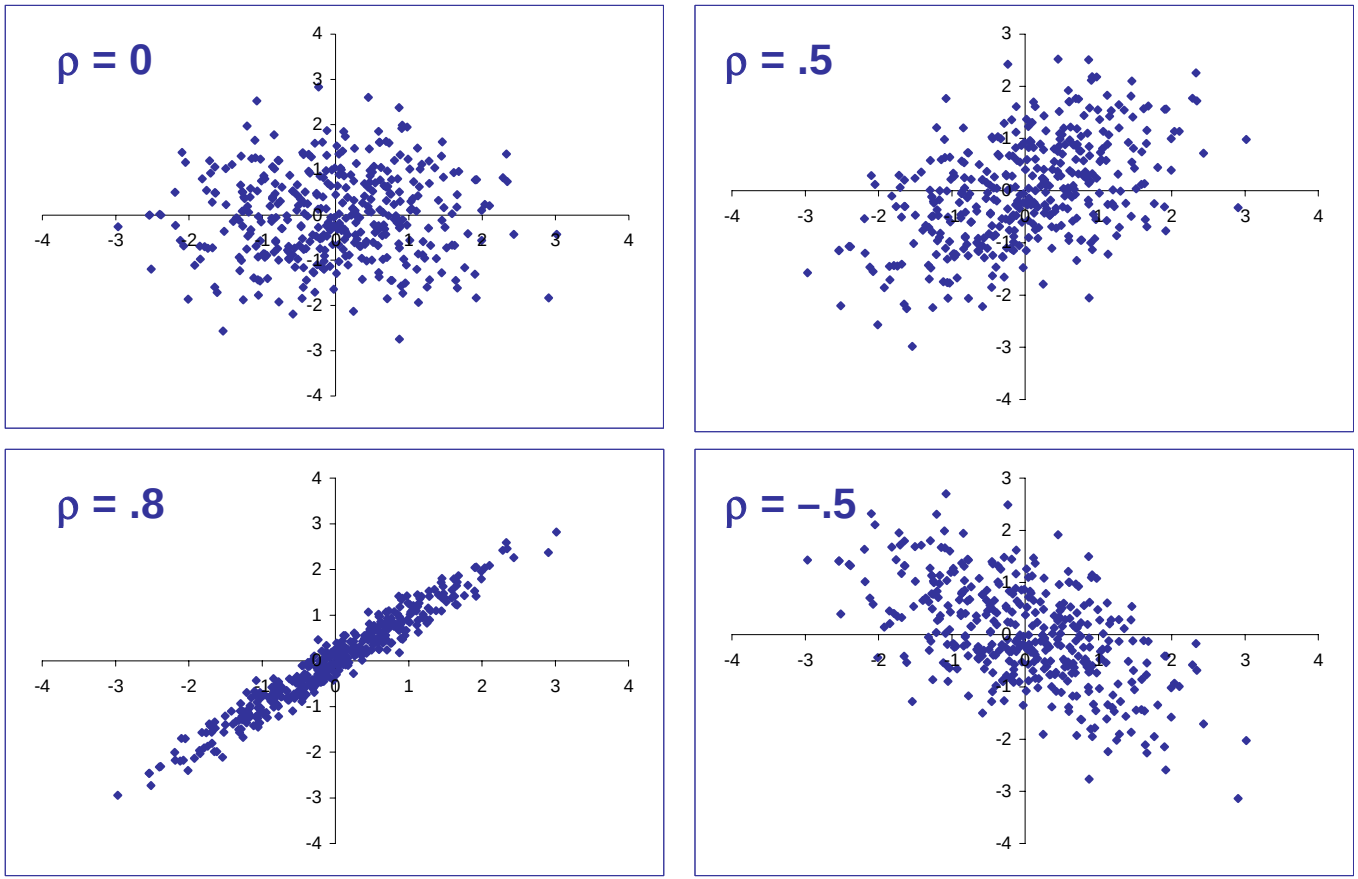
$$\text{Cov}[R_{it}, R_{jt}] \equiv \text{E}[(R_{it} - \mu_i)(R_{jt} - \mu_j)] \quad \text{Covariance}$$

$$\text{Corr}[R_{it}, R_{jt}] \equiv \frac{\text{E}[(R_{it} - \mu_i)(R_{jt} - \mu_j)]}{\sigma_i \sigma_j} \quad \text{Correlation}$$

Statistical Background



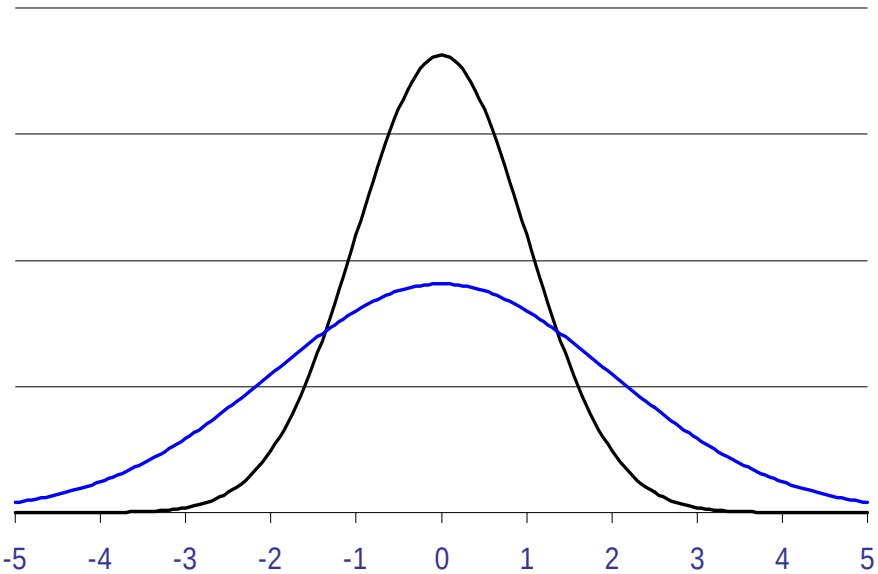
Examples of Correlation Between Two Random Variables



Statistical Background

Normal Distribution

- Bell-shaped, symmetric
- A model of randomness
- Central Limit Theorem

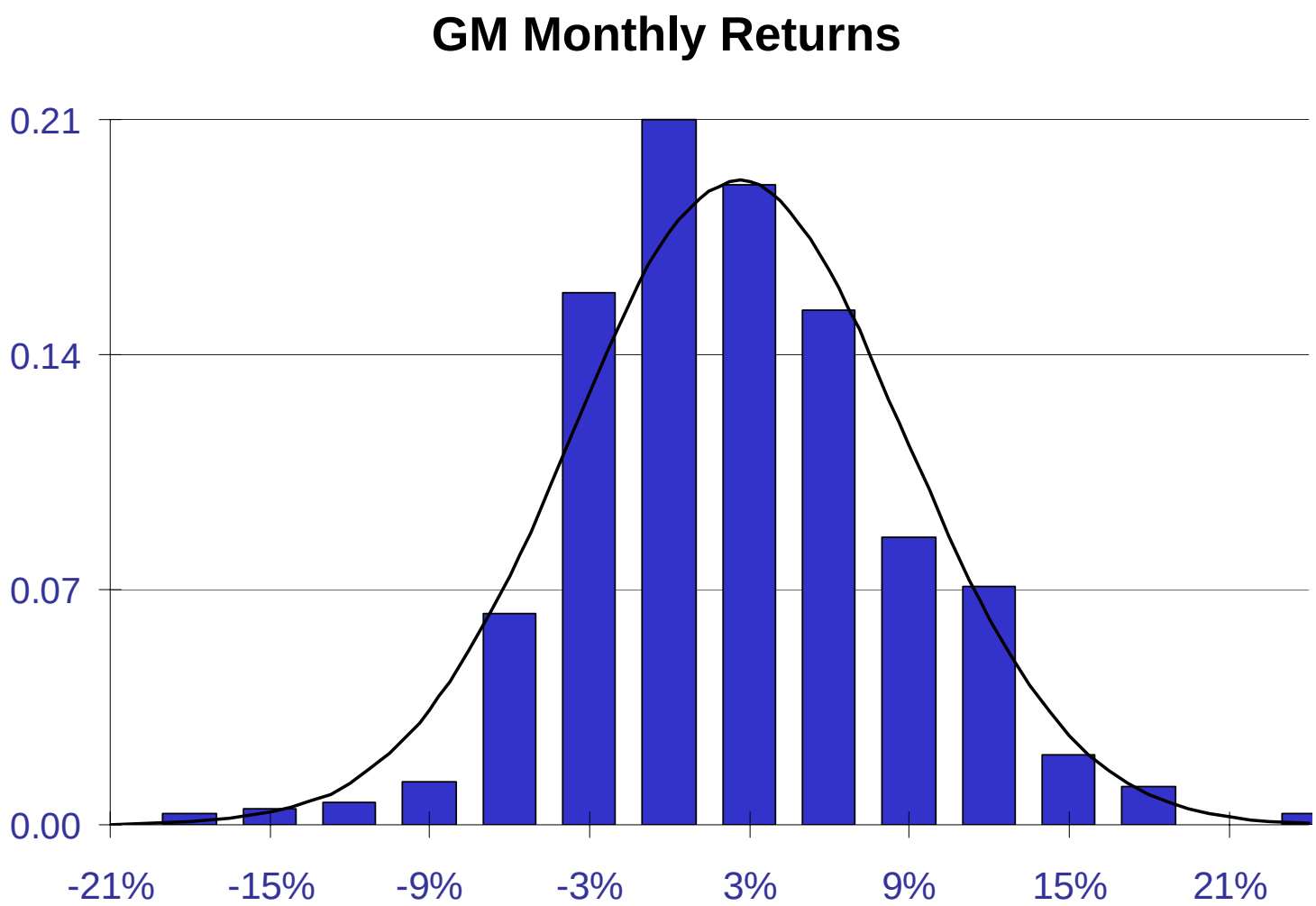


Confidence Intervals

If R is normally distributed, then …

- **68%** of observations fall within +/–**1.00** std. deviations from mean
- **90%** of observations fall within +/–**1.65** std. deviations from mean
- **95%** of observations fall within +/–**1.96** std. deviations from mean
- **99%** of observations fall within +/–**2.58** std. deviations from mean

