PROFESSOR: --equal forces of light. But before we get started with it, do you have any question about the last lecture, which was on open quantum system-- Quantum Monte Carlo methods-- and in particular, the conceptual things we discussed is what happens when you do a measurement, or when you don't detect a photon, how does it change the wave function? And this was actually at the heart of realizing similar quantum systems, and simulating them using quantum Monte Carlo methods. Any question? Any points for discussion? OK.

Today is, actually conceptually, a simple lecture, because I'm not really introducing some subtleties of quantum physics. We know our Hamiltonian-- the dipole Hamiltonian-- this is the fully quantized Hamiltonian, how atoms interact with the electromagnetic field. And, of course, it's fully quantized because E perpendicular is the operator of the quantized electromagnetic field, and it's the sum of a plus a [? dega. ?] But when it comes to forces, it's actually fairly simple. We don't introduce anything else. This is our Hamiltonian. This is the energy. And the force is nothing else than the gradient of this energy.

So I decided it today-- it's rather unusual-- but I will have everything pre-written, because I think the concentration of new ideas is not so high today, so I want to go a little bit faster. But I invite everybody if you slow me down if you think I'm going too fast and if you think I'm skipping something. Also another justification is at least the first 30%, 40% of this lecture, you've already done as a homework assignment, when you looked at the classical limit of mechanical forces. You actually had already, as a preparation for this class, a homework assignment on the spontaneous scattering force and the stimulated dipole force.

PROFESSOR: Well, the only subtlety today is how do we deal with a fully quantized electromagnetic field? But what I want to show you in the next few minutes is in the end, we actually do approximations that the quantized electromagnetic field pretty much disappears from the equation. We have a classical field. And then, almost everything you have done in your homework directly applies. The only other thing we do today is the dipole moment. Well, we use, for the dipole moment of the atom, the solution of the Optical Bloch Equation. So I think now you know already everything we do in the first 30, 40 minutes.

PROFESSOR: So we get rid of all of the quantum character of the electromagnetic field in two stages. One is the following, well, we want to describe the interactions of atoms with laser fields. And that means the electromagnetic field is in a coherent state. And you know from several weeks ago, the coherent state is a superposition of a [INAUDIBLE] states, coherent superposition and such, so they're really photons inside. It's a fully quantized description of the laser beam. But now-- and this is something if you're not familiar, you should really look up-- there is an exact canonical transformation where you transform your Hamiltonian-- you do, so to speak, a basis transformation-- to another Hamiltonian. And the result is, after this unitary transformation, the coherent state is transformed to the vacuum state.

So in other words, the quantized electromagnetic field is no longer in the coherent state, it's in the vacuum state. But what appears instead is a purely classical electromagnetic field. A c number. So you can sort of say-- I'm waving my hands now-- the coherent state is the vacuum, with a displacement operator. All the quantumness of the electromagnetic field is in the vacuum state, and that's something we have to keep because this is responsible for spontaneous emission quantum fluctuations and such. But the displacement operator, which gives us an arbitrary coherent state, that can be absorbed by completely treating the electromagnetic field classically.

So therefore, we take the fully quantized electromagnetic field and we replace it now by a classical field, and simply the vacuum fluctuation. So there are no photons around anymore. There is the vacuum ready to absorb the photons, and then there's a classical electromagnetic field, which drives the atoms. And, of course, the classical electromagnetic field is not changing because of absorption emission, because it's a c number in our Hamiltonian. So this is an important conceptual step, and it's the first step in how we get rid of the quantum nature of the electromagnetic field when it interacts with atoms mechanically. OK.

So, by the way, everything I'm telling you today-- actually, in the first half of today-- can be found in *Atom-Photon Interaction*. Now, so this takes care actually, of most of the electromagnetic field. We come back to that in a moment. The next thing is we want to use the full quantum solution for the atomic dipole operator, and that means we want to remind ourselves of the solution of the Optical Bloch Equations.

