Lecture 4 Game Plan

- February Madness
- Games with multiple Nash equilibria
  - … which equilibrium does evolution select?
  - … mixed strategies or pure strategies?
- Sequential move games
  - … escaping from Annoying Servant Game

Mixed Strategies

“Ni bhionn an rath ach mar a mbionn an smacht”

“There is no luck except where there is discipline”

- old Irish proverb

Analysis of Bluffing Game

- You get Good Card 15/48, about 1/3
- What do you do with Bad Card?
  - If you *never raise*, player B will always Fold when you have a Good Card.
    - get +100 when Good, -100 when Bad
    - average payoff about **–33**
  - If you *always raise*, player B will always Call you on it (even worse!)
    - get + 200 when Good, -200 when Bad
    - average payoff about **-67**

How Often to Raise in Eqm?

- Need to Raise enough for Player B to be indifferent between Fold and Call
  - B gets –100 if Folds
  - B gets either –200 or +200 if Calls
    - By Call, B "risks 100 to gain 300" relative to Fold
    - So we need Prob(Bluff | Raise) = 25%
  - 15 Good Cards so we <u>Bluff on 5 Bad Cards</u>
    - So, Raise with 5/33 Bad Cards
    - When 1/3 chance of Good Card, Bluff with prob. 1/6

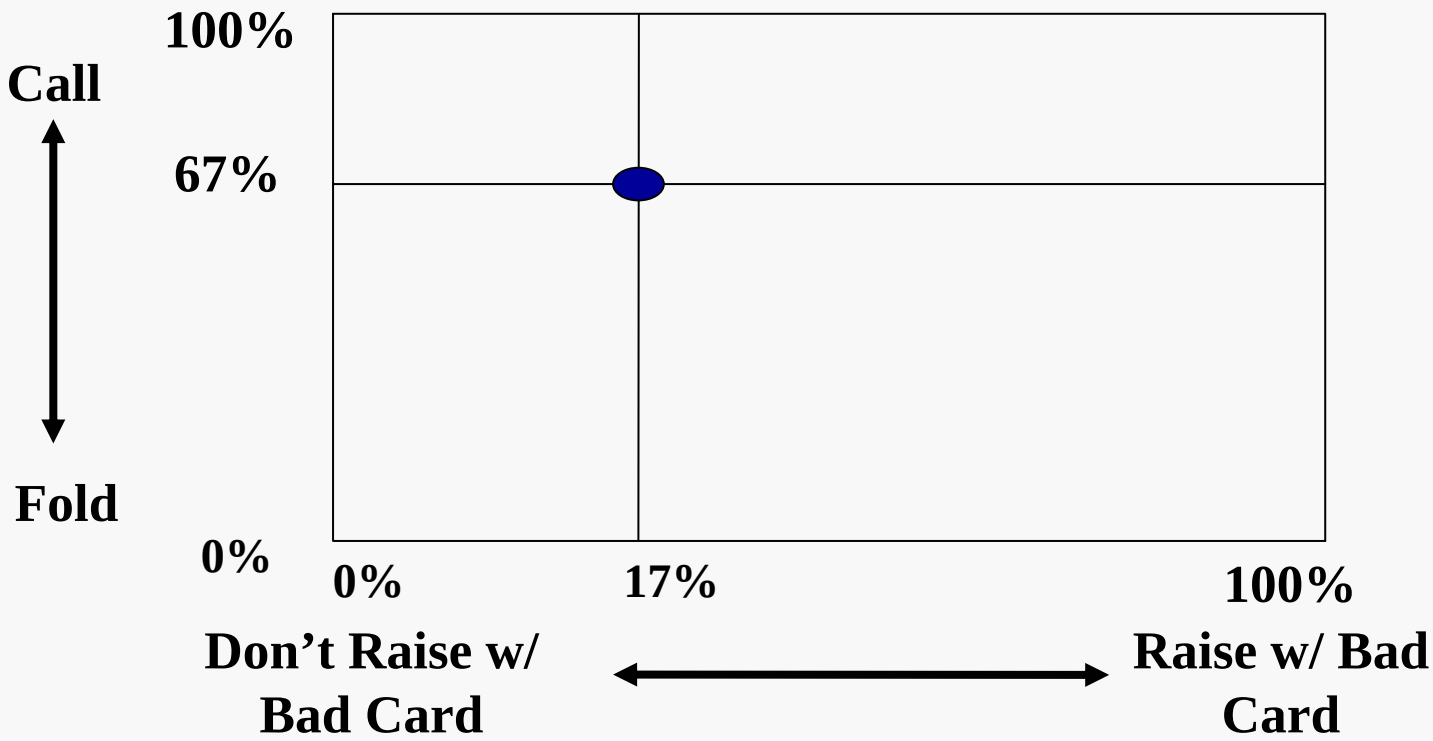
How Often to Fold in Eqm?