Statistical Background



Empirical Properties of Stock Returns

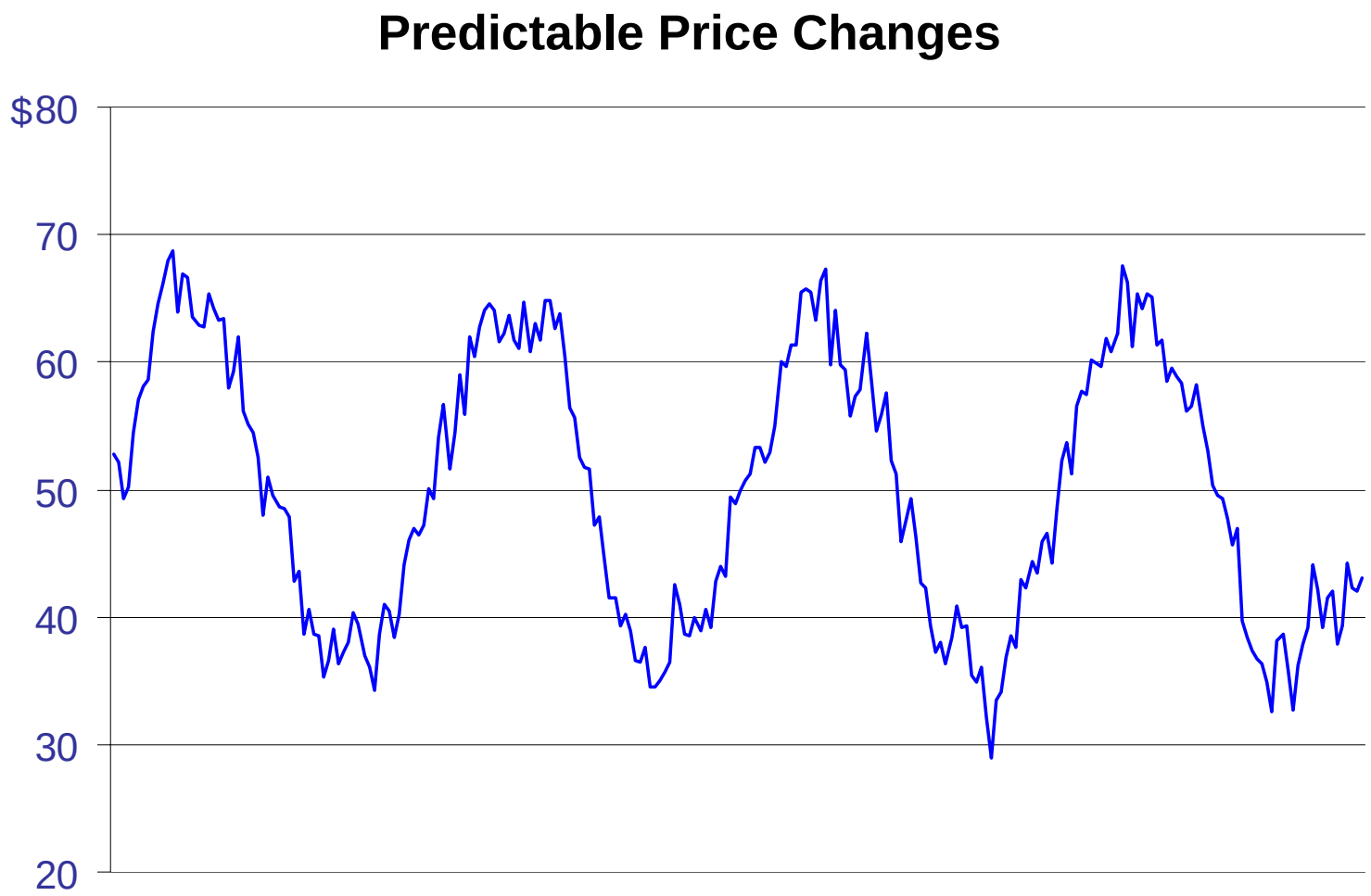
What Characterizes U.S. Stock Returns?

- How volatile are stock returns?
- Are returns predictable?
- How does volatility change over time?
- What types of stocks have the highest returns?

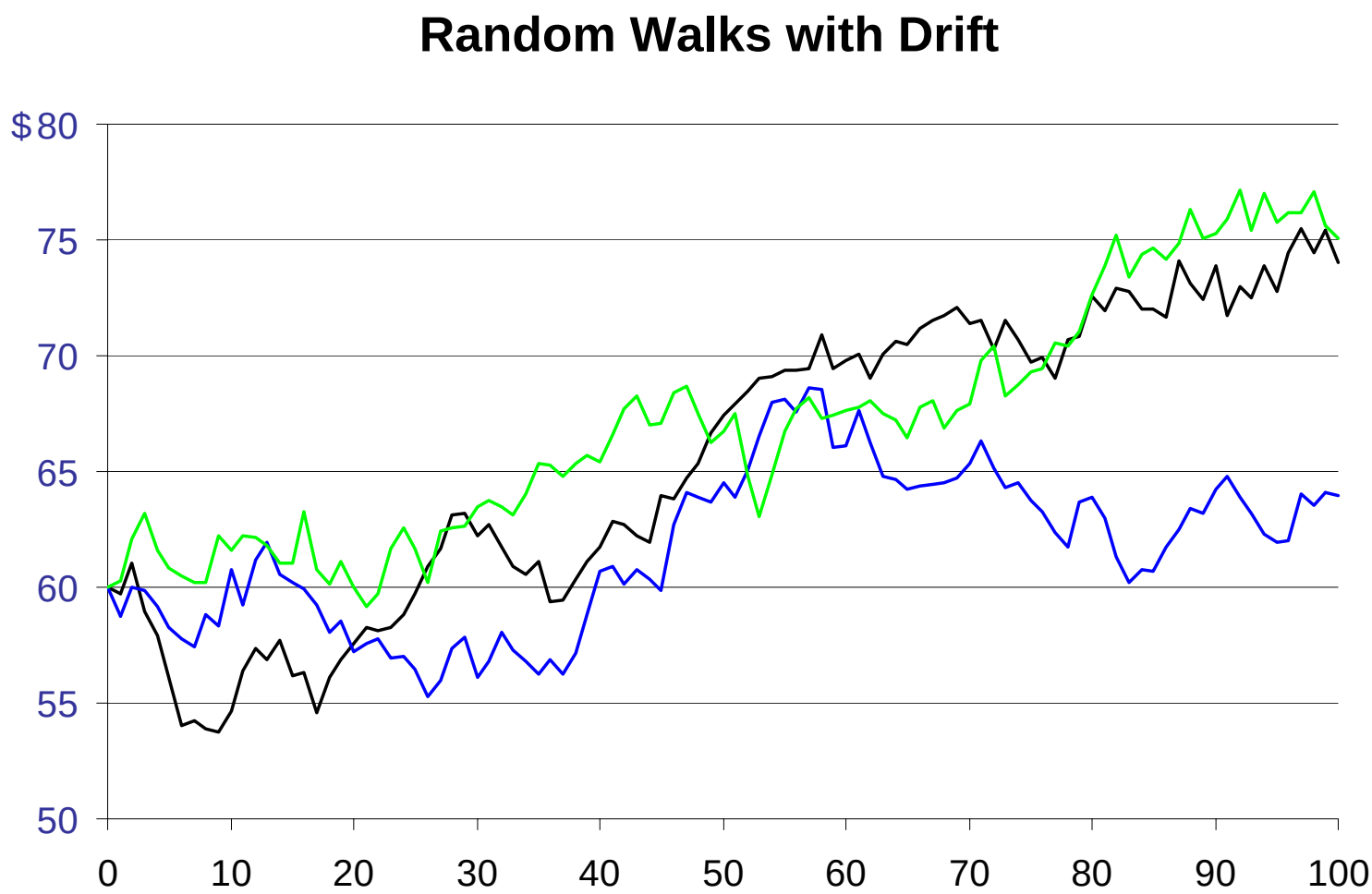
What Properties Should Stock Prices Have In "Efficient" Markets?

- Random, unpredictable
- Prices should react quickly and correctly to news
- Investors cannot earn abnormal, risk-adjusted returns (or at least it shouldn't be easy)

Empirical Properties of Stock Returns



Empirical Properties of Stock Returns



Empirical Properties of Stock Returns

Four facts from history of U.S. financial markets:

1. Real interest rate has been slightly positive on average.
2. Return on more risky assets has been higher on average than return on less risky assets.
3. Returns on risky assets can be highly correlated to each other.
4. Returns on risky assets are (usually) serially uncorrelated.