PROFESSOR: Now, talking about the Optical Bloch Equations, I just have to remind you of what we did in class, but I have to apologize it's a little bit exercise in notation. Because in *Atom-Photon Interaction* they use a slightly different notation then you find in other sources. On the other hand, I think I've nicely prepared for you, and I give you the translation table. In *Atom-Photon Interaction*, they regard the Bloch vector, and we have introduced the components, u, v, and w, as a fictitious spin, 1/2. Therefore, you find with this kind of old-fashioned letters, Sx, Sy, Sc, in *Atom-Photon Interaction*.

The other thing, if you open the book, *Atom-Photon Interaction* is that their operators, density matrix-- the atomic density matrix-- sigma, and sigma head. One is in the rotating frame, and one is in the lab frame. So we have to go back and forth to do that, but the result is fairly trivial. So this is simply the definition. First of all, the density matrix is the full description of the atomic system, but instead of dealing with matrix elements of the density matrix, it's more convenient for two level system to use a fictitious spin or the Bloch vector. So the important equation-- the are the Optical Block Equations-- is the equation A20 in *API.* And it's simply the first order differential equation for the components of the Optical Bloch vector, u v, and w. Notation is as usual, the laser detuning, delta L. Omega 1 is the [INAUDIBLE] frequency. And this is the parametrization of the classical electromagnetic field which drives the system.

Now, this equation is identical to the equation we discussed in the unit on solutions of Optical Bloch Equations. We just called the [INAUDIBLE] frequency, g, delta L was called delta. And then, vectors of 2r was confusing. The components, [? r,x,y,z, ?] are two times u, v, w. And the reason for that is some people prefer that it represents spin 1/2, so they want to normalize things to 1/2. Or if you want to have unit vectors, your throw in a vector of two. But that is all, so if you're confused about a vector of two, it's exactly this substitution which has been made.

So these are the Optical Bloch Equations, and what we need now is the solution. What we need now is the solution for the dipole moment, because it is the dipole moment which is responsible for the mechanical forces. The dipole operator is, of course, the non-diagonal matrix elements, the coherencies of the density matrix. So all we want to take now from the previous unit, the expectation value of the dipole moment, which is nothing else than the trace of the statistical operator with the operator of the dipole moment.

At this point, just to guide you through, we go from the lab frame-- from the density matrix in the lab frame-- to the density matrix in the rotating frame, and this is when the laser frequency appears, is into the i omega l t. And therefore, we want to describe things in the lab frame. If you use the u and v part of the Optical Bloch vector obtained in the rotating frame, in the lab frame everything rotates at the laser frequency. If you drive an atom, if you drive a harmonic oscillator with a laser frequency, the harmonic oscillator responds at the laser frequency, not at the atomic frequency. We discussed that a while ago. So therefore, the dipole moment oscillates at the laser frequency. And their two components, u and v-- since our laser field was parametrized by definition-- epsilon 0 times cosine omega t.

We have now nicely separated, through the Optical Bloch vector the in phase part of the dipole moment, and the in quadrature phase of dipole moment. If you didn't pay attention what I said the last five minutes, that's OK, you can just start here, saying the expectation value of the dipole moment oscillates with a laser frequency. And as for any harmonic oscillator, or harmonic oscillator type system, there is one component which is in phase, and one component which is in quadrature.

Fast forward. OK. And you know from any harmonic oscillator, when it comes to the absorbed power, that it's only the quadrature component which absorbs the power. And you can immediately see that when we say-- just use what is written in black now-- the energy is nothing else than the charge times the displacement. When we divide by delta t, you'll find that the absorbed power is the electric field times the derivative of the dipole moment. Well, if you ever reach over one cycle, we are only interested in d dot, which oscillates with cosine omega t. But that means since it's d dot, d oscillates with sine omega t. And this was the in quadrature component, v.

So it is only the part, v, of the Optical Bloch vector which is responsible for exchanging energy with the electromagnetic field. And we can also, by dividing by the energy of a photon, find out what is the number of absorbed photons. But we did that already when we discussed Optical Bloch Equations.

OK, so that was our look back on the Optical Bloch Equation, the solution for the oscillating dipole moment, and now we are ready to put everything together.