- Need to Fold enough for Player A to be indifferent between Raise and Not with Bad Card
  - A gets –100 if Not Raise
  - A gets either –200 or +100 if Raise
    - By raising, A “risks 100 to gain 200”
  - So we <u>Fold 33%</u>

Payoffs in Equilibrium

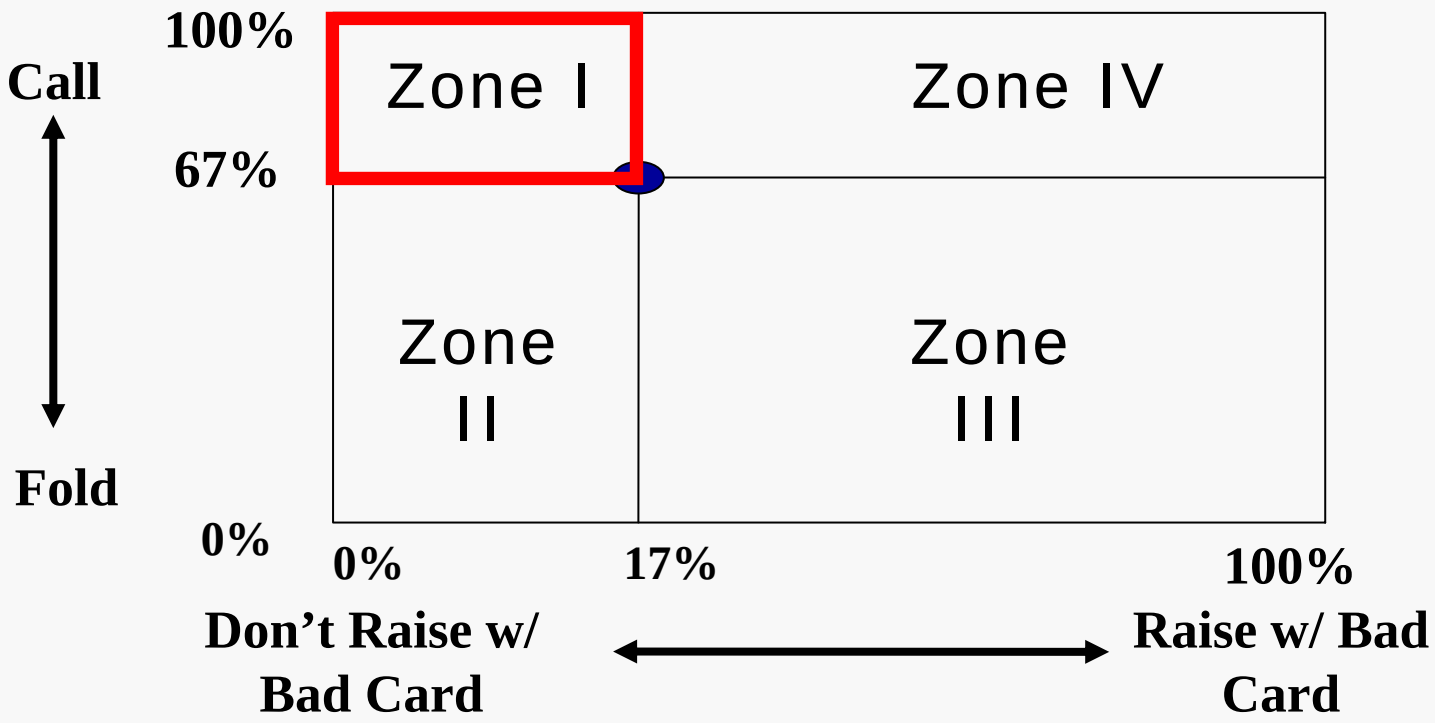
- Player B Folds 33% of time
  - Good Card → 33%(+100)+67%(+200), so get *167 when Good Card*
- … & Player A indifferent to Raise or Not given a Bad Card
  - *–100 when Bad Card*
- Overall payoff is about **–11** for A
  - much better than always/never bluffing

Best responses in bluffing



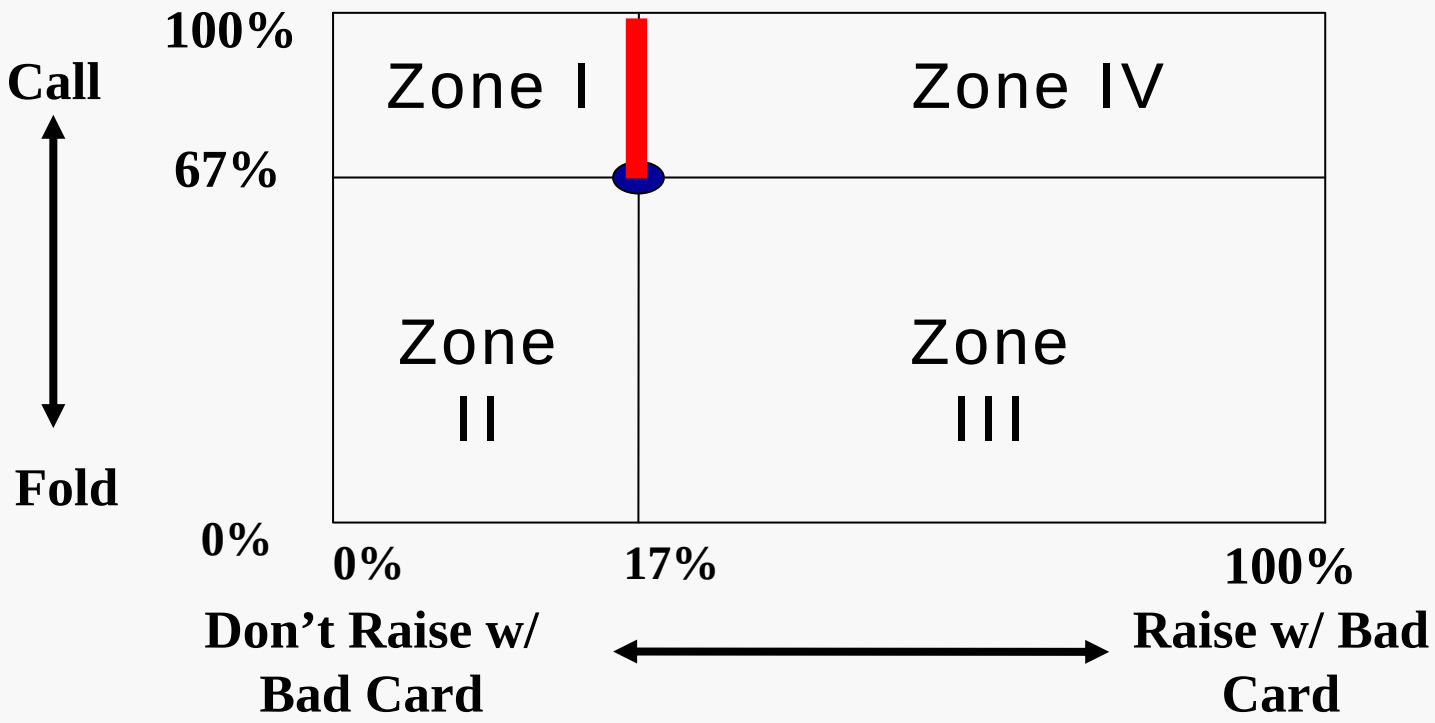
- Suppose other Raises & Folds
  - What's your best response?