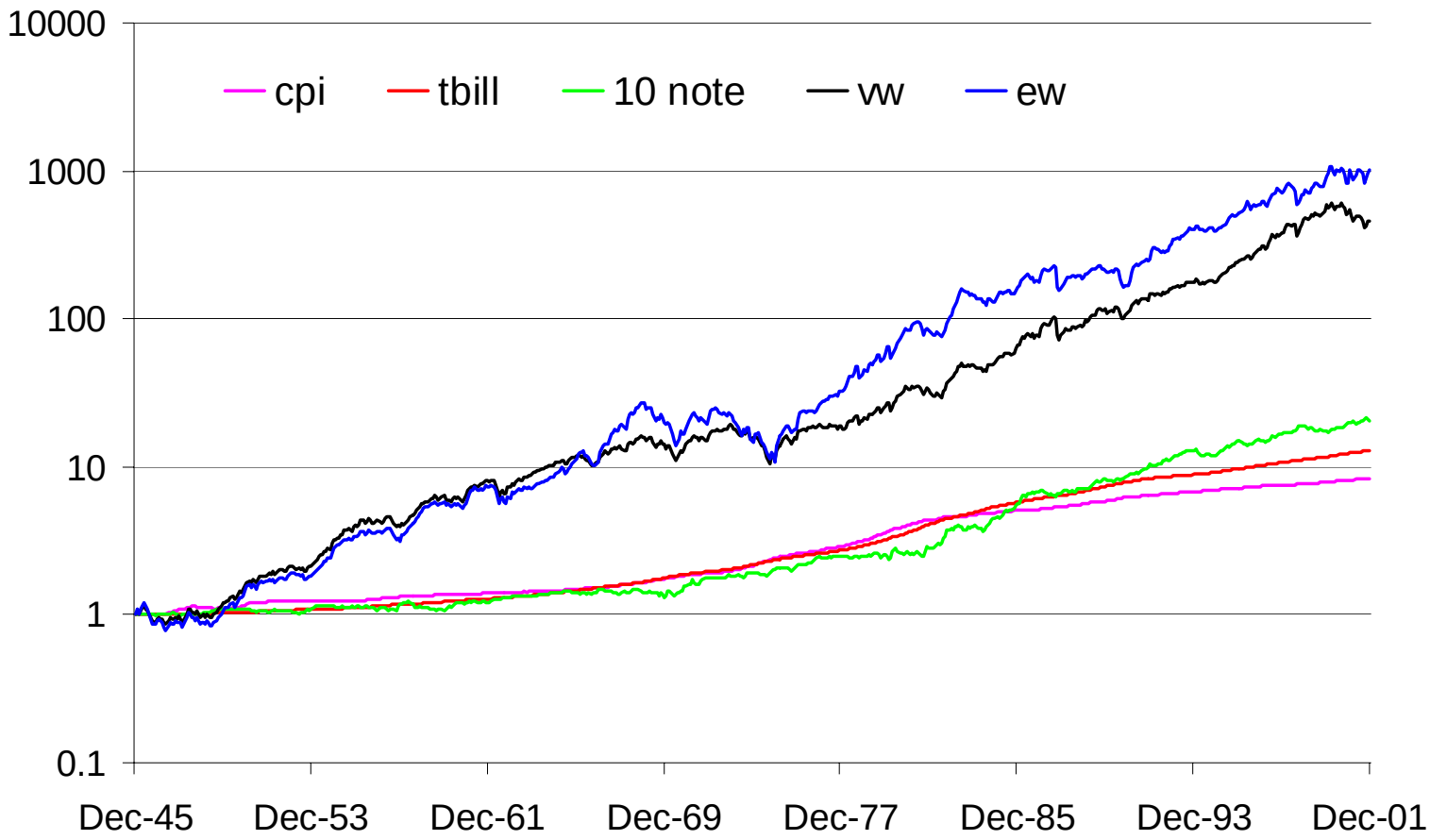
Basic Statistics, U.S., 1946 – 2001 (monthly,in percent)

	Avg	Stdev	Skew	Min	Max
Inflation	0.32	0.36	0.82	-0.84	1.85
Tbill (1 yr)	0.38	0.24	0.98	0.03	1.34
Tnote (10 yr)	0.46	2.63	0.61	-7.73	13.31
VW stock index	1.01	4.23	-0.47	-22.49	16.56
EW stock index	1.18	5.30	-0.17	-27.09	29.92
Motorola	1.66	10.02	0.01	-33.49	41.67

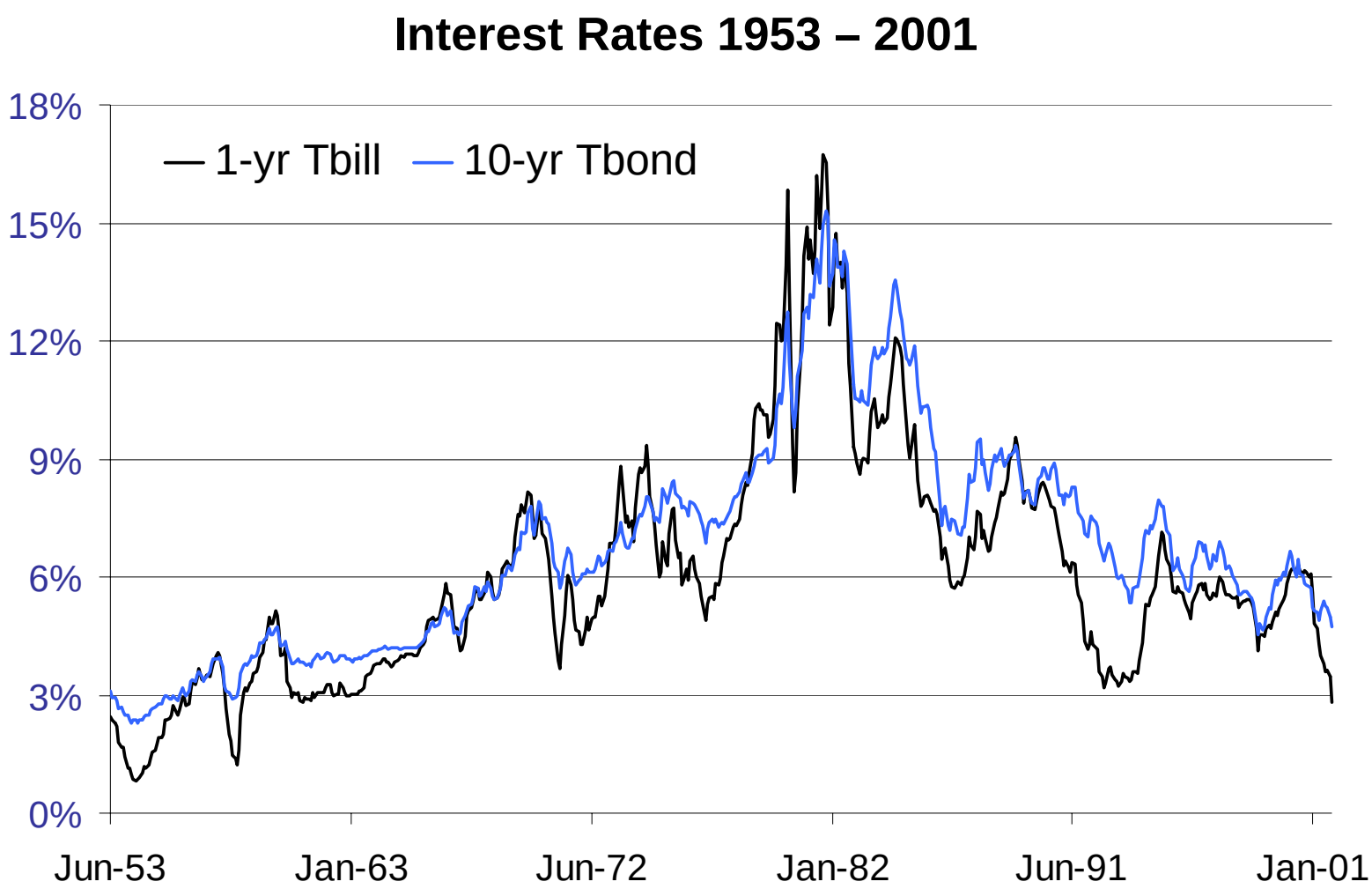
NYSE, Amex, NASDAQ: 6,700 firms, $16.4 trillion market cap

Empirical Properties of Stock Returns

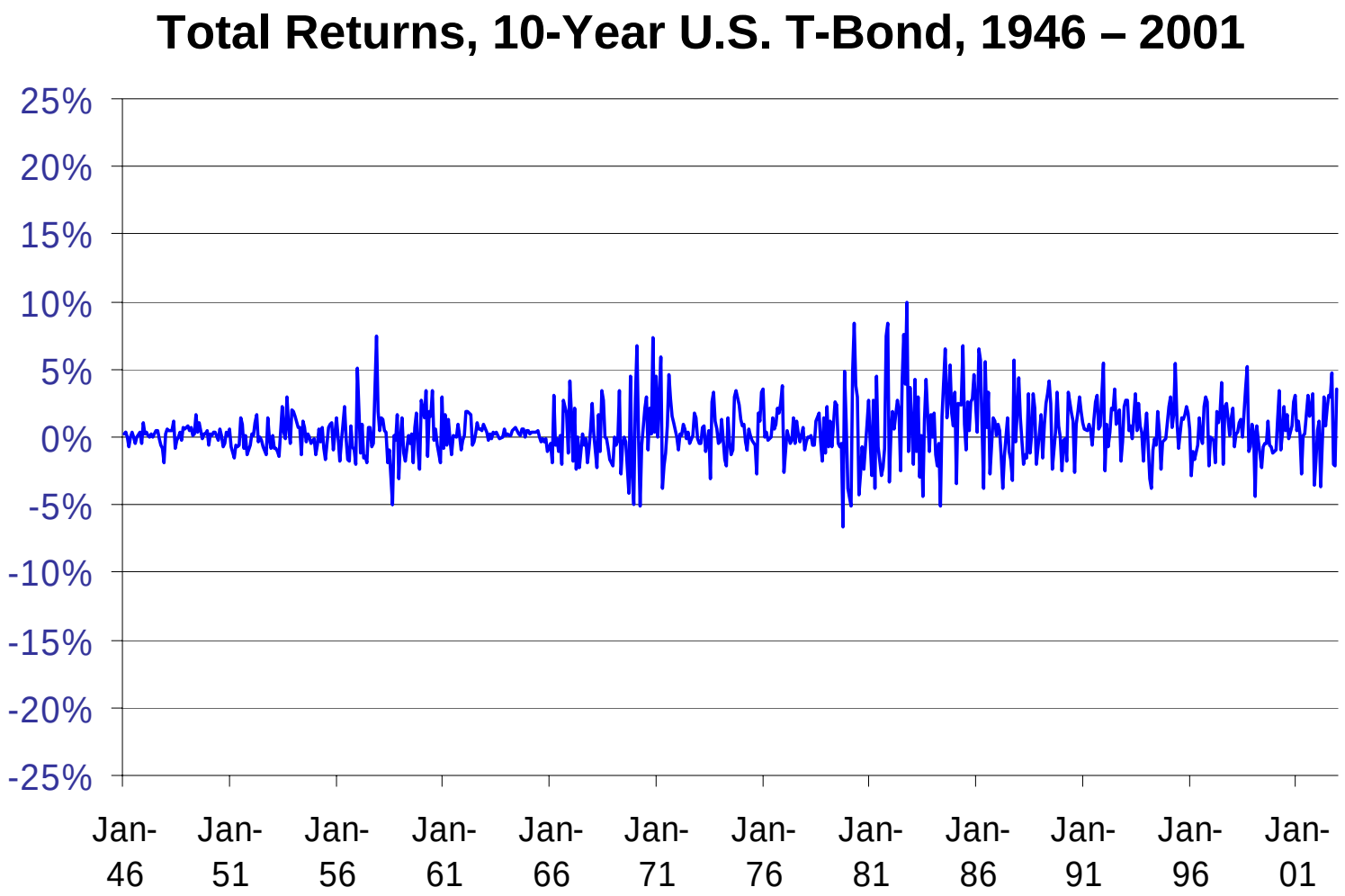
Total Return of Stocks, Bonds, Bills and Inflation 1946 – 2001



