# Portfolio Risk Management



Summer 2003

# Hot Off the Press

- Please see the article:

  Daniel McGinn, "Let's Make a (Tough) Deal", *Newsweek*, June 23, 2003, pp. 48-49.

# *An Investment Decision*

*Question:* Given the choice to invest *$1 million* in these three firms, which one(s) would you choose?

| Asset | Expected Return (% per year) | Risk (% per year) |
|-------|:---:|:---:|
| Exxon | 11.2 | 13.9 |
| Boeing | 9.1 | 16.5 |
| GM | 12.1 | 15.8 |

# *Outline*

- Covariance and correlation

- Sums of random variables

- Asset diversification in investing

# Covariance and Correlation

How do we describe the relationship between two random variables?

*Example:* Chain of upscale cafés sells gourmet <u>hot</u> coffees and <u>cold</u> beverages. Let X = # hot coffees, Y = # cold beverages sold per day. Historical data at their Harvard Square café results is the following joint probability distribution p(X,Y) for X and Y.

| Probability $p_i$ | Number of Hot Coffees Sold $x_i$ | Number of Cold Drinks Sold $y_i$ |
|---|---|---|
| 0.10 | 360 | 360 |
| 0.10 | 790 | 110 |
| 0.15 | 840 | 30 |
| 0.05 | 260 | 90 |
| 0.15 | 190 | 450 |
| 0.10 | 300 | 230 |
| 0.10 | 490 | 60 |
| 0.10 | 150 | 290 |
| 0.10 | 550 | 140 |
| 0.05 | 510 | 290 |
| **Mean** | E(X)=457.0 | E(Y)=210.0 |
| **Standard Deviation** | SD(X)=244.3 | SD(Y)=145.6 |

# Scatter Plot of Daily Sales for
# Hot Coffees and Cold Beverages
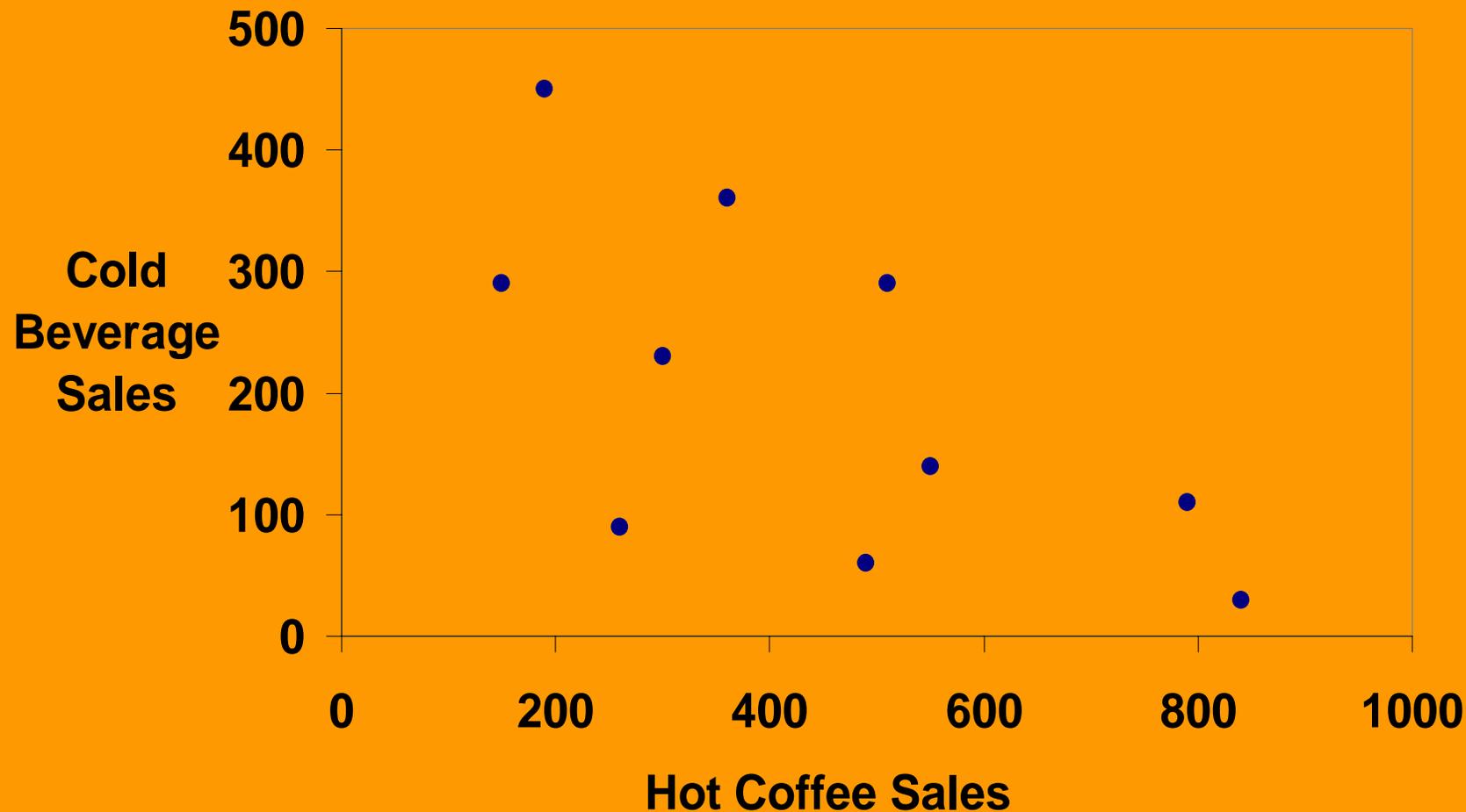


*Comment:* It seems that smaller sales of hot coffees are often accompanied by larger sales of cold beverages.