Best responses in bluffing



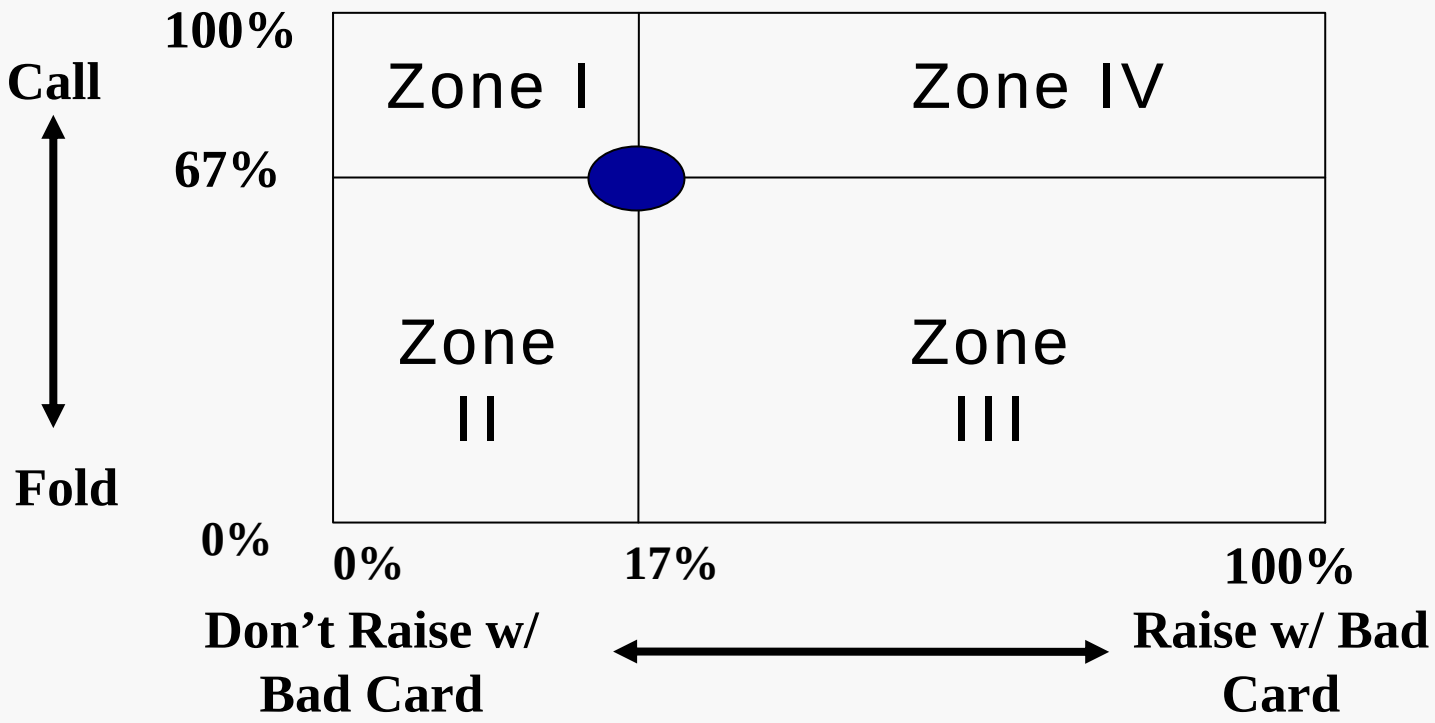
- Who will you beat if you choose strategy in Zone I?

Bluffing on a boundary



- Who will you beat if you choose on boundary of Zone I and Zone IV?

Bluffing by equilibrium



- Who will you beat if you choose the equilibrium strategy?