Empirical Properties of Stock Returns



Empirical Properties of Stock Returns



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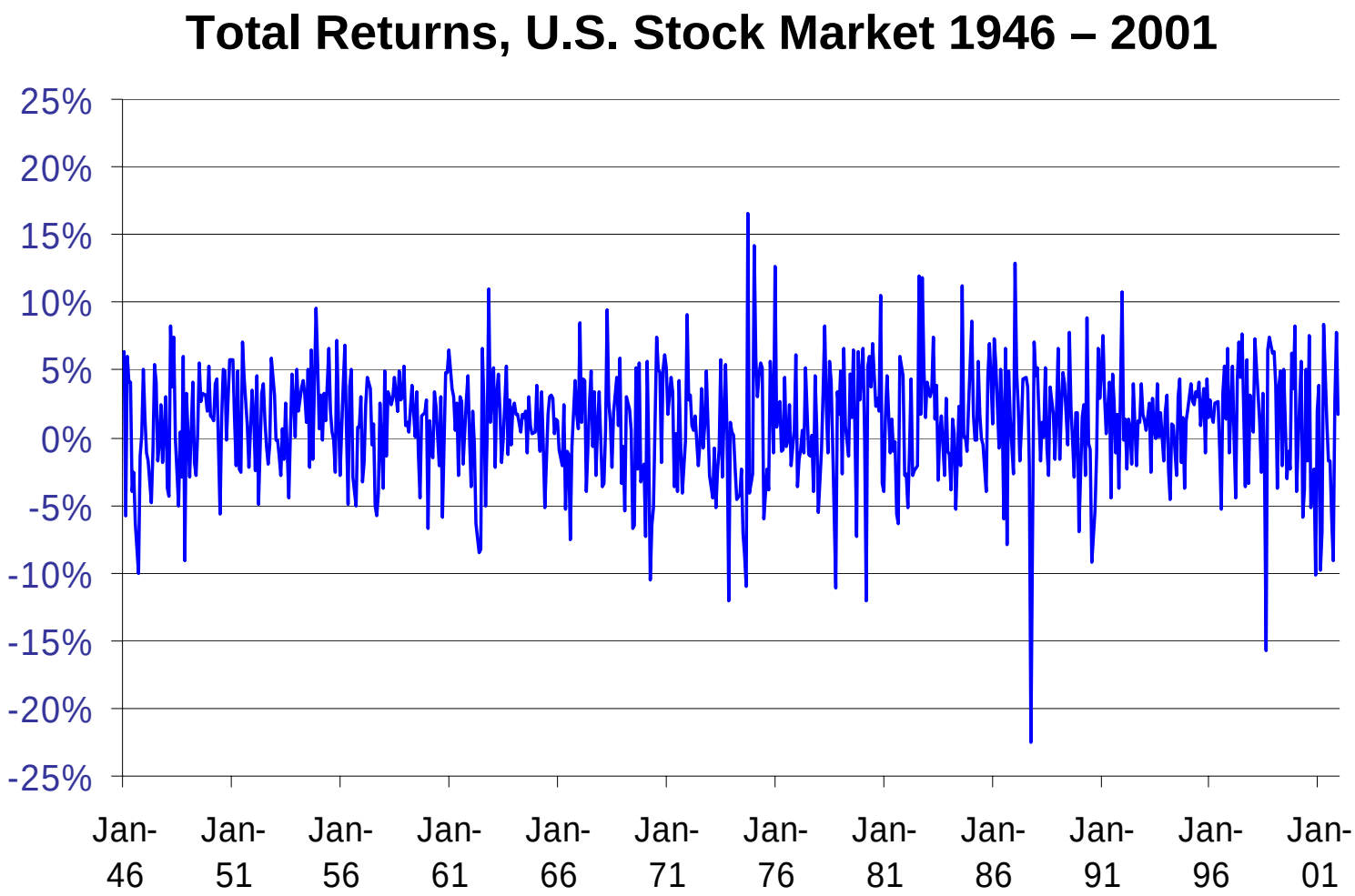
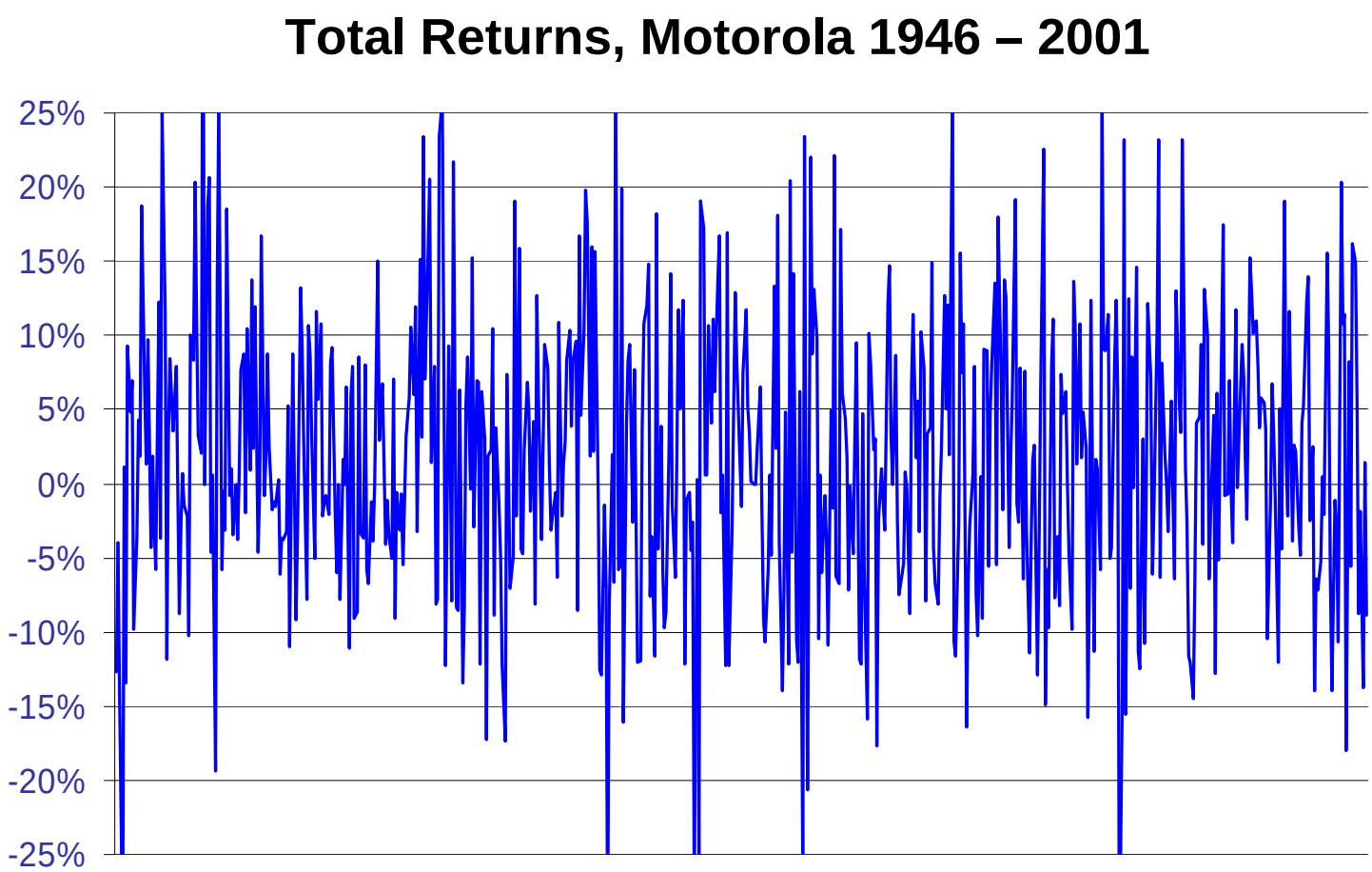
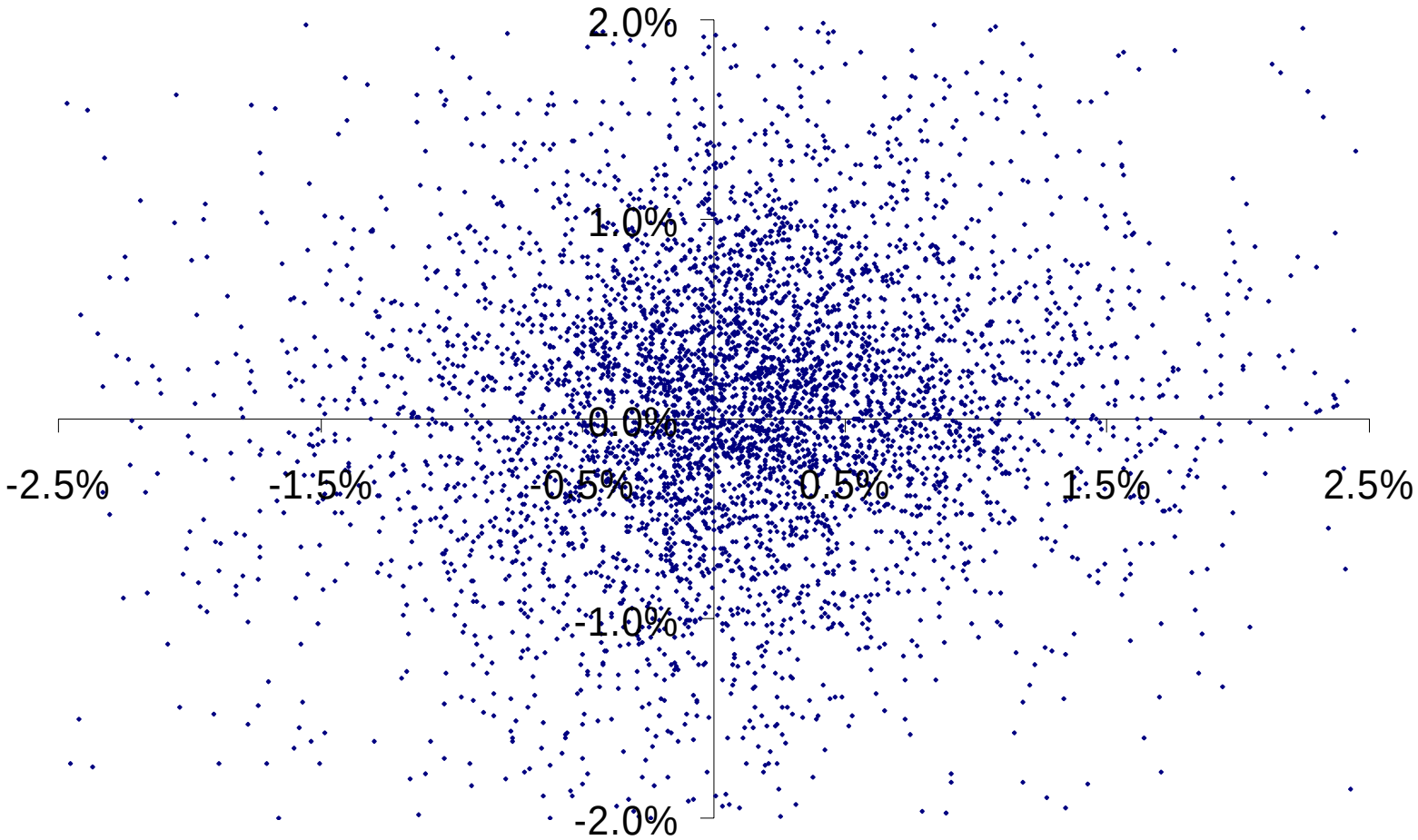


