In this video, we'll discuss how radiation therapy can be framed as an optimization problem.

The data's collected in the treatment planning process, which starts from a CT scan, like the one you see here, on the right.

Using a CT scan, a radiation oncologist contours, or draws outlines around the tumor and various critical structures.

In this image, the oncologist would contour structures like the parotid glands, the largest of the saliva glands, and the brain.

Then, each structure is discretized into voxels, or volume elements, which are typically four millimeters in dimension.

The second image here shows a closer view of the brain.

You can see the small squares, or voxels.

Here, they're two-dimensional, but in reality they would be three-dimensional.

Now, we can compute how much dose each beamlet, or piece of the beam, delivers to each voxel.

We'll start with a small example.

Suppose we have nine voxels and six beamlets.

Our voxels can be categorized into three types: the tumor voxels, which are colored pink here; the spinal cord voxel, colored dark green; and other healthy tissue voxels, colored light green.

So we have four tumor voxels, one spinal cord voxel, and four other healthy tissue voxels.

We have two beams that are each split into three beamlets.

Beam 1 is composed of beamlets 1, 2, and 3, and comes in from the right.

Beam 2 is composed of beamlets 4, 5, and 6, and comes in from the top.

Our objective is to minimize the total dose to healthy tissue, both to the spinal cord and to the other healthy tissue.

We have two types of constraints.

The first is that the dose to the tumor voxels must be at least 7 Gray, which is the unit of measure for radiation.

Our second constraint is that the dose to the spinal cord voxel can't be more than 5 Gray, since we want to be careful to protect the spinal cord.

We know the dose that each beamlet gives to each voxel at unit intensity.

This table shows the dose that each beamlet in Beam 1 gives to the voxels.

Remember that this is at unit intensity.

If we double the intensity of the beamlet, we double the doses.

The dose to each voxel can depend on how far the beamlet has to travel, or the type of tissue that the beamlet has to travel through.

Similarly, we know the dose that each beamlet in Beam 2 gives to each voxel, again at unit intensity.

The dose depends on the direction of the beam and what it travels through.

Putting these tables together, we can write out our optimization problem.

Our decision variables are the intensities of each beamlet.

We'll call them x_1, the intensity for beamlet 1, x_2, the intensity for beamlet 2, x_3, the intensity for beamlet 3, etc., all the way up through x_6.

As we mentioned before, our objective is to minimize the total dose to the healthy tissue, including the spinal cord.

So we want to minimize the total dose beamlet 1 gives to healthy tissue, which is (1 + 2)*x_1, plus the total dose beamlet 2 gives to healthy tissue, which is (2 + 2.5)*x_2, plus the total dose beamlet 3 gives to healthy tissue, which is 2.5*x_3.

Now for beamlets 4, 5, and 6, beamlet 4 just gives one dose to healthy tissue, beamlet 5, 2*x_5, and then beamlet 6, we have (1 + 2 + 1)*x_6.

Now for our constraints.

First, we need to make sure that each voxel of the tumor gets a dose of at least 7.

Let's start with the first tumor voxel in the top row.

So $2*x_1 + x_5$ needs to be greater than or equal to 7.

Now the tumor voxel in the second row, we have $x_2 + 2*x_4$, also greater than or equal to 7.

Now for the two tumor voxels in the bottom row, we have $1.5*x_3 + x_4$, greater than or equal to 7.

And $1.5*x_3 + x_5$, greater than or equal to 7.

Then for the spinal cord, we need to make sure that $2*x_2 + 2*x_5$ is less than or equal to 5.

And lastly, we just need to make sure that all of our decision variables are non-negative.

So they should all be greater than or equal to 0.

Now that we've set up our optimization problem, we'll solve it in LibreOffice in the next video.