Some Prototypical Games

- Prisoner's Dilemma
- Loyal Servant
- Hunter and Hunted
- **Assurance**

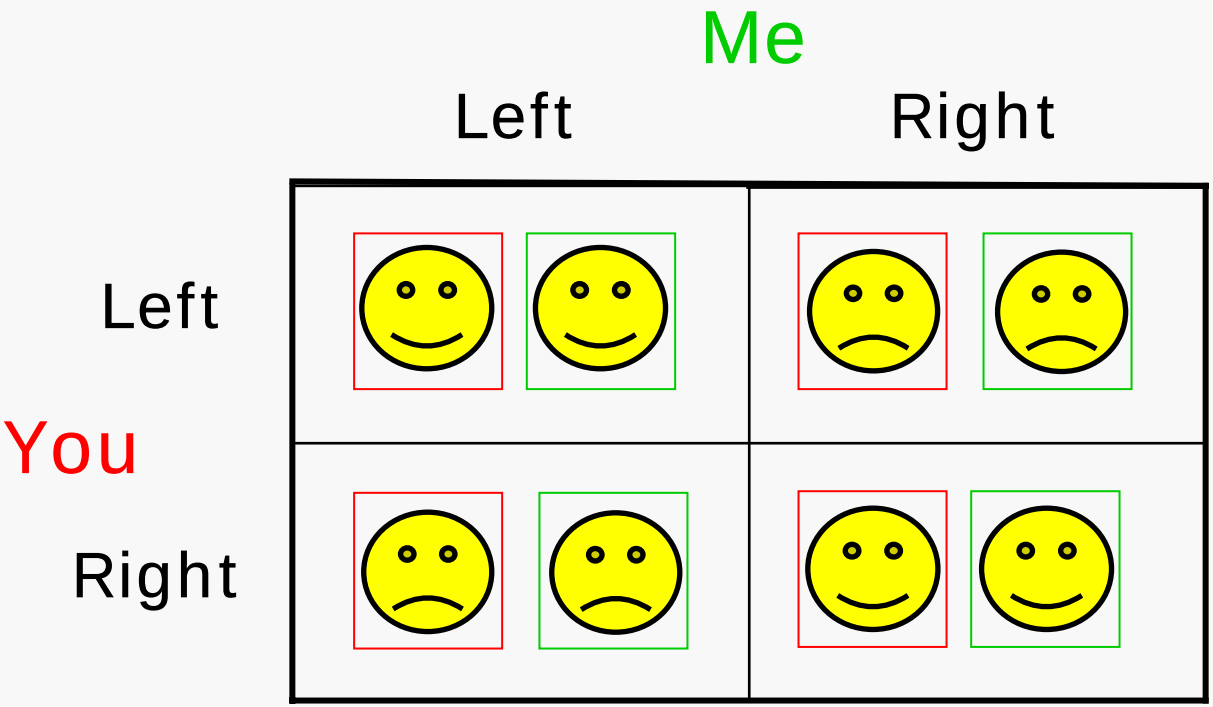
- price war
- defensive innovation
- audits, bluffing
- **driving, cooperation**

Which Side of the Road Should We Drive On?

- Map of the world showing which countries drive on the right (the majority) and which drive on the left.
- Photograph of a 1967 pamphlet explaining Sweden's change to driving on the right.
- Photograph of a car with a sign "Keep to the Right" on its dashboard reminding the driver of the new law. From Nova Scotia, 1923.

These three images are available at http://www.brianlucas.ca (accessed July 14, 2004).