Empirical Properties of Stock Returns



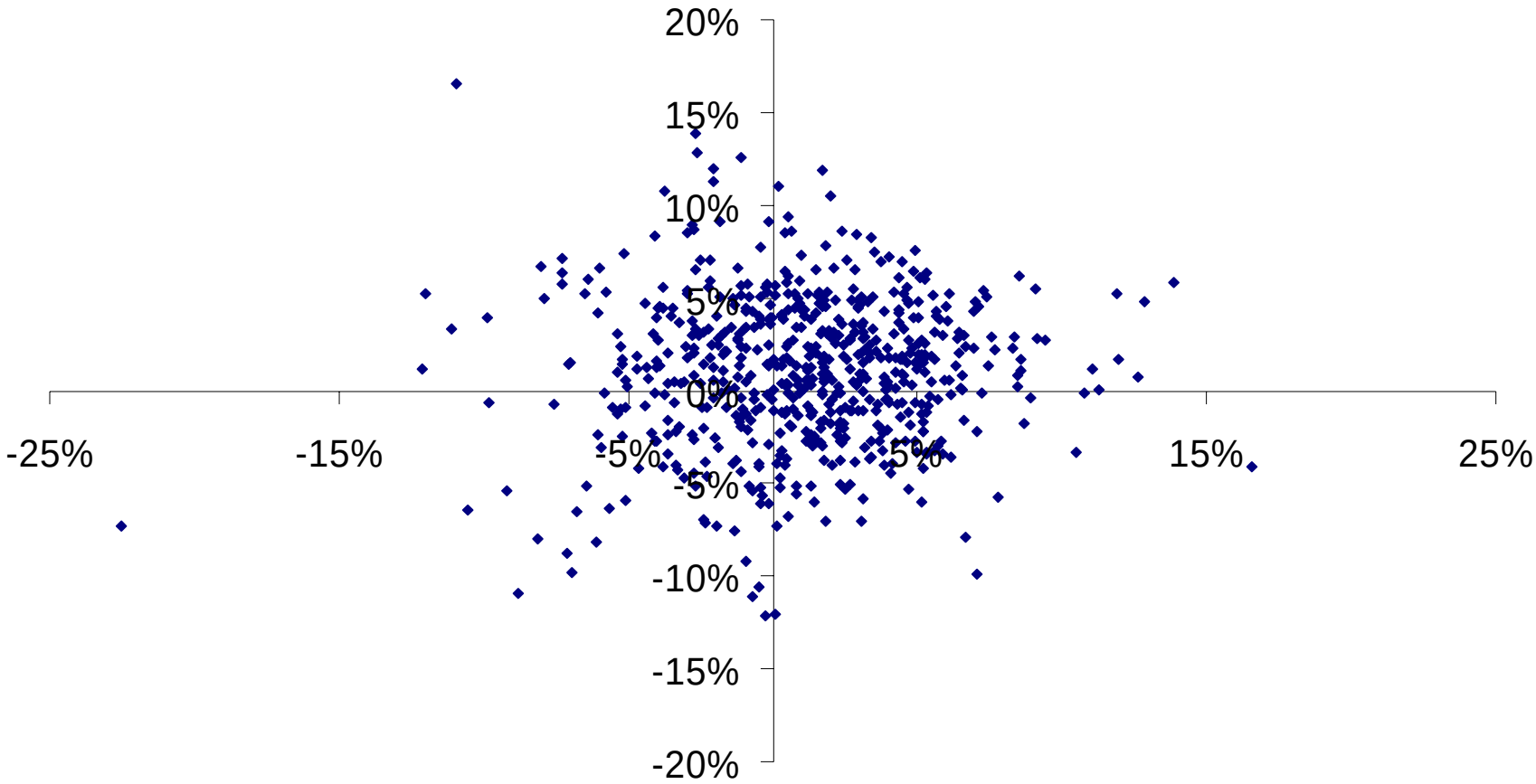
Empirical Properties of Stock Returns

Scatterplot, VWRETD Today vs. Yesterday ,1980 – 1999



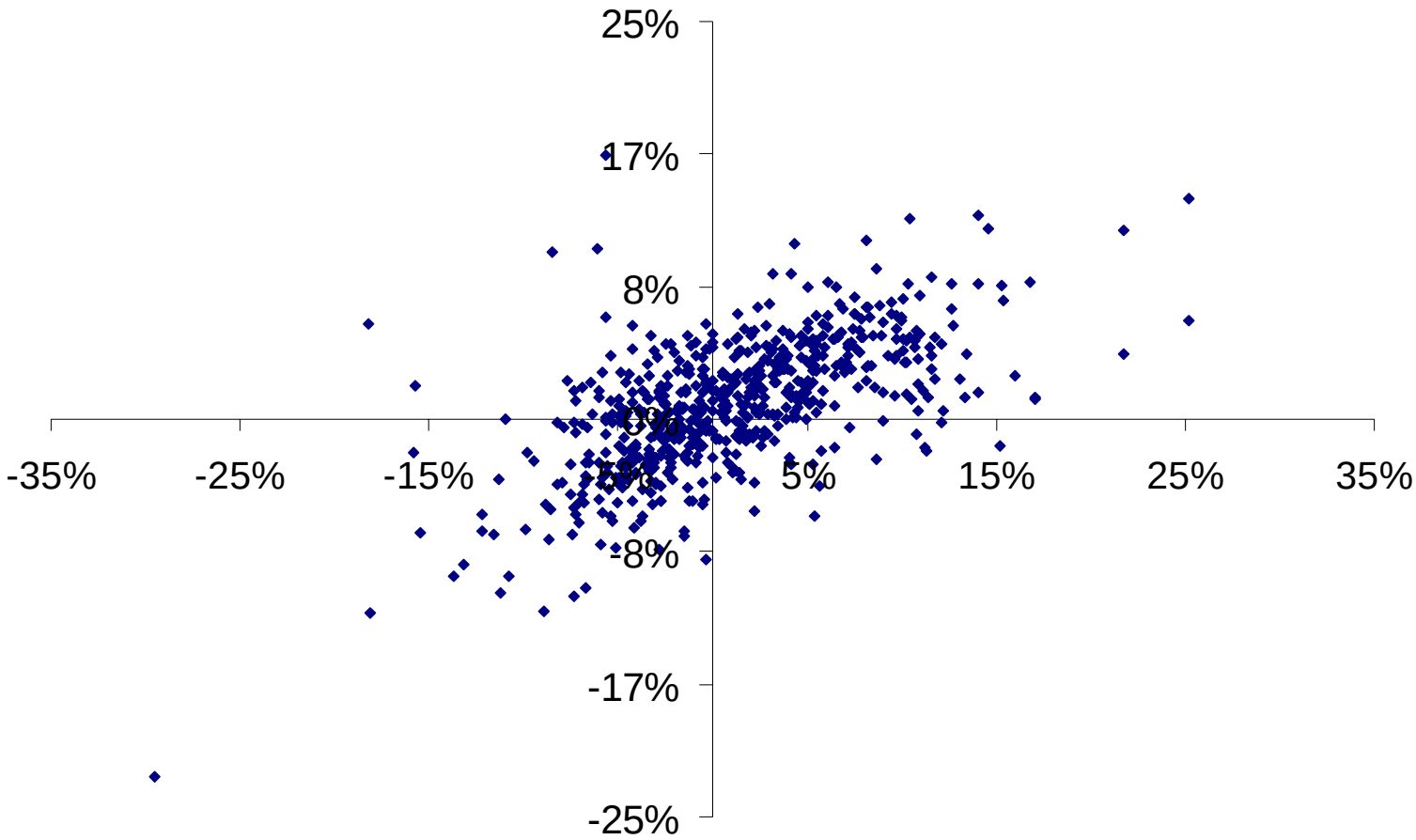
Empirical Properties of Stock Returns

Scatterplot, S&P 500 This Month vs. Last Month, 1926 to 1997



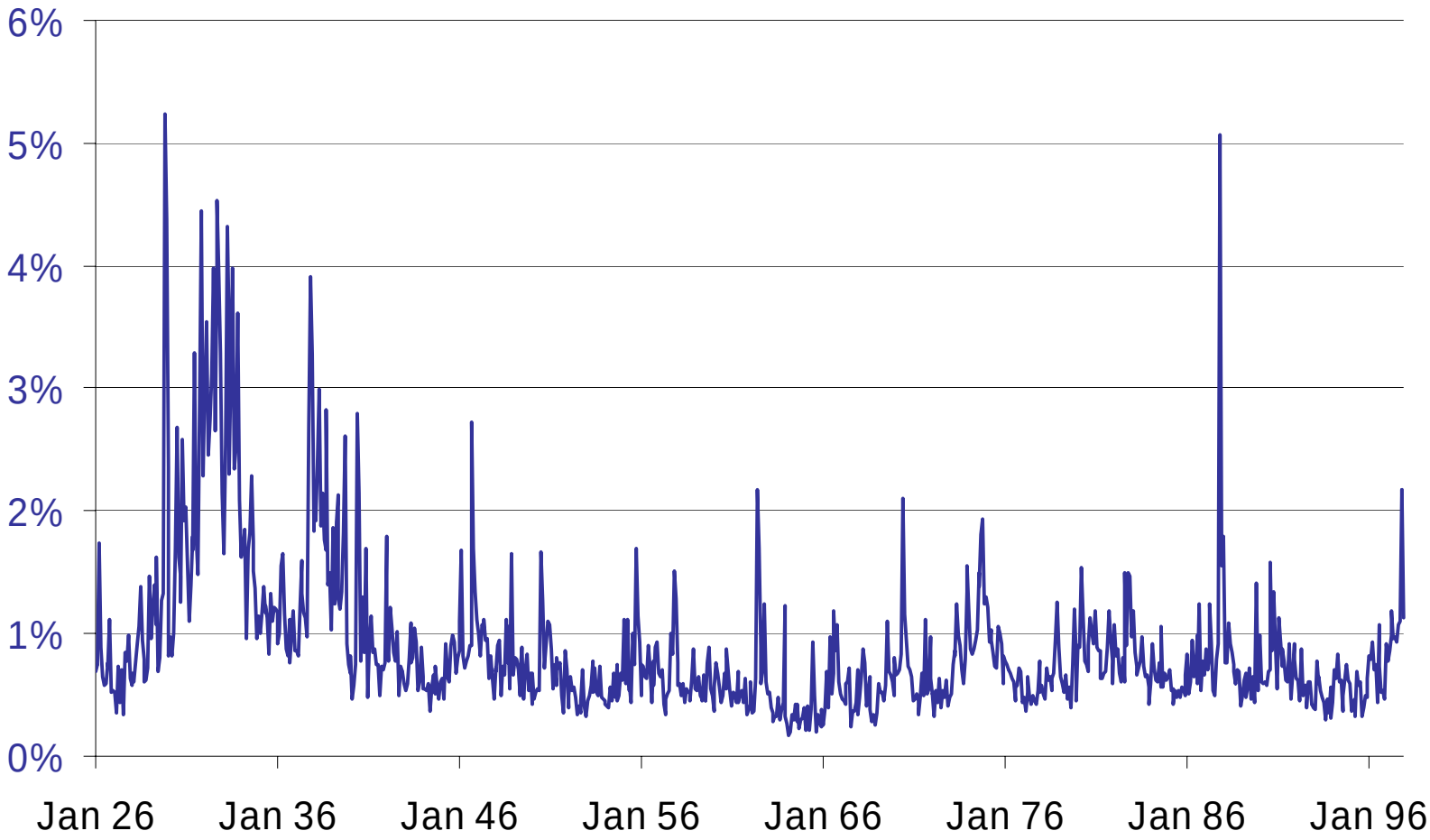
Empirical Properties of Stock Returns

Scatterplot GM vs. S&P 500 Monthly Returns, 1946 – 1997



