15.401 Sample Final Exam – Fall 2008

Please make sure that your copy of the examination contains 25 pages (including this one). Write your name and MIT ID number on *every* page.

- You are allowed two $8\frac{1}{2}'' \times 11''$ sheets of notes and one non-programmable non-PDA calculator.

- Answer these examination questions *without* consulting anyone. No scratch paper is allowed; do all your work on these examination pages.

- You have 180 minutes to complete this examination. Credit for each question is proportional to the amount of time you should spend on it. Therefore, do not agonize over a 10-point question without having tackled a 30-point question.

- Use only the space provided.

- Be neat and show your work. You will receive no credit for answers without work. You may receive partial credit for wrong answers with partially correct work.

Good luck!

Some Useful Formulas

$$\text{NPV} = \sum_{k=1}^{n} \frac{C}{(1+r)^k} = \frac{C}{r}\left[1 - \frac{1}{(1+r)^n}\right] \quad \text{(Annuity)} \tag{1}$$

$$\text{NPV} = \sum_{k=1}^{\infty} \frac{C}{(1+r)^k} = \frac{C}{r} \quad \text{(Perpetuity)} \tag{2}$$

$$\text{Price} = \sum_{k=1}^{\infty} \frac{D}{(1+r)^k} = \frac{D}{r} \quad \text{(DDM)} \tag{3}$$

$$\text{Price} = \sum_{k=1}^{\infty} \frac{D(1+g)^{k-1}}{(1+r)^k} = \frac{D}{r-g} \quad \text{(DDM with growth)} \tag{4}$$

$$R_p = \sum_{i=1}^{n} \omega_i R_i \quad \text{(Portfolio Return)} \tag{5}$$

$$\text{E}[R_p] = \sum_{i=1}^{n} \omega_i \text{E}[R_i] \quad \text{(Portfolio Expected Return)} \tag{6}$$

$$\sigma_p^2 = \omega_a^2 \sigma_a^2 + \omega_b^2 \sigma_b^2 + 2\omega_a \omega_b \sigma_a \sigma_b \rho_{ab} \quad \text{(Portfolio Variance)} \tag{7}$$

$$\beta_p = \sum_{i=1}^{n} \omega_i \beta_i \quad \text{(Portfolio Beta)} \tag{8}$$

$$\text{E}[R_i] = R_f + \beta_i(\text{E}[R_m] - R_f) \ , \quad \beta_i \equiv \frac{\text{Cov}[R_i, R_m]}{\text{Var}[R_m]} \quad \text{(CAPM)} \tag{9}$$

15.401 Final Examination 2008 Grade Sheet

1.	_____ /	40
2.	_____ /	15
3.	_____ /	25
4.	_____ /	15
5.	_____ /	10
6.	_____ /	20
7.	_____ /	25
8.	_____ /	30
9.	_____ /	35
10.	_____ /	35
Total	_____ /	250

Question 1 (40 points):

True, false or uncertain? Please explain your answers carefully and fully. No points will be rewarded for a true/false-only answer.

a. (5 points, Ch2Q1a) The duration of a bond maturing at date T is always less than the duration of a zero-coupon bond maturing on the same date.

b. (5 points, Ch3Q2) The market price of a share of stock equals the discounted value of the stream of future earnings per share.

Question 1 (continued):

c. (5 points) Growth stocks must have a plowback ratio > 1.

d. (5 points) If a commercial airline wants to hedge its risk against oil prices, it should go short in oil futures.

Question 1 (continued):

e. (5 points) The value of an American call option is always equal to the value of a European call option.

f. (5 points) Holding everything else constant, the price of a European call option is increasing with increasing risk free interest rate.

Question 1 (continued):

g. (5 points) Investors do not get rewarded for bearing idiosyncratic risk.

h. (5 points, CH7Q7) CAPM implies that all risky assets must have a positive risk premium.

Extra Space for Question 1 only:

Question 2 (15 points, Ch1Q17):

The annual membership fee at your health club is \$750 per year and is expected to increase at 5% per year. A life membership is \$7,500 and the discount rate is 12%. In order to justify taking the life membership, what would your minimum life expectancy need to be?

Question 3 (25 points):

The current prices of three U.S. treasury bonds are as follows:

Maturity	Coupon Rate	Price
1	0%	$97.474
2	5%	$99.593
3	6%	$100.148

Assume that coupons paid yearly and all bonds have a PAR value of $100.

  a. (10 points) What are the 1-, 2- and 3-year spot rates?

  b. (6 points) What are the year 1 to 2 and year 1 to 3 forward rates?

Question 3 (continued):

c. (9 points) What is the price of a three-year bond with a 8% annual coupon.

Extra Space for Question 3 only:

Question 4 (10 points, Ch7Q16):

Consider three stocks: Q, R and S.

	Beta	STD (annual)	Forecast for Nov 2009	
			Dividend	Stock Price
Q	0.45	35%	$0.50	$45
R	1.45	40%	0	$75
S	-0.20	40%	$1.00	$20

Use a risk-free rate of 2.0% and an expected market return of 9.5%. The market's standard deviation is 18%. Assume that the next dividend will be paid after one year, at t = 1.

a. (5 points) According to the CAPM, what is the expected rate of return of each stock?

b. (5 points) What should today's price be for each stock, assuming the CAPM is correct?