Driving Game

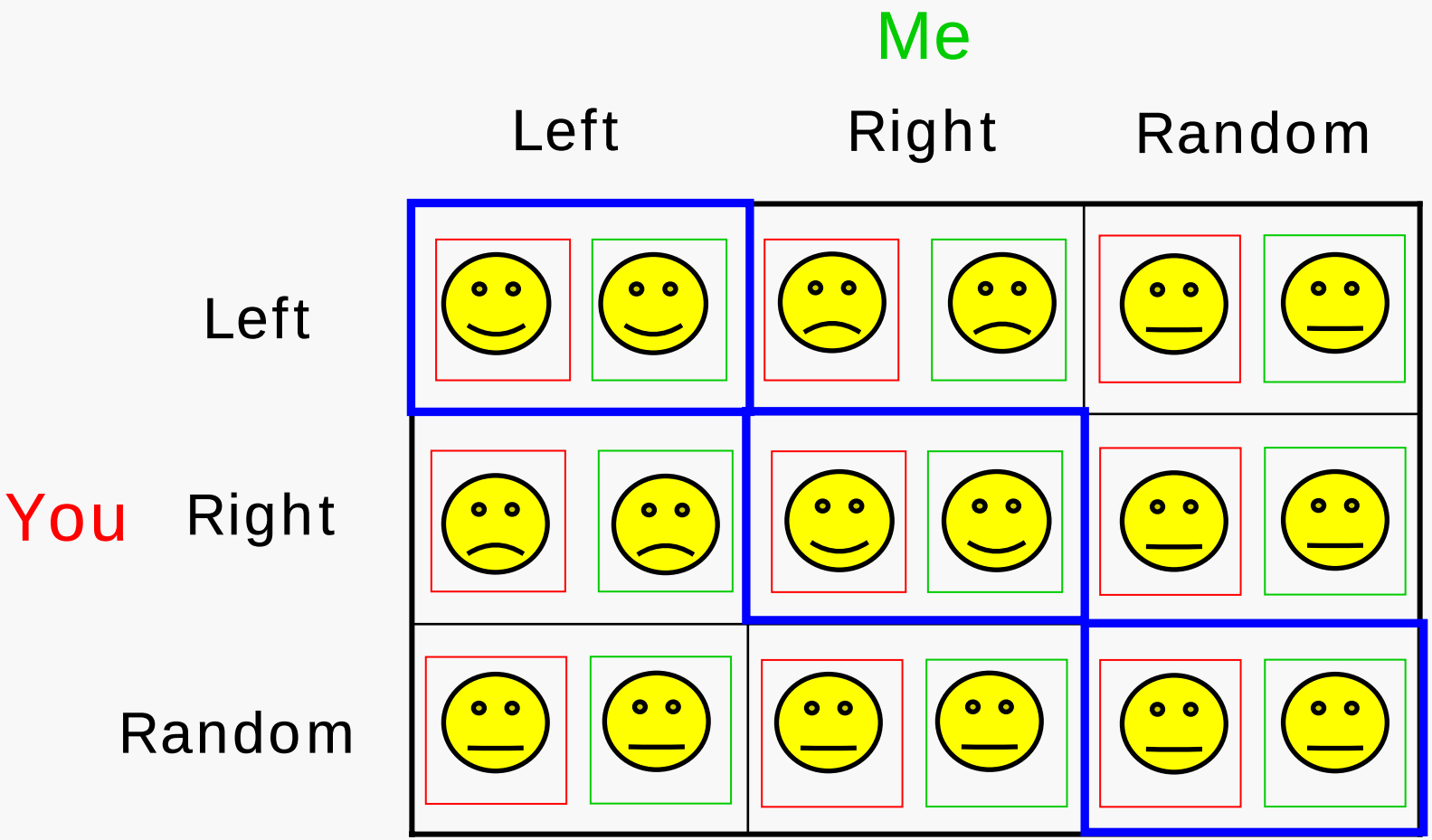


- What are the Nash equilibria in this game?

Mixed Strategies in the Driving Game

- (Left, Left) and (Right, Right) are the two *pure strategy equilibria*
- But there is also a *mixed strategy equilibrium*: each goes Left and Right half of the time
  - "Driving Chaos" is a possibility
  - You have no reason to go left vs right since *I'm* driving randomly!
  - … so *you* might as well drive randomly

Mixed Strategies in the Driving Game

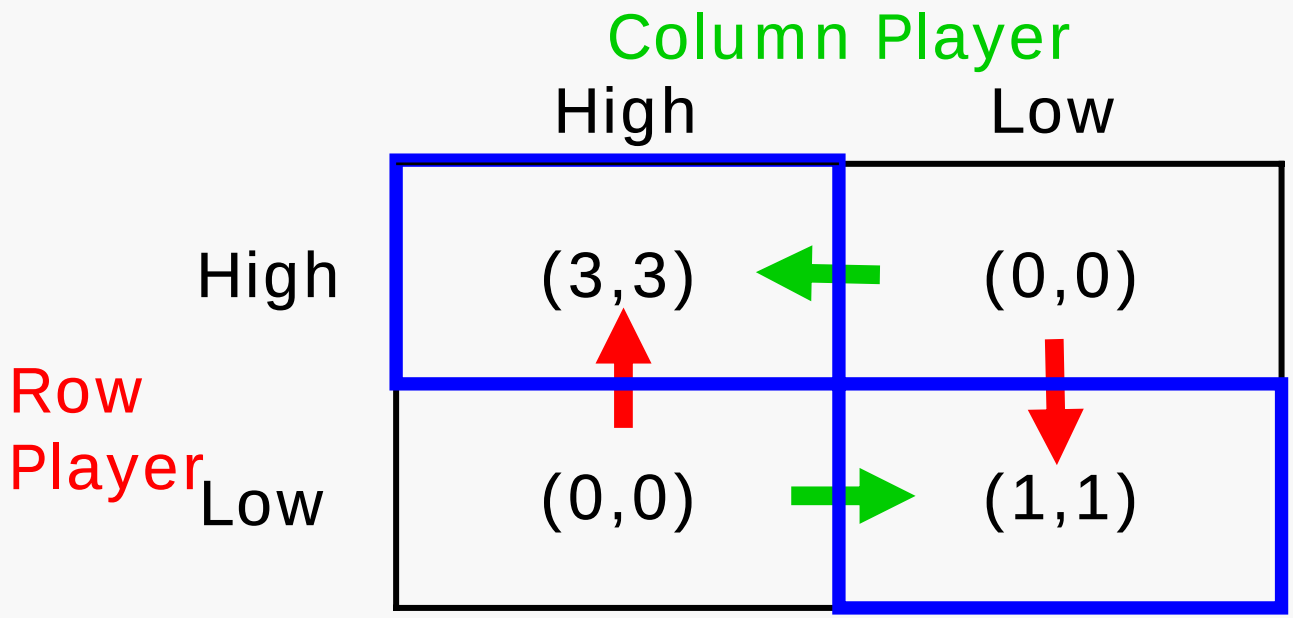


Assurance Game

		Column Player: High	Column Player: Low
Row Player	High	(3,3)	(0,0)
	Low	(0,0)	(1,1)

- Key features:
  - Each wants to do the same thing as the other
  - Both better off if both choose High