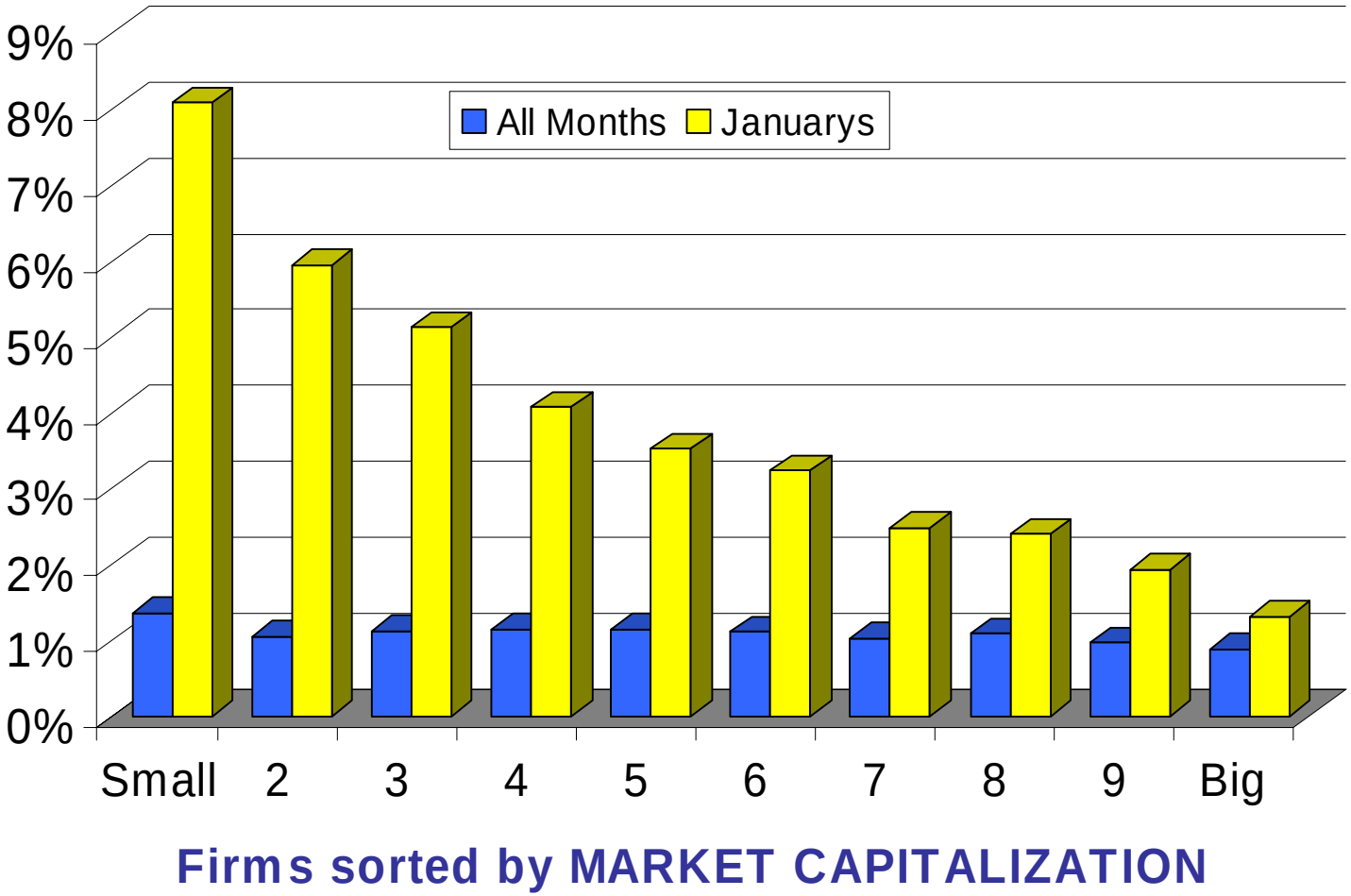
Empirical Properties of Stock Returns

Monthly Estimates of U.S. Stock Market Daily Volatility 1926 – 1997



Anomalies: The Size Effect, 1964 – 2004

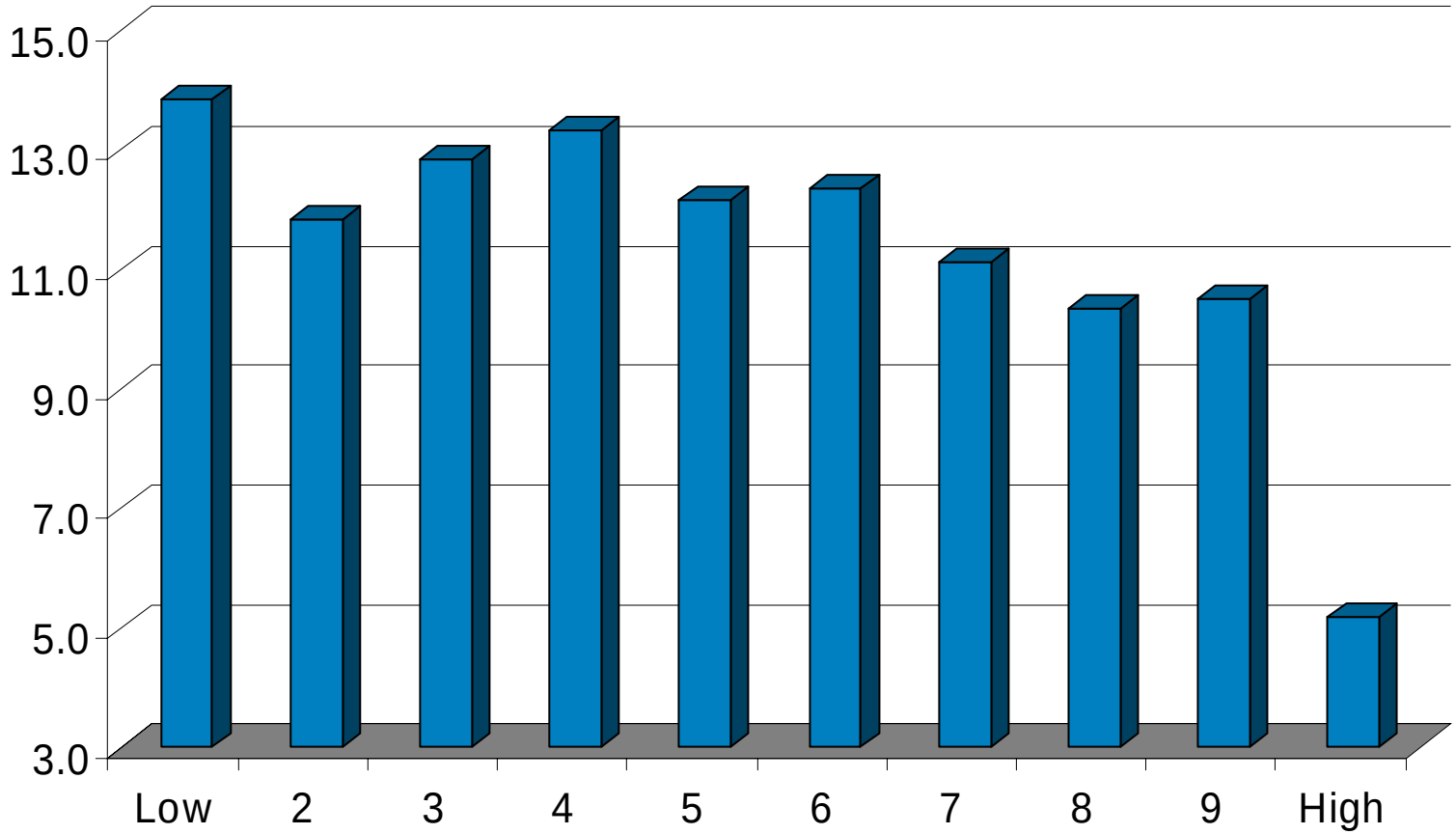
Anomalies: The January Effect, 1964 – 2004



Anomalies: The Value Premium, 1964 – 2004

Anomalies: Momentum, 1964 – 2004

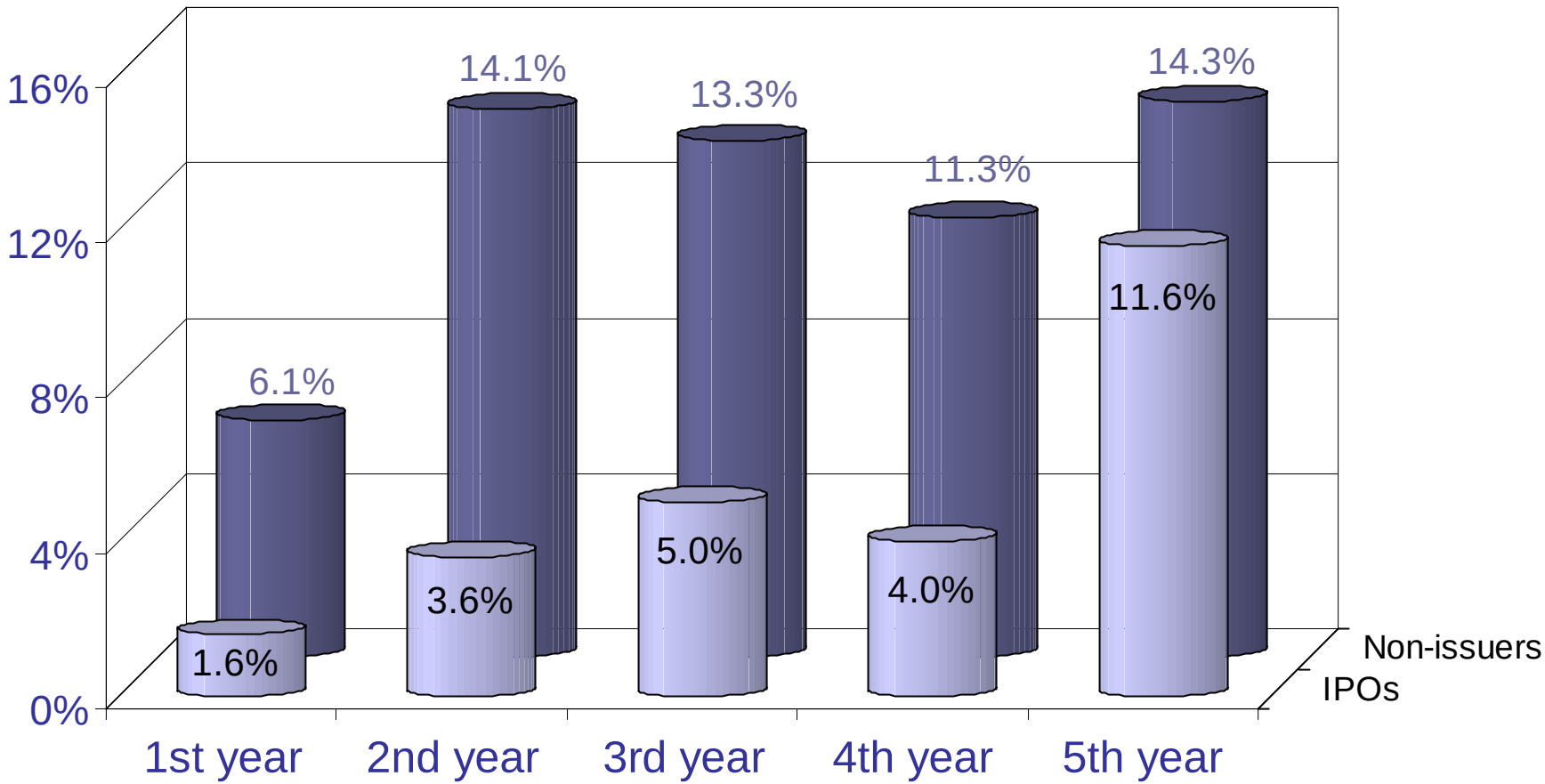
Anomalies: The Accrual Effect, 1964 – 2004