Scatter Plot of Daily Sales for Hot Coffees and Cold Beverages
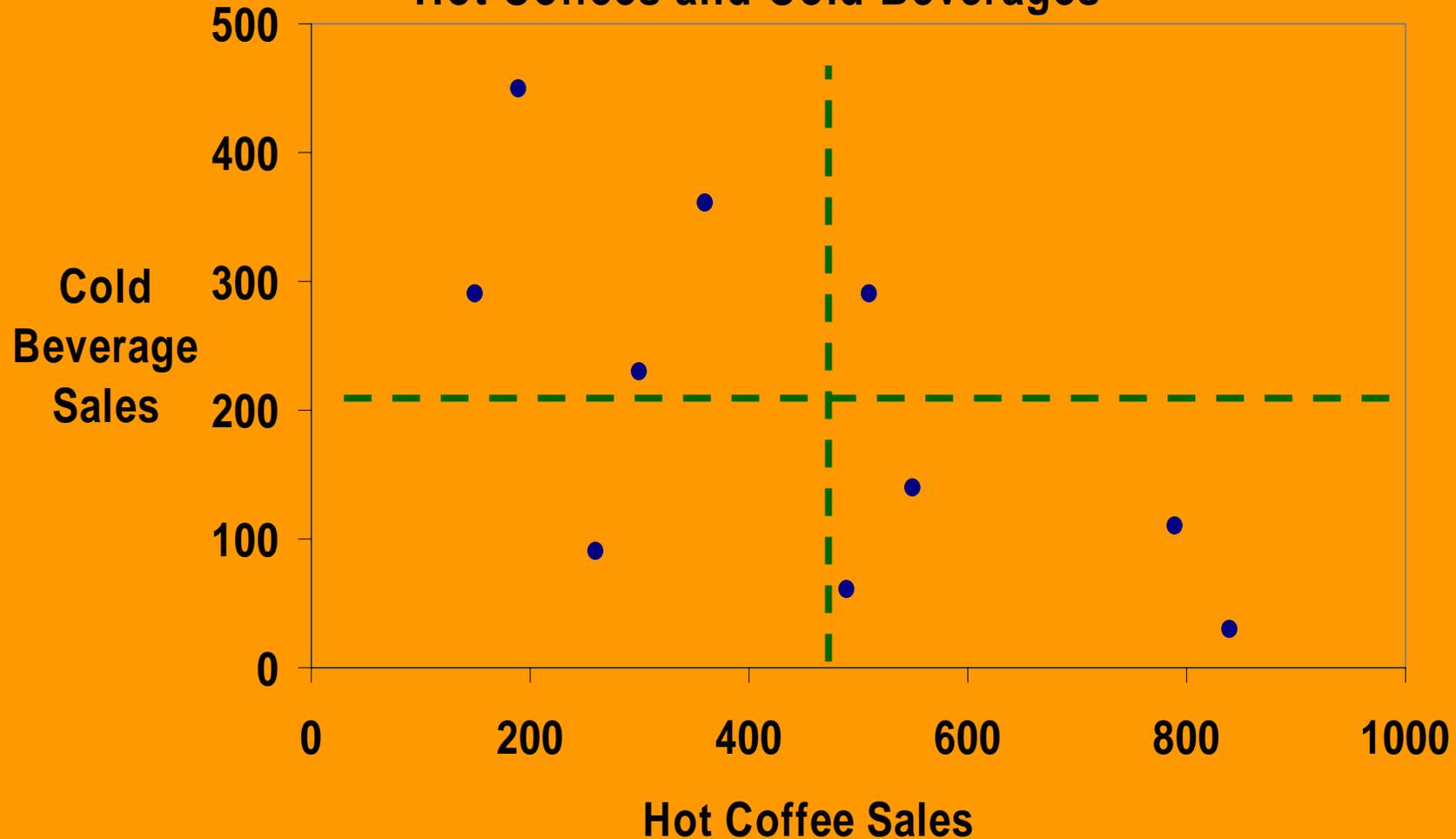
*Comment:* Hot coffee sales *greater* than the average number sold per day are typically accompanied by cold beverage sales that are *smaller* than the average sold per day.