Extra Space for Question 4 only:

Question 5 (15 points, Ch3Q20):

Company Ts current return on equity (ROE) is 16%. It pays out one-quarter of earnings as cash dividends (payout ratio = .25). Current book value per share is $35. The company has 5 million shares outstanding.

Assume that ROE and payout ratio stay constant for the next four years. After that, competition forces ROE down to 10% and the company increases the payout ratio to 60%. The company does not plan to issue or retire shares. The cost of capital is 9.5%.

 a. (10 points) What is stock T worth?

 b. (5 points) How much of stock Ts value is attributable to growth opportunities (PVGO)?

Extra Space for Question 5 only:

Question 6 (20 points, Ch4Q5 modified: a) b) identical c) new):

Spot and futures prices for Gold and the S&P in September 2007 are given below.

	07-September	07-December	08-June
COMEX Gold ($/oz)	$693	$706.42	$726.7
CME S&P 500	$1453.55	$1472.4	$1493.7

Table 1: Gold and S&P 500 Prices on September 7, 2007

a. (12 points) Use prices for gold to calculate the effective annualized interest rate for Dec 2007 and June 2008. Assume that the convenience yield for gold is zero.

b. (4 points) Suppose you are the owner of a small gold mine and would like to fix the revenue generated by your future production. Explain how the futures market enables such hedges.

Question 6 (continued):

c. (4 points) Calculate the convenience yield on the S&P index between September 07 and December 07.

Question 7 (25 points, Ch8Q5):

You have developed the technology to use gold to produce high capacity fiber optic switches. The technology has cost $ 5 million to develop. You need $50 million of initial capital investment to start production. Sales of the switch sales will be $20 million per year for the next 5 years and then drop to zero. The main cost of production is gold. Each year, you need 20,000 ounces of gold. Gold is currently selling for $250 per ounce. Your supplier thinks that the gold price will appreciated at 5% per year for the next 5 years. The cost of capital is 10% for the fiber-optics business. The tax rate is 35%. The capital investment can be depreciated linearly over the next 5 years.

    a. Calculate the after-tax cash flows of the project.

    b. Should you take the project?

Extra Space for Question 7 only:

Question 8 (30 points, Ch7Q18):

It is November, 2007. The following variance-covariance matrix, for the market (S&P 500) and stocks T and U, is based on monthly data from November 2002 to October 2007. Assume T and U are included in the S&P 500. The betas for T and U are T = 0.727 and U = 0.75.

	$S\&P500$	T	U
$S\&P500$	0.0256	0.0186	0.0192
T	0.0186	0.1225	0.0262
U	0.0192	0.0262	0.0900

Average monthly risk premiums from 2002 to 2007 were:

$S\&P500 : 1.0\%$
$T : 0.6\%$
$U : 1.1\%$

Assume the CAPM is correct, and that the expected future market risk premium is 0.6% per month. The risk-free interest rate is 0.3% per month.

 a. (10 points) What were the alpha's for stocks T and U over the last 60 months?

Question 8 (continued):

   b. (10 points) What are the expected future rates of return for T and U?

   c. (10 points) What are the optimal portfolio weights for the S&P 500, T and U? Explain qualitatively.

Question 9 (35 points, Ch6Q17modified: a) identical, b) c) new):

Expected returns and standard deviations of three risky assets are as follows:

	Expected Return	Standard Deviation	Correlations A	B	C
A	11%	30%	1.0	.3	.15
B	14.5%	45%	.3	1.0	.45
C	9%	30%	.15	.45	1.0

a. (10 points) Calculate the expected return and standard deviation of a portfolio of stocks A, B and C. Assume an equal investment in each stock.

b. (15 points) Compute the Sharpe ratio of a portfolio that has 30% in A, 30% in B and 40% C. The risk-free interest rate is 4%.

Question 9 (continued):

c. (10 points) Assume a portfolio of asset B and C. Determine the weight in asset B, such that the total portfolio risk is minimized.

Extra Space for Question 9 only:

Question 10 (35 points, Ch5Q25 modified : a) b) identical c) d) new):

You are asked to price options on KYC stock. KYC's stock price can go up by 15 percent every year, or down by 10 percent. Both outcomes are equally likely. KYC does not pay dividend. The risk free rate is 5% (EAR), and the current stock price of KYC is $100.

    a. (15 points) Price a European put option on KYC with maturity of 2 years and a strike price of 100.

Question 10 (continued):

b. (5 points) Price an American put option on KYC with the same characteristics. Is the price different? Why or why not?

c. (5 points) Given the price of the put option that you calculated in a), specify the ranges of KYC share price at the option's maturity date for which you will be making a net profit.

Question 10 (continued):

    d. (10 points) Suppose you expect the price of KYC stock to have little variance in the future. How would you design a strategy (using options) to take advantage of this?

Extra Space for Question 10 only:

15.401 Finance Theory I
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