Assurance Game

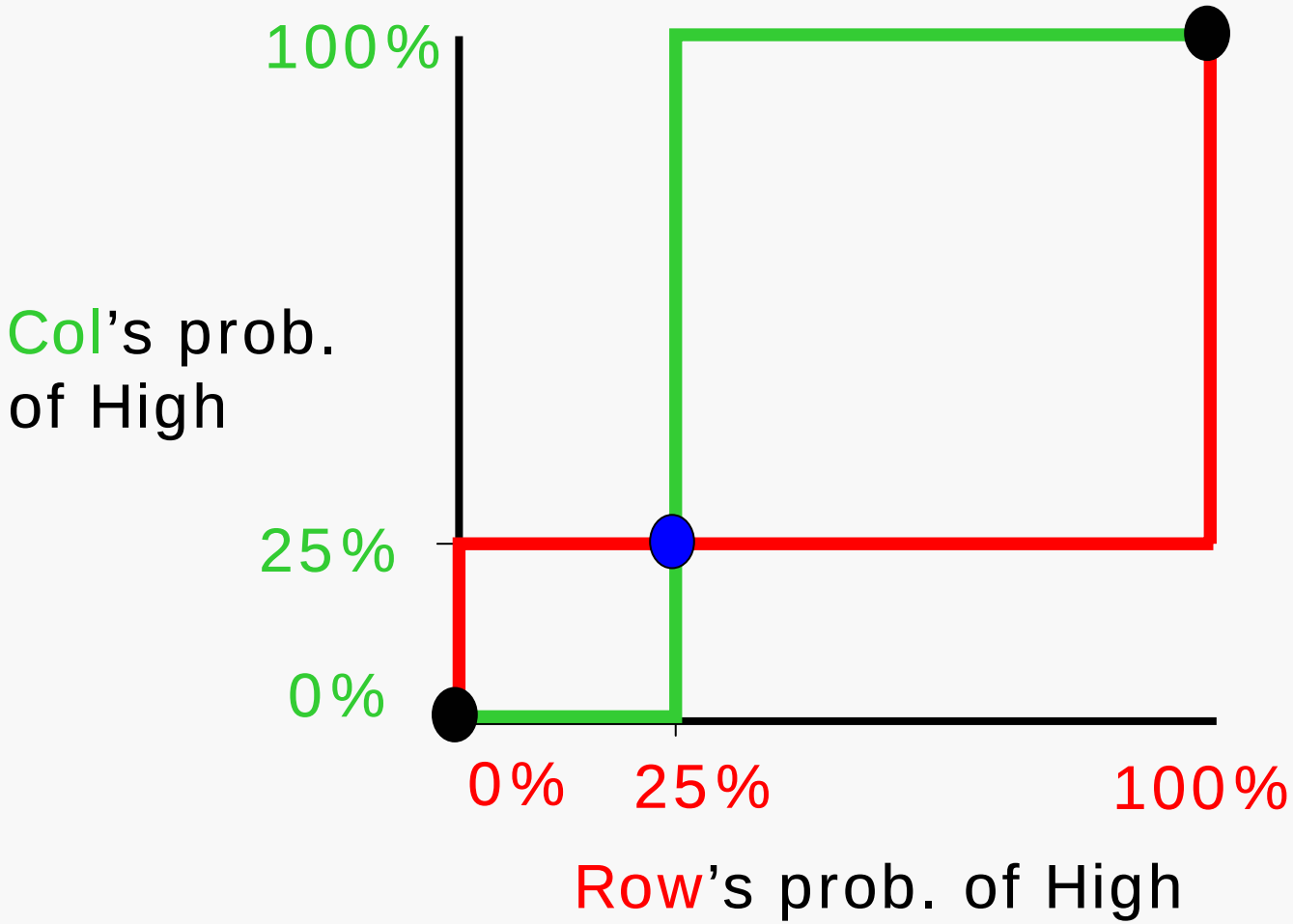


- Two pure strategy equilibria
- PLUS a mixed strategy equilibrium in which Prob(High) = 1/4, Prob(Low) = 3/4

Mixed Strategies in the Assurance Game

- Any player who mixes between two actions **must be indifferent** between those actions
  - This requires that **the other player** mix with just the right probabilities to create this indifference
- If Prob(High)=1/4 & Prob(Low)=3/4 is *my* mixture, then *you* get payoff 3/4 no matter what you do.
  - Each player adopts the better action **less often (!!)** in the mixed-strategy eqm

Reaction Curves in Assurance Game



Evolution & Stability of Play

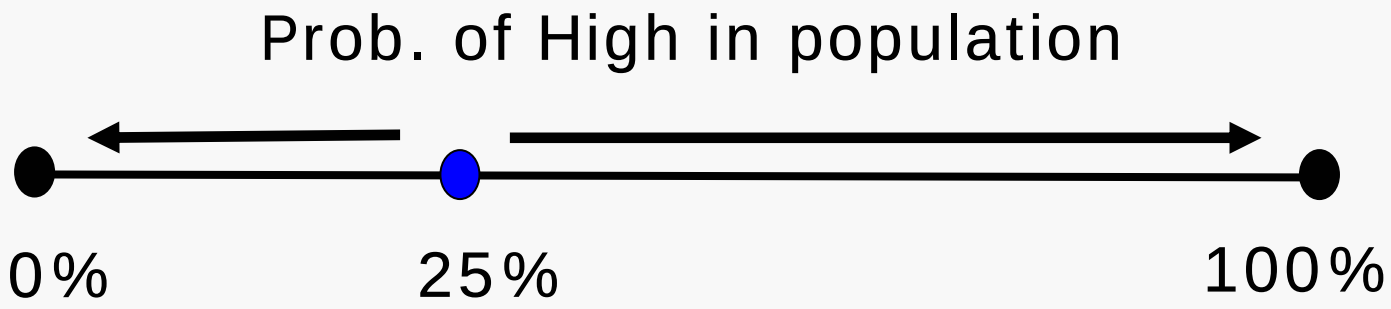
- Suppose that (1) players are “hard-wired” for either High or Low and (2) those who get higher payoffs become relatively more numerous.