# Describing Joint Relationships

*Covariance:*

$$\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$
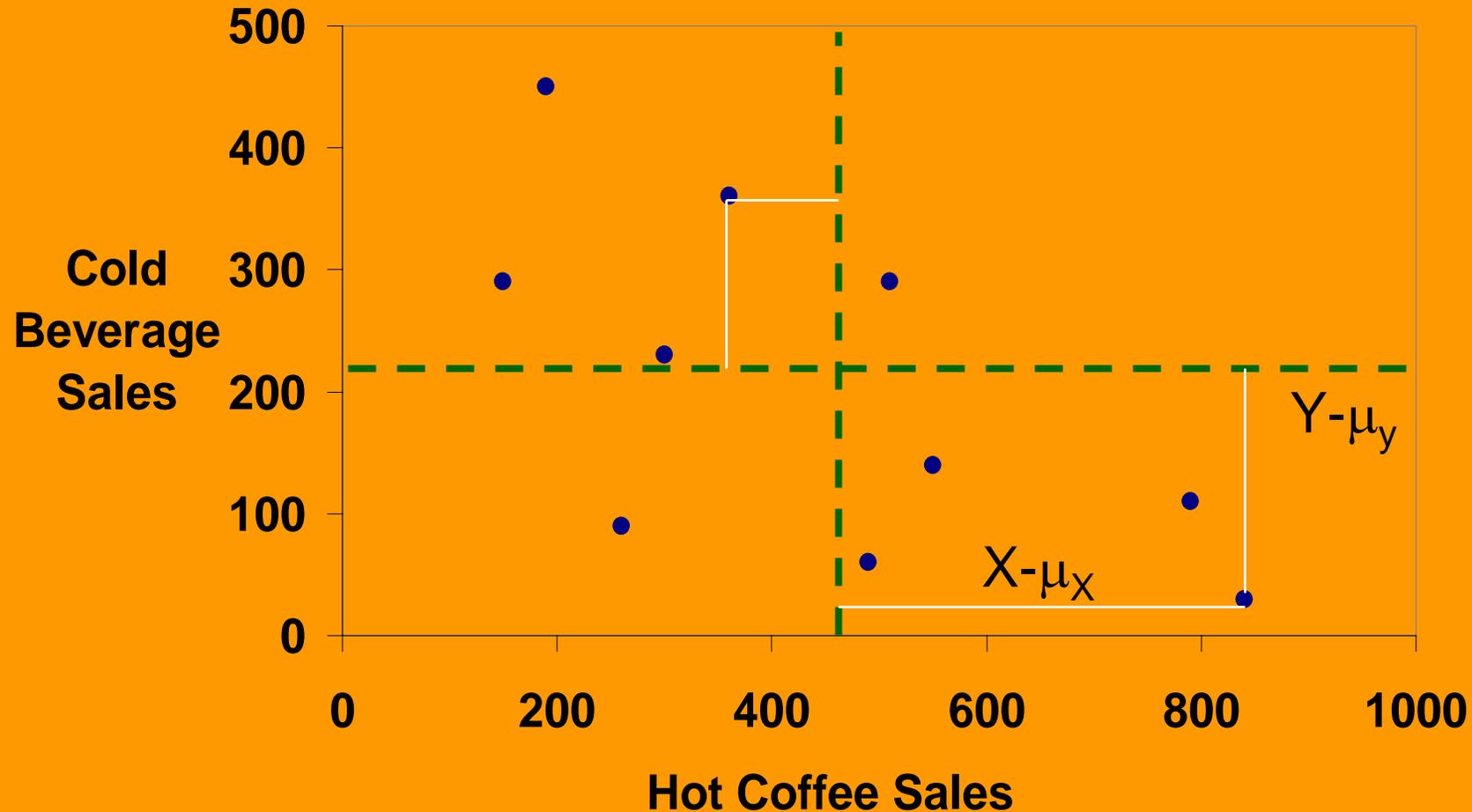$$= \sum_i P(X = x_i, Y = y_i)[(x_i - \mu_X)(y_i - \mu_Y)]$$

*Correlation:*

$$\text{CORR}(X, Y) = \frac{\text{COV}(X, Y)}{\sigma_X \sigma_Y}$$

Comments . . .
- The measure of correlation is unit-free.
- CORR(X, Y) is always between -1.0 and 1.0 .

**Scatter Plot of Daily Sales for Hot Coffees and Cold Beverages**

Cold Beverage Sales (y-axis: 0, 100, 200, 300, 400, 500)

Hot Coffee Sales (x-axis: 0, 200, 400, 600, 800, 1000)

$Y-\mu_y$

$X-\mu_X$

*Comment:* Hot coffee sales *greater* than the average number sold per day are typically accompanied by cold beverage sales that are *smaller* than the average sold per day.

# X = # hot coffees, Y = # cold beverages sold per day

| Probability $p_i$ | Number of Hot Coffees Sold $x_i$ | Number of Cold Drinks Sold $y_i$ |
|---|---|---|
| 0.10 | $*$ (360 -457 ) | $*$ (360 -210) |
| 0.10 | $*$ (790 -457 ) | $*$ (110 -210) |
| 0.15 | 840 | 30 |
| 0.05 | . 260 | . 90 |
| 0.15 | 190 | 450 |
| 0.10 | . 300 | . 230 |
| 0.10 | . 490 | . 60 |
| 0.10 | 150 | 290 |
| 0.10 | . 550 | . 140 |
| 0.05 | $*$ (510 -457 ) | $*$ (290 -210 ) |
| **Mean** | E(X)=457.0 | E(Y)=210.0 |
| **Standard Deviation** | SD(X)=244.3 | SD(Y)=145.6 |

$$Cov(X,Y) = 0.10*(360-457)*(360-210) + 0.10*(790-457)*(110-210)$$

$$+ . . . + 0.05*(510-457)*(290-210) = -23{,}702$$

Corr(X,Y) = -23,702/{(244.3)*(145.6)} = -0.67

# *Comments . . .*

If higher than average values of X are apt to occur with higher than average values of Y, then COV(X, Y)>0 and CORR(X,Y)>0.  i.e., X and Y are positively correlated.

If higher than average values of X are apt to occur with lower than average values of Y, then COV(X, Y) < 0 and CORR(X, Y)<0. i.e.,  X and Y are negatively correlated.

# Linearly related random variables have perfect correlation

If $Y=aX+b$, with $a>0$,  then $Corr(X,Y)=1$

If $Y=aX+b$, with $a<0$,  then $Corr(X,Y)=-1$

"Perfect correlation" !