So we want to know the force. The force, using Heisenberg's Equation of Motion for the momentum derivative is the expectation value of the operator, which is the gradient of the Hamiltonian. OK. So now we have to take the gradient of the Hamiltonian. This is the only expression, actually, today, I would've liked to just write it down, and build it up piece by piece. But let me step you through.

We want to take the gradient. The dipole operator of the atom does not have an r dependence, it's just x on the atom, wherever it is. But the gradient with respect to the center of mass position from the atoms comes from the operator of the electromagnetic field. So in other words, what this involves is the gradient of the operator of the fully quantized electromagnetic field. But we have already written at the electromagnetic field as an external electromagnetic field-- classical electromagnetic field-- plus a vacuum fluctuations. But the vacuum fluctuations are symmetric. There is nothing which tells vacuum fluctuations what is left and what is right. So therefore, the derivative only acts on the classical electromagnetic field.

So this is how we have now, you know, we first throw out the coherent state, and now we throw out the vacuum state. And therefore, it is only the classical part of the electromagnetic field which is responsible for the forces. Now we make one way assumption which greatly simplifies it. R is the center of mass position of the atom, and we have to take the electromagnetic field as the center of mass position. And if the atom is localized-- wave packet with a long [? dipole ?] wave-- we sort of have to involve the atomic wave function when we evaluate this operator of the electromagnetic field.

However, if the atom is very well localized-- you would actually want the atom to be localized to within an optical wave length-- then, under those circumstances, you can replace the parameter, r, which is a position of the atom, by the center of the atomic wave packet. So therefore, the kind of wave nature of the atoms-- which means the atom is smeared out-- as long as this wave packet is small, compared to the wave lengths, you can just evaluate the electromagnetic field at the center of the wave packet. This assumption requires-- since the scale of the electromagnetic field is set by the wave lengths-- this requires that the atom is localized better than an optical wave length. And that would mean that the energy of the atom, or the temperature, has to be larger than the recoil in it.

And if you're localized the, to within a wave length, your momentum spread is larger, by Heisenberg's Uncertainty Relation than h bar k, and that means your energy is larger than the recoil energy. So if you want to have a description or light forces in that limit, you may have to revise this point. But for most of laser cooling, when you start with a hot cloud and cool it down to micro-Kelvin temperatures, this is very [INAUDIBLE]. Colin?

AUDIENCE: Vacuum field doesn't contribute in sort of in the sense that the expectation value is zero. But wouldn't the rms value be non-zero? So what's the--?

PROFESSOR: Exactly. I mean, in the end, you will see that in a moment, when we talk about cooling limits, fluctuations, spontaneous emission, provides heating. It doesn't contribute if you want to find the expectation value for the force. The vacuum and the fluctuations-- I mean, that's also logical-- the vacuum has only fluctuations, it has no net force. But the fluctuation's heat. And for heating, it will be very important. But we'll come to that in a little bit later.

AUDIENCE: So what is [INAUDIBLE]?

PROFESSOR: If the light is squeezed, yes. That's a whole different thing. We really assumed here that the light is in a coherent state. Now--

AUDIENCE: The transformation that we had [INAUDIBLE].

PROFESSOR: Well, you know, you can squeeze the vacuum. And then it's squeezed. But there's so few photons, you can't really laser cool with that. So the only way how you could possibly laser cool is if you take the squeezed vacuum, and displace it. So now you have, instead of a little circle, which is a coherent state, you have an ellipse, which is displaced. I haven't done the math-- maybe that would be an excellent homework assignment for the next time I teach the course-- you could probably show that the displacement operator, which is now displacing the squeezed vacuum, again can be transformed into a c number, and the ellipse this squeezed vacuum fluctuation-- the ellipse has an even symmetry, has an 180 degree rotation symmetry. And I could imagine that it will not contribute to the force.

So in other words, if you have a displaced, squeezed vacuum, my gut feeling is nothing would change here. But what would change, aren't the fluctuations, but which would possibly change is the heating.

OK, so we within those approximations, Heisenberg's Equation of Motion for the force becomes an equation for the acceleration for the atomic wave packet. And it involves now, two terms. One is the atomic dipole moment as obtained from the Optical Bloch Equations, and the other one is nothing else than the gradient of the classical part of the electromagnetic field