- What are evolutionarily stable strategies (ESS)?
  - Any ESS must be Nash equilibrium!
  - Not all Nash equilibria are ESS

ESS in Assurance Game

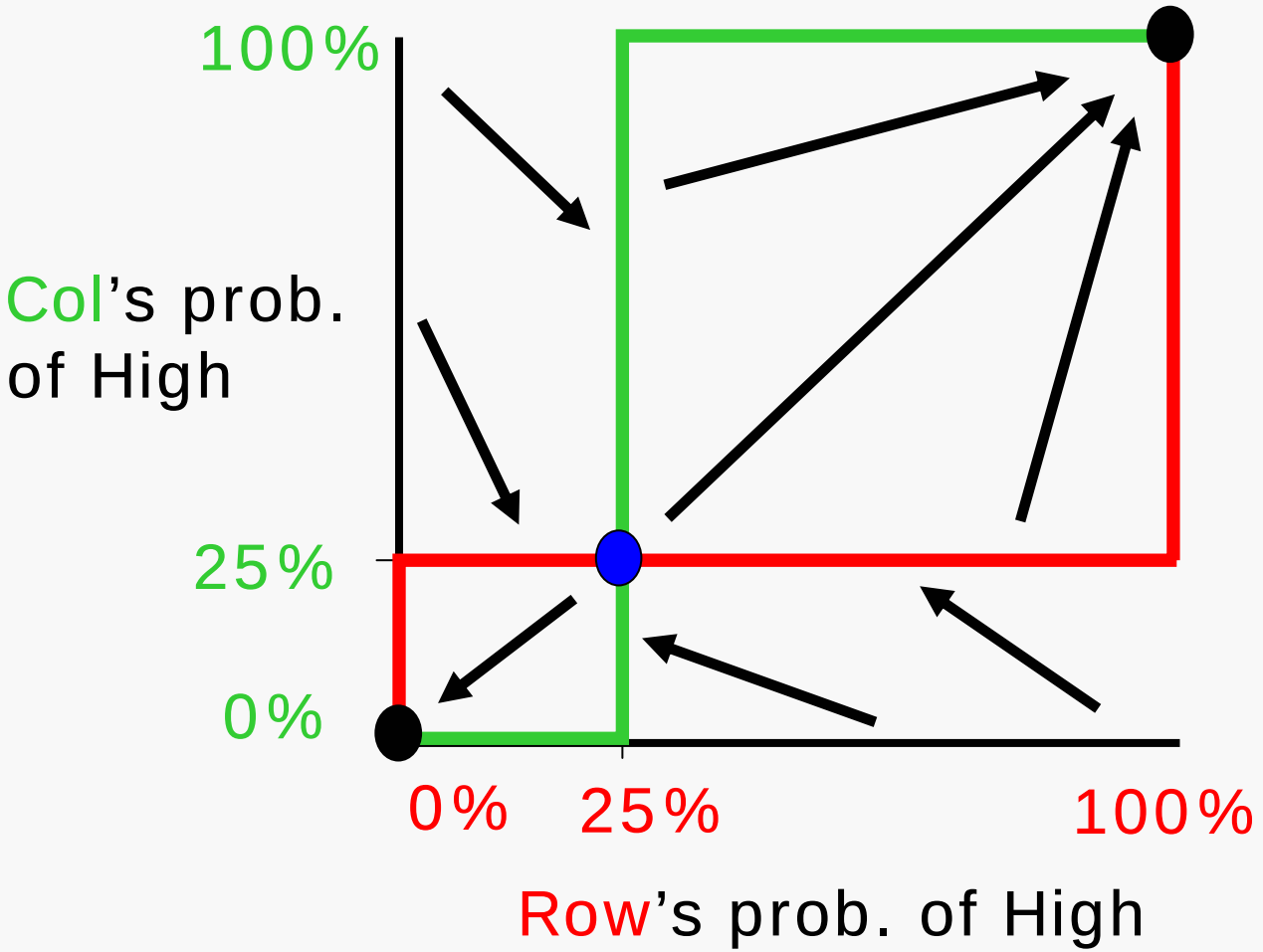
- Both pure strategy equilibria are evolutionarily stable
- The mixed-strategy equilibrium (MSE) is *not* evolutionarily stable
  - In the MSE, 25% of population plays High and 75% plays Low
  - If a few extra people are born (say) High, that will increase payoff to High and decrease payoff to Low, putting Low-people at a disadvantage
  - This will push everyone toward playing High