Examples:  X= temperature in F; Y= temperature in C

X= #hot coffees sold in a day;

Y= revenue from hot coffee/day

# Caution: Correlation is not the same as Causality!

(See "Cause or Correlation", *The Economist*, October 1998.)

# *Sums of Random Variables*

Example:       cold beverages are $2.50/glass;
                       (Y: the number of cold beverages sold in a day )
                       hot coffees are $1.50/cup.
                        (X: the number of hot coffees sold in a day )

*Find:*

- The mean and standard deviation of daily $ sales of cold beverages?

$$E(2.5Y) = ? \qquad Var(2.5Y) = ?$$

- The mean and standard deviation of daily $ sales of hot coffees?

$$E(1.5X) = ? \qquad Var(1.5X) = ?$$

- The mean and standard deviation of total daily $ sales of both beverages?

$$E(1.5X + 2.5Y) = ? \qquad Var(1.5X + 2.5Y) = ?$$

# *Mean of the Sum of Random Variables*

"Expectation of a weighted sum of random variables equals the weighted sum of the expectations"

Rule 1:   $E(aX + bY) = aE(X) + bE(Y)$

# *Variance of the Weighted Sum of Random Variables*

Rule 2:

$$VAR(aX + bY) = a^2VAR(X) + b^2VAR(Y) + 2abCOV(X, Y)$$

or, equivalently:

$$VAR(aX + bY) = a^2VAR(X) + b^2VAR(Y) + 2ab\sigma_X\sigma_Y CORR(X, Y)$$

If X and Y are independent then Cov(X,Y)=0 and Corr(X,Y)=0

# Comments:

If X and Y are independent, then VAR(X + Y) = VAR(X) + VAR(Y)

"Variance of a sum of independent random variables is the sum of the variances"

Note that in particular, that these rules imply:

$$E(aX+b) = aE(X)+b$$

$$VAR(aX+b) = a^2 VAR(X)$$
$$\text{and}$$
$$SD(aX+b) = |a|\, SD(X)$$

The constant b shifts the distribution but does not change its shape

Hot coffees (X)
$1.50/cup
Cold beverages (Y)
$2.50/glass

$E(X) = 457.0 \quad SD(X) = 244.3$

$E(Y) = 210 \quad\quad SD(Y) = 145.6$

- The mean and SD of daily sales of cold beverages?

$E(2.5Y) = 2.5*E(Y) = 2.5*210 = \$525, \quad SD(2.5Y) = 2.5*145.6 = \$364$

- The mean and SD of daily sales of hot coffees?

$E(1.5X) = 1.5*E(X) = 1.5*457 = \$685.5, \quad SD(1.5X) = 1.5*244.3 = \$366.45$

- The mean and SD of total daily sales of all beverages?

$E(1.5X+2.5Y) = 1.5*E(X)+2.5*E(Y) = 685.5+525 = \$1,210.5$

$Var(1.5X+2.5Y) = (1.5)^2*(244.3)^2 + (2.5)^2*(145.6)^2 + 2*1.5*2.5*244.3*145.6*(-0.6663) = 89,029$

$SD(1.5X+2.5Y) = sqrt\{89,029\} = \$298.38$

| | Cold | Hot | Total |
|---|---|---|---|
| $\mu$ | $525 | $685.5 | $1,210.5 |
| $\sigma$ | $364 | $366.45 | $298.38 |

# Variance Reduction

- Notice the surprise in the previous analysis

- The variance and standard deviation of the sum of hot and cold drink sales is ***less*** than the variance and standard deviation of either one alone

- This is the principle of diversification to reduce variation and risk

# *Asset Diversification*

Asset diversification is the cornerstone of modern finance.

The simple analysis that follows extends to yield the CAPM  (Capital Asset Pricing Model) and portfolio models and portfolio theory.  This is a key business concept.

The Nobel Prize in economics (1990) was awarded to Merton Miller, William Sharpe and Harry Markowitz for their work on portfolio theory and portfolio models (and the implications for asset pricing).

# *Expected Return and Risk in Investing*

- The future rate of return on investment in an asset is a random variable.

- Other things being equal, investors prefer higher expected return.

- Other things being equal, investors prefer lower risk.

- We measure risk with the standard deviation of the rate of return.

| Asset | Expected Return (% per year) | Risk (% per year) | Correlation | | |
|---|---|---|---|---|---|
| | | | Exxon | Boeing | G M |
| Exxon | 11.2 | 13.9 | ----- | 0.41 | 0.13 |
| Boeing | 9.1 | 16.5 | 0.41 | ---- | -0.22 |
| GM | 12.1 | 15.8 | 0.13 | -0.22 | ---- |

- Given the choice to invest all money in one of these three, rational investors would not choose Boeing: it has lower expected return and more risk.

- Most of us are risk-averse about certain investment decisions, some more than others (as we have seen in various decision examples).

