$\vec{W}$, Properties of a massive spin=1 particle

- Massive Maxwell Eqs:

$$\partial^\alpha \partial_\alpha W^\mu - \partial^\mu \partial_\alpha W^\mu + m_W^2 W^\mu = 0$$

Let $W^\mu(x) = \int d^4k \, e^{ik_\nu x^\nu} w^\mu(k)$

and $W^\mu(k) = \sum_i \varepsilon_i^\mu(k)\, a(k)$ where $\varepsilon_i \cdot \varepsilon_j = -\delta_{ij}$

We have $\left[-k^2 g^{\mu\alpha} + k^\mu k^\alpha + m_W^2 g^{\mu\alpha}\right] W_\alpha(k) = 0$

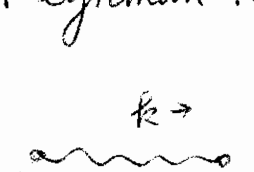
The propagator is the solution of

$$\left[-k^2 g^{\mu\alpha} + k^\mu k^\alpha + m_W^2 g^{\mu\alpha}\right] S_{\alpha\beta}(k) = i\, g_{\mu\beta}$$

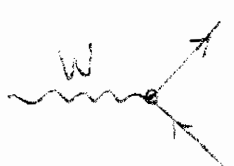
$$\Rightarrow \quad S^{\alpha\beta}(k) = \frac{i}{k^2 - m_W^2}\left(-g^{\alpha\beta} + \frac{k^\alpha k^\beta}{m_W^2}\right)$$

Same as s=1   Summation over polarizat

Feynman rules:



$$\frac{i}{k^2 - m_W^2}\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{m_W^2}\right)$$



$i g_W \gamma^\mu (1-\gamma_5) \longleftrightarrow$

$i Q_f \gamma^\mu = -ie\gamma^\mu$

(for e^-)

Polarization vectors of γ and the W's ($S=1$)

- For γ, we have Maxwell's Eqs: ($m_\gamma = 0$, spin $= 1$)

$$\nabla \cdot \vec{E} = \rho \quad \text{— (1)} \qquad \nabla \cdot \vec{B} = 0 \quad \text{— (2)} \qquad \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad \text{— (3)}$$

$$\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j} \quad \text{— (4)}$$

- Use vector potential $(\vec{A}, \varphi)$ where

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \phi \quad \text{and} \quad \vec{B} = \nabla \times \vec{A}$$ Eqs (2) & (3) are satisfied.

- Let $j^\mu = (\rho, \vec{j})$, we have the covariant Maxwell Eqs.

$$\Box^2 A^\mu - \partial^\mu \partial_\nu A^\nu \overset{+m_\gamma^2 A^\mu \text{ if } m_\gamma \neq 0}{\downarrow} = j^\mu \quad \text{or} \quad \partial_\mu F^{\mu\nu} = j^\nu, \quad \text{where } F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

- For $\mu = 0$, we have Eq.(1); and $\mu = 1,3$ we have Eq (4).

- Gauge transformation $A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda$ or $\left\{ \begin{array}{l} A'_\mu = A_\mu + \partial_\mu \Lambda(\vec{x}, t) \\ \phi' = \phi - \partial \Lambda / \partial t \end{array} \right.$ Take $\Lambda = i a e^{-i q_\mu x}$

- Define polarization vector $\varepsilon_\mu(\vec{q})$ by $A_\mu = \varepsilon_\mu(\vec{q}) e^{-i q_\nu x^\nu}$

  Solution of $\Box^2 A^\mu = 0$

- Lorentz condition $\partial_\mu A^\mu = 0 \longrightarrow q_\mu \varepsilon^\mu = 0$

  Gauge Inv. $\varepsilon'_\mu \Leftrightarrow \varepsilon_\mu + a q_\mu$

  For $m_\gamma = 0$, take $\varepsilon_0 = 0$, $\vec{\varepsilon} \cdot \vec{q} = 0$ i.e. transverse. For $\vec{P} = P_z \hat{z}$

$$\therefore \varepsilon_1 = (0, 1, 0, 0), \quad \varepsilon_2 = (0, 0, 1, 0) \quad \text{or}$$

$$\varepsilon_{RL} = (\varepsilon_1 \pm i \varepsilon_2) / \sqrt{2}$$

- Spin $= 0$ particles obey Klein Gordon Eq. $i(\Box^2 + m^2)\phi = -i V \phi$

$$\Box^2 \to -p^2 \qquad \therefore \text{propagator} = \frac{i}{\cdots} = \frac{\cdot}{\cdots}$$

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Unitarity limit and the $W^{\pm}$ mass limit

We have

$$\sigma(\bar{\nu}_e + e^- \to \mu^- + \bar{\nu}_\mu) = \frac{G^2 S}{3\pi}$$

In the $J=1$ channel, $\bar{\nu}_e$ with RH (helicity) only will contribute.

$$\sigma(\bar{\nu}_e \text{ RH} + e^- \to \mu^- + \bar{\nu}_\mu) \leq \frac{8\pi}{S}(2J+1) = \frac{24\pi}{S}$$

$$\therefore \frac{G^2 S}{3\pi} \leq \frac{24\pi}{S} \qquad \therefore \sqrt{S}_{\text{violation}} = \left(\frac{6\pi\sqrt{2}}{G}\right)^{\frac{1}{2}} = 1516 \text{ GeV}$$

If we would save unitarity by using a form factor $G \to G(s)$

which would mean leptons are not elementary or point-like

Introduction of $W^{\pm}$

$$\frac{G}{\sqrt{2}} = \frac{1}{2v^2} = \frac{i g_W^2}{q^2 - m_W^2} \qquad v = (\sqrt{2}\, G)^{-\frac{1}{2}} = 246 \text{ GeV}$$

Consequences:

1) For $q^2 \ll m_W^2$, Fermi W.I. is recovered if $\frac{g_W^2}{m_W^2} = \frac{G}{\sqrt{2}}$

Perturbative calculation (which works well for W.I.) requires

$g_W^2 \ll 1$, i.e. $m_W \ll 350$ GeV !

From $e^+e^- \to W^+W^-$: $m_W = 80.22 \pm 0.$ [illegible]

Use similar argument, we'll set limit on m_H !!!

(2) $\sigma(\bar{\nu}_e + e^- \rightarrow \mu^- + \bar{\nu}_\mu) = \dfrac{2g_w^4}{3\pi} \dfrac{s}{(s - m_w^2)^2}$

$= \dfrac{2g_w^4}{3\pi m_w^4} s = \dfrac{G^2 s}{3\pi}$ same as Fermi's W.I. as $s << m_w$

As $s \rightarrow \infty >> m_w^2$

$$\sigma = \frac{2g_w^4}{3\pi} \frac{1}{s} \leq \frac{24\pi}{s}$$

unitarity o.k. if $g_w^4 \leq 36\pi^2$

$m_w < 1516$ GeV