Evolution in Assurance Game with One Population



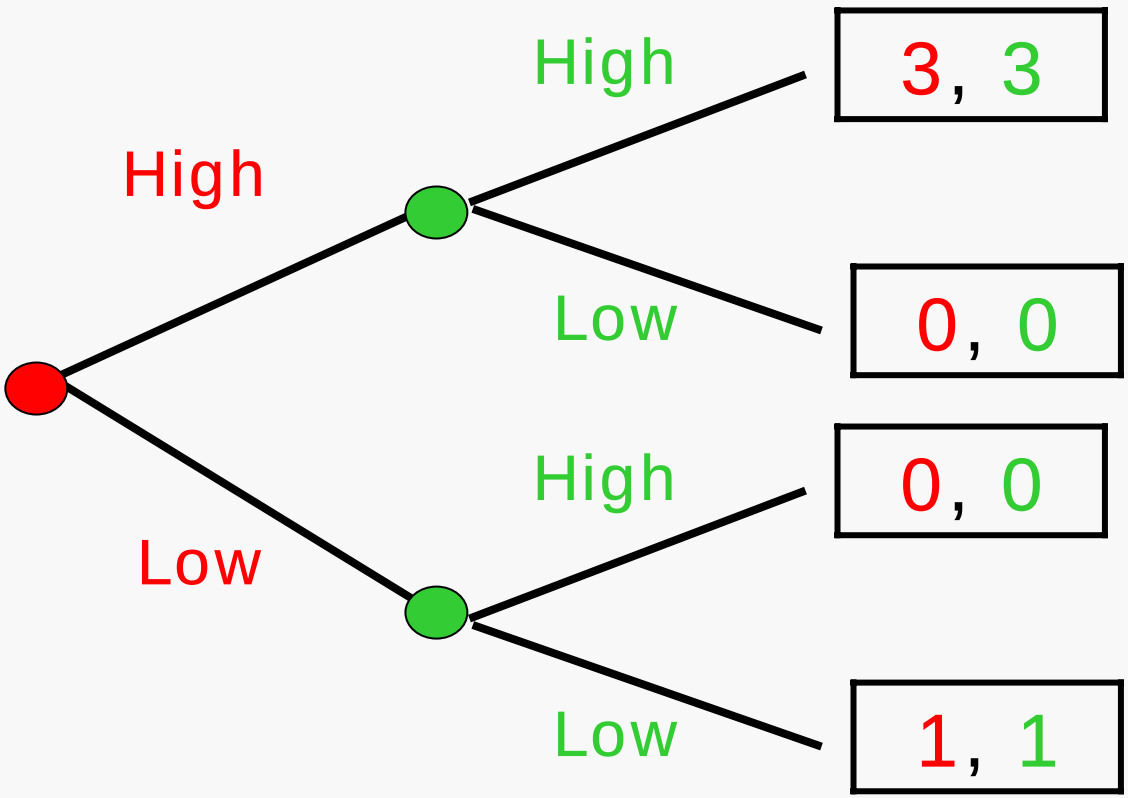
- Row and Col players are drawn from the same population
- If initial condition < 25% High-types, then evolution will push us to Low eqm, and vice versa

Evolution in Assurance Game with Two Populations



Now Row is drawn from "reds" and Col from "greens", where these populations *evolve separately*

Sequential Moves in Assurance Game

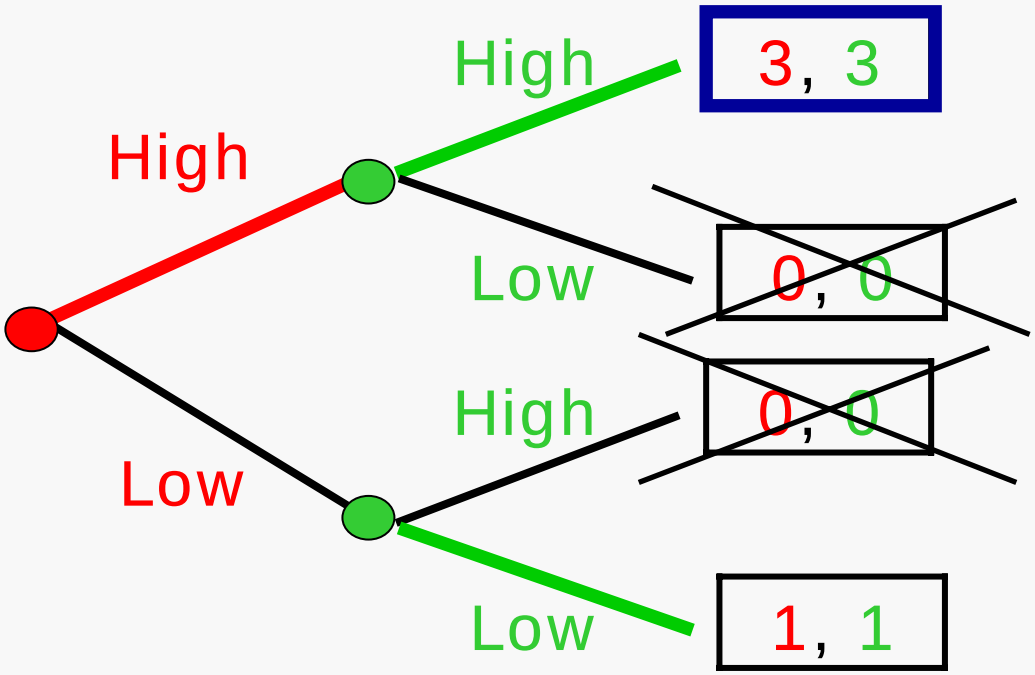


How to Find Subgame-Perfect Equilibrium

- Early movers make choices assuming that later movers will make whatever choice is in their best interest

- “Rollback procedure”
  - start at the *terminal decision nodes* in the game tree, and work backwards thru the tree

Sequential Moves in Assurance Game



- Players *coordinate* on (High, High) in the subgame-perfect equilibrium

On-line Game #3

Entrant Game

Equilibria in Sequential Move Games

- The meaning of “equilibrium” is entirely different in a sequential move game.
  - Before: Each player chooses a best response to others’ *fixed* strategies
    - since simultaneous moves, you can’t change others’ choice by your choice
  - Now: Each chooses a best response to others’ *responsive* strategies

Equilibria in Sequential Move Games

- In Assurance and Chicken Games, the outcome of the sequential-move version has been same as in a Nash equilibrium
- This need not be the case!

Summary

- How to play in Hunter & Hunted game
  - use equilibrium probabilities as benchmark
  - assess whether other player is likely Evolution may lead to pure or mixed strategy equilibrium.
- Assurance Game
  - with and without pre-emptive moves
- *Next time*: more on commitment

Online Game #6 (New Market Game)

- Play Online Game #6 prior to midnight before next lecture.
- Note: We are *not* playing the games in their numerical order!!