**Risk and Expected Return**
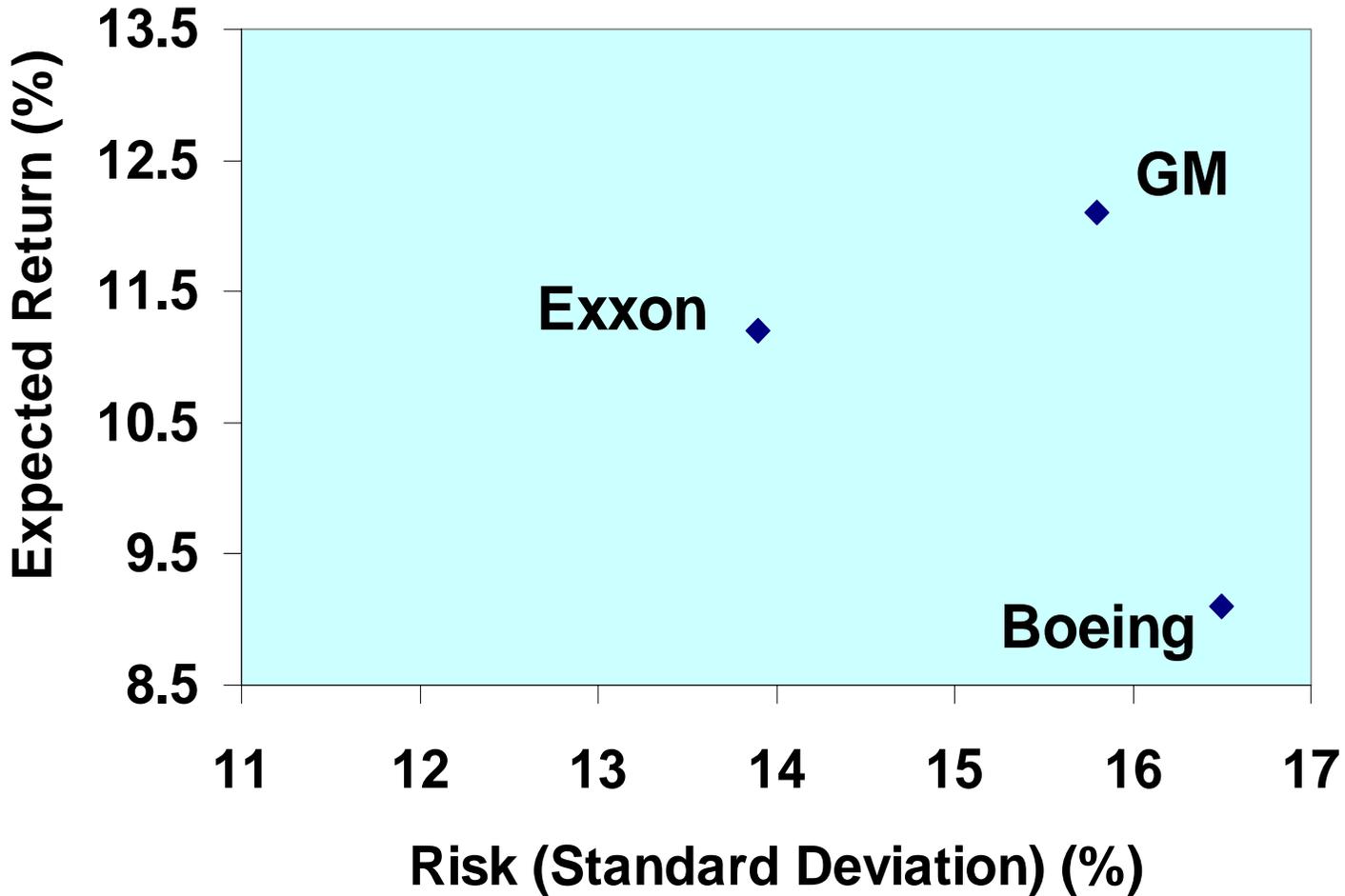
# Diversified Portfolio

Let  X (for Boeing) and  Y (for GM) be the r.v.s that denote the annual return (in %) on these two assets.  Suppose we have reliable estimates for $m_X$,  $m_Y$,  $s_X$,  $s_Y$, and  Corr(X,Y)  as given in the table.

Suppose that we invest a fraction f in Boeing and a fraction 1-f in GM
Let W = annual return on our diversified portfolio of the two assets. Then,

W = f X + (1-f ) Y

$m_W$ = E(W)  =  f E(X)  +  (1 - f) E(Y)

$\boxed{m_W = E(W)  =  9.1 \, f + 12.1 \, (1\text{-}f)}$

$$\boxed{\begin{array}{l} \mu_X = 9.1, \ \mu_Y = 12.1, \\ \sigma_X = 16.5, \ \sigma_Y = 15.8 \\ (\text{ in } \%) \\ Corr(X,Y) = -0.22 \end{array}}$$

Var(W) =  $f^2$ Var(X) + $(1- f)^2$ Var(Y) + 2 f (1-f) $s_X$ $s_Y$ Corr(X, Y)

Var(W) =  $16.5^2 f^2$ +  $15.8^2 (1\text{-}f)^2$ +  2 f (1-f) (16.5)(15.8)(-0.22)

$$\boxed{\sigma_W = SD(W) = \sqrt{272.25 \, f^2 + 249.64 \, (1\text{-}f)^2 + 2 f (1\text{-}f)(16.5)(15.8)(-0.22)}}$$