*Operating income minus operating cashflows

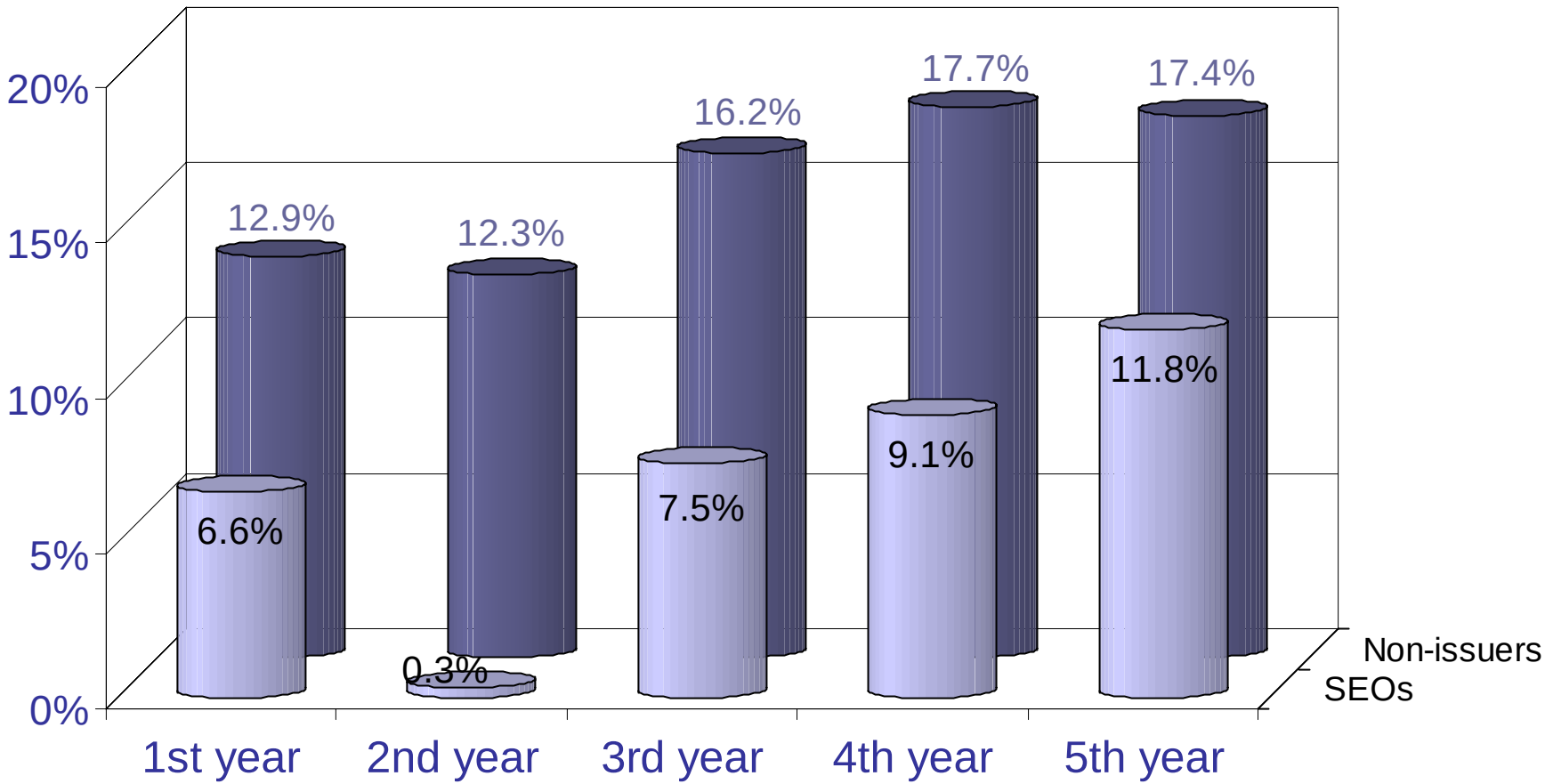
Anomalies: IPO Returns, 1970 – 1990

Average Annual Returns, 1 – 5 Years After IPO



Anomalies: SEO Returns, 1970 – 1990

Average Annual Returns, 1 – 5 Years After SEO



Anomalies: Takeover Announcements

Stock price of TARGET

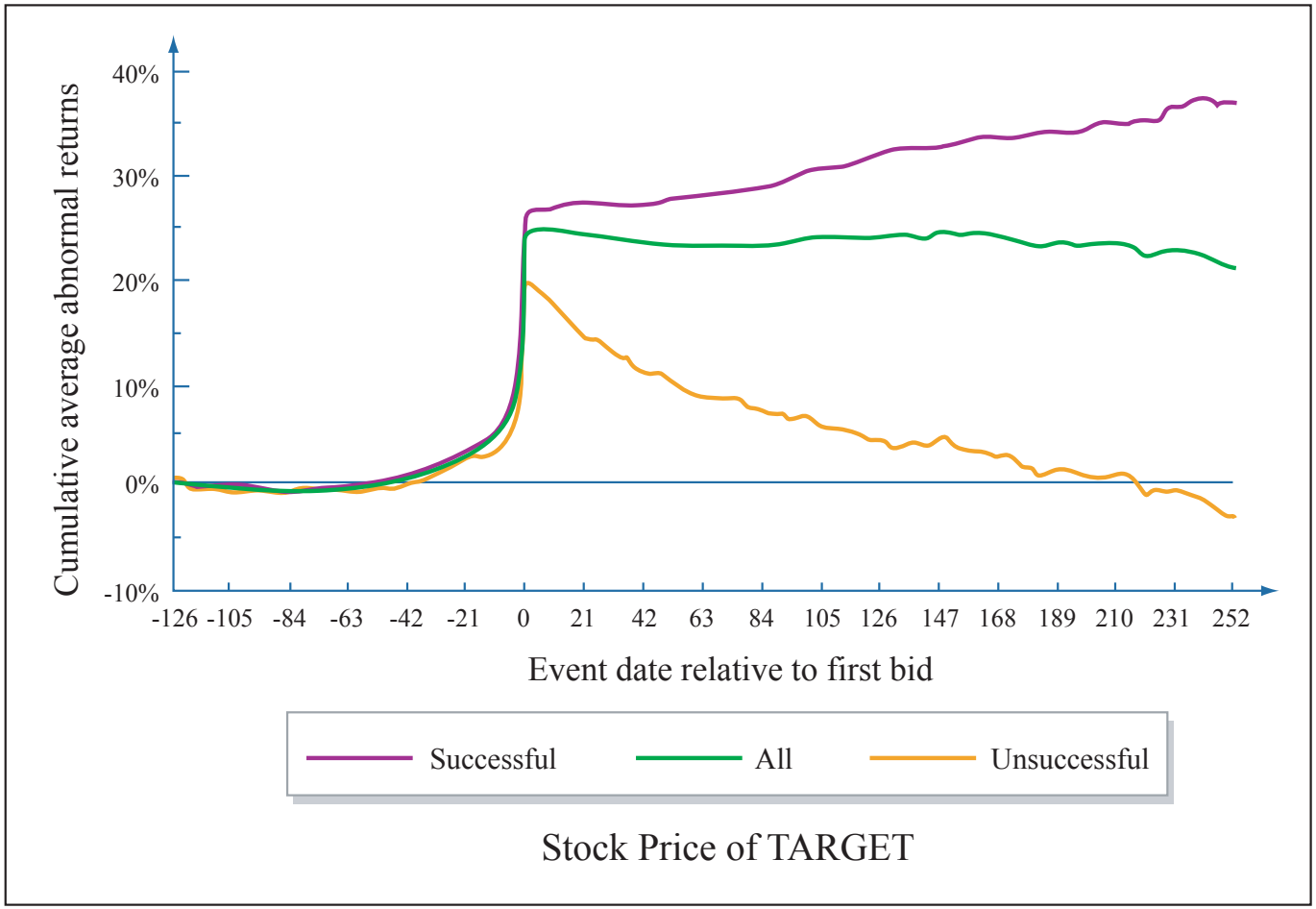
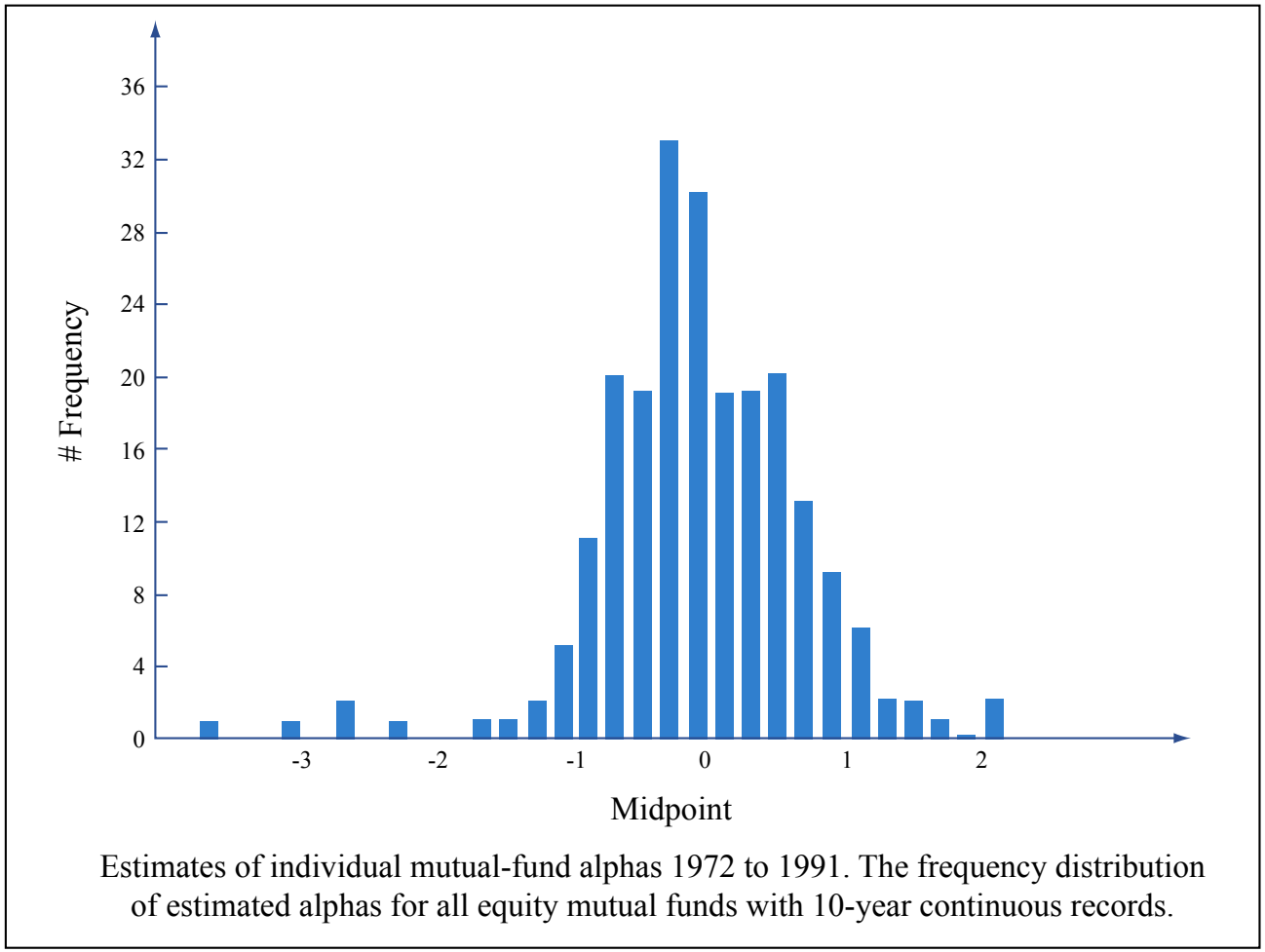


Image by MIT OpenCourseWare.

Anomalies: Performance of Mutual Funds



Estimates of individual mutual-fund alphas 1972 to 1991. The frequency distribution of estimated alphas for all equity mutual funds with 10-year continuous records.

Key Points

Observations

- **The average annual return on U.S. stocks from 1926 – 2004 was 11.2%.** The average risk premium was 7.8%.
- **Stocks are quite risky**. The standard deviation of returns for the overall market is 4.5% monthly (16.4% annually).
- **Individual stocks are much riskier**. The average monthly standard deviation of an individual stock is around 17% (or 50% annually).
- **Stocks tend to move together over time**: when one stock goes up, other stocks are likely to go up as well. The correlation is far from perfect.
- **Stock returns are nearly unpredictable**. For example, knowing how a stock does this month tells you very little about what will happen next month.
- **Market volatility changes over time**. Prices are sometimes quite volatile. The standard deviation of monthly returns varies from roughly 2% to 20%.
- **Financial ratios like DY and P/E ratios vary widely over time**. DY hit a maximum of 13.8% in 1932 and a minimum of 1.17% in 1999. The P/E ratio hit a maximum of 33.4 in 1999 and a minimum of 5.3 in 1917.

Key Points

Anomalies:

- **Size Effect**: Smaller stocks typically outperform larger stocks, especially in January.
- **January Effect**: Returns in January tend to be abnormally high.
- **Value Effect**: Low P/B (value) stocks typically outperform high P/B (growth) stocks.
- **Momentum**: Stocks with high returns over the past 12 months typically continue to outperform stocks with low past returns.
- **Accruals and Issuances**: Stocks with high past accruals and/or recent stock offerings typically underperform stocks with low past accruals and no stock offerings.

Additional References

- Lefevre, E., 2006, *Reminiscences of a Stock Operator*. New York: John Wiley & Sons.
- Malkiel, B., 1996, *A Random Walk Down Wall Street: Including a Life-Cycle Guide to Personal Investing*. New York: W.W. Norton.

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