$\mu_X = 9.1, \ \mu_Y = 12.1$

All GM

All Boeing

**Expected Return (%)**

13

12

11

10

9

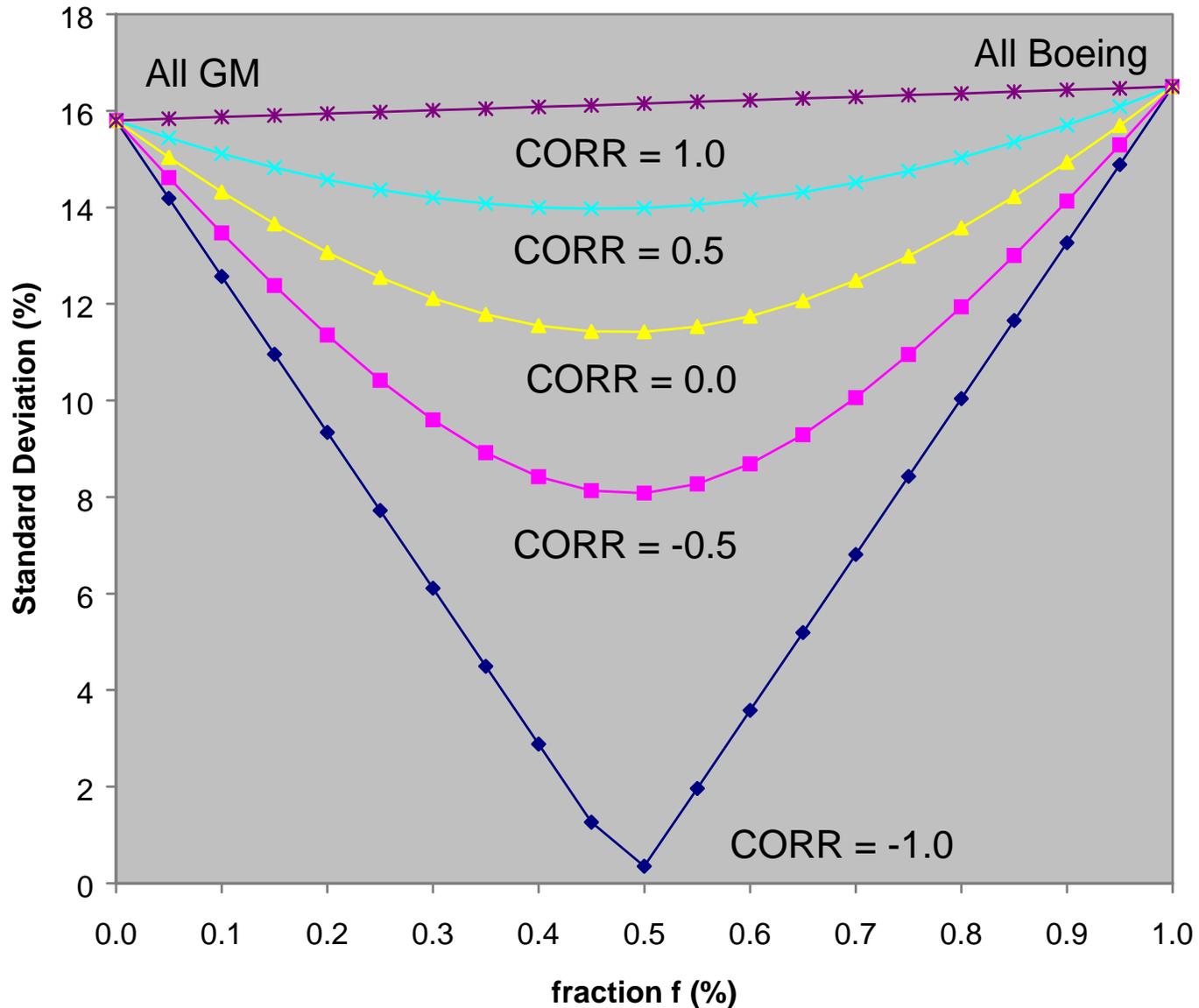0    0.2    0.4    0.6    0.8    1    1.2

**fraction f**
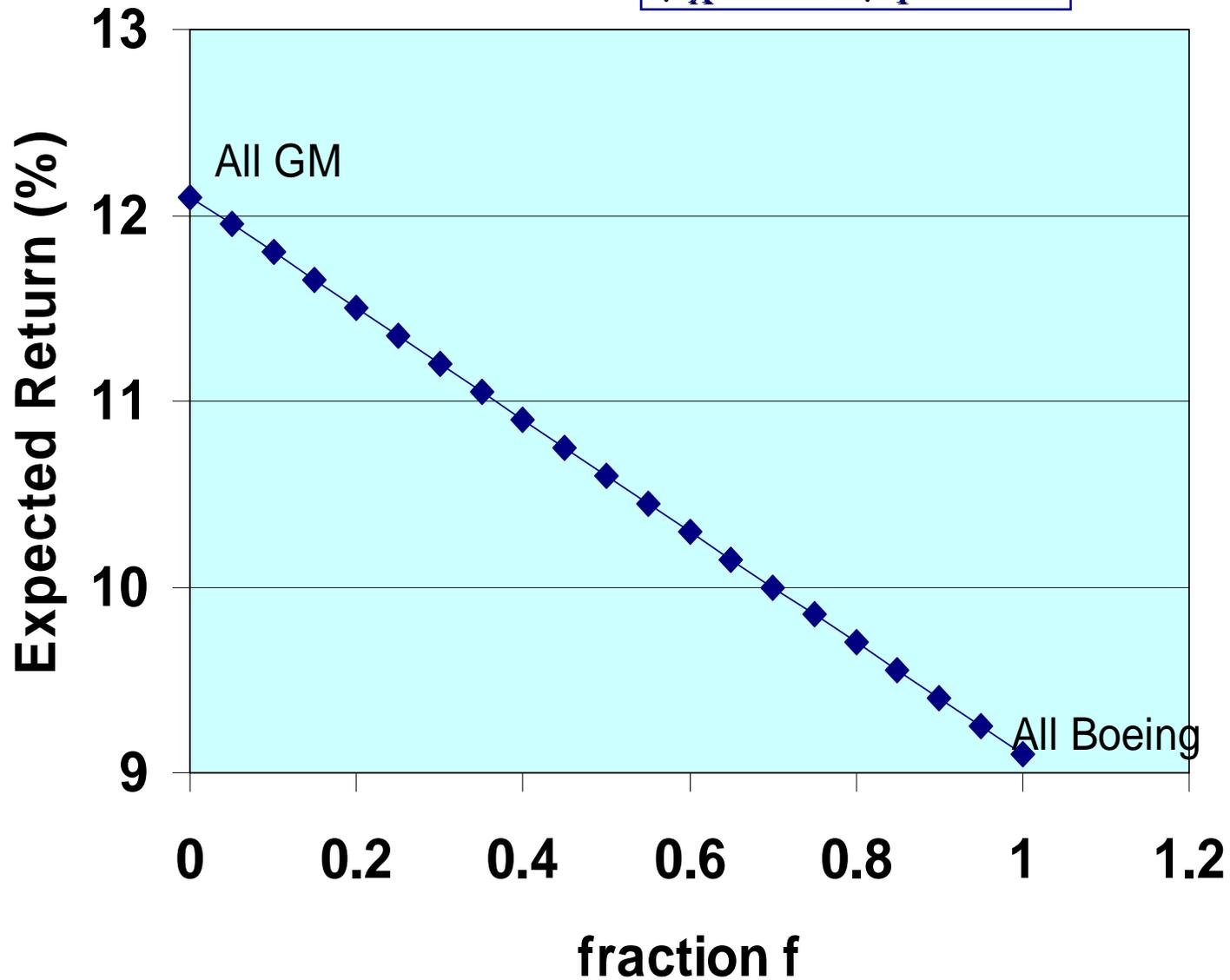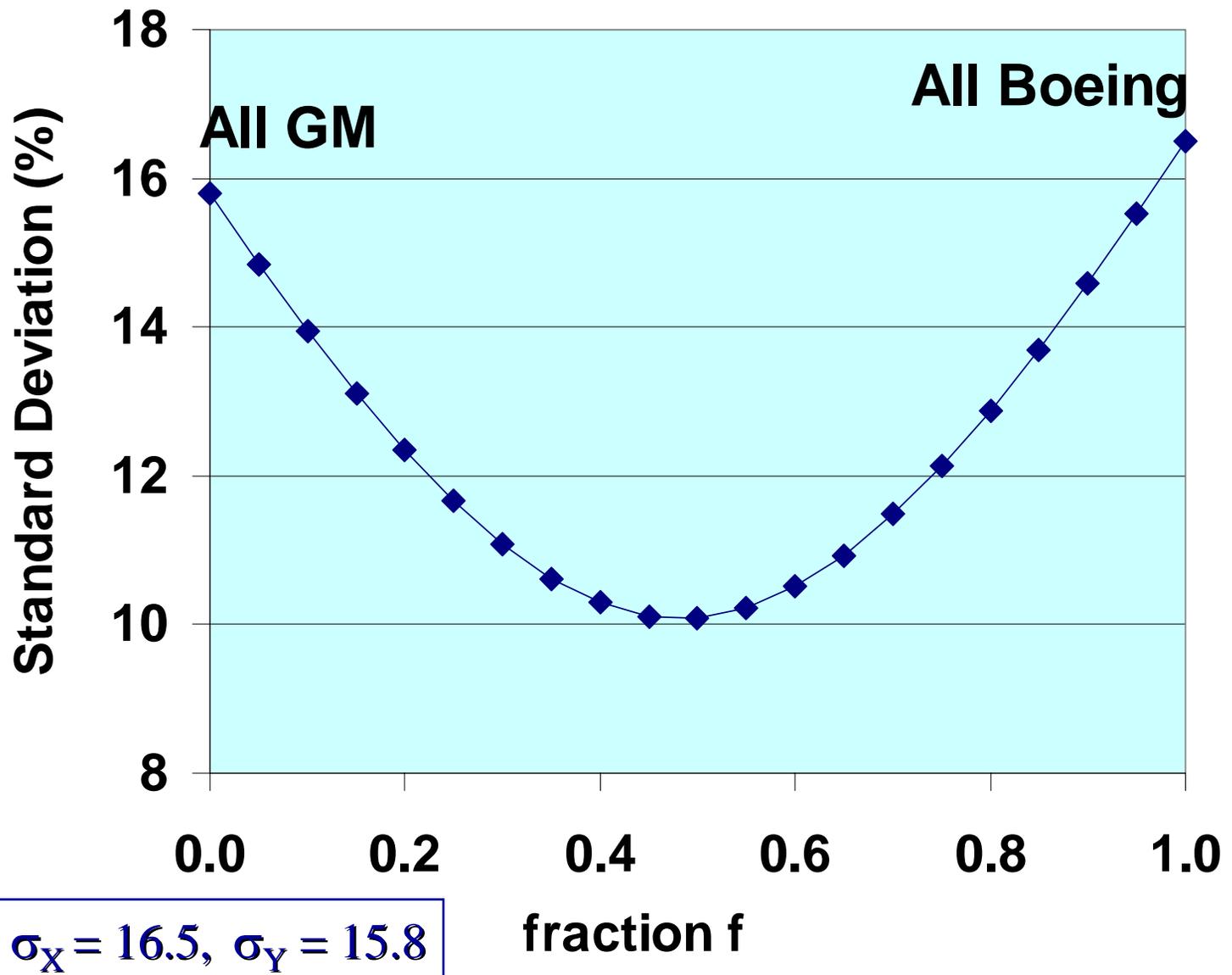
# Risk and Expected Return
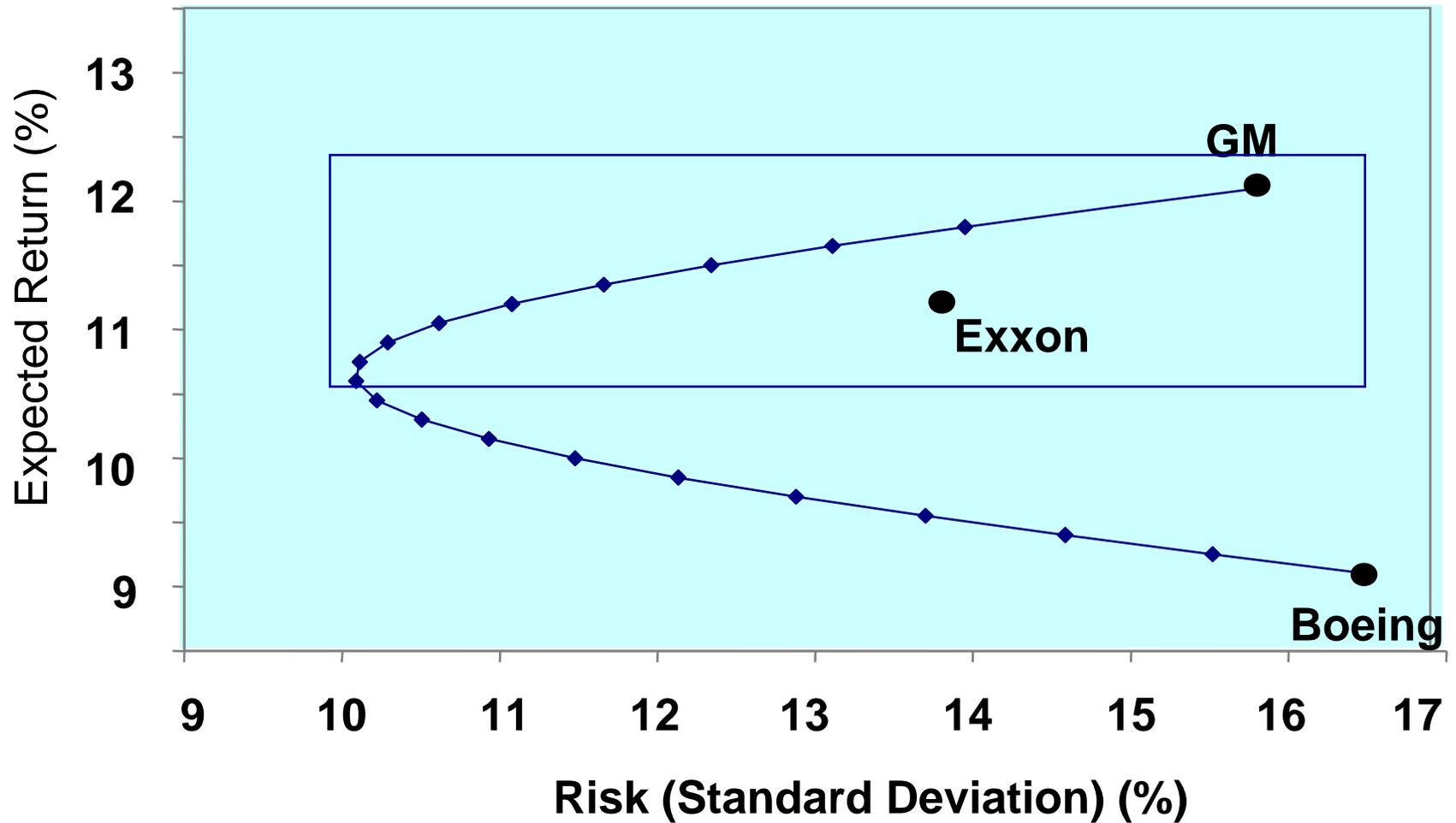
# Effect of Correlation on Portfolio Risk Levels

# Examples of Diversification

- Stock portfolios
- Pharmaceutical companies managing multiple drugs with expected returns and risks
- Pratt & Whitney commercial and military divisions:  1990s commercial is the profit engine, but post 9/11…
- Others?

# Many companies sell software packages for asset diversification, also known as "portfolio optimization."

- BARRA  (www.barra.com)
  Aegis System-Optimizer
- Wilson Associates International  (www.wilsonintl.com)
  Power Optimizer
  RAMCAP
  Xpress
- LaPorte Asset Allocation System  (www.laportesoft.com)

Typical features of these systems include:
* Historical databases
* Graphical capabilities
* Reporting capabilities
* Technical support

Typical prices are $ 2,000 to $ 10,000 for an initial license
 plus $ 1,000 to $ 4,000 for software and/or database upgrades.

# Summary and Look Ahead

- Portfolio Risk Management is one of the most important financial concepts of the last two decades, underlying all kinds of investment decisions

- Yet it is based on very simple statistical principles

- In Lecture 9, we look at the ubiquitous Normal distribution, adding some other very useful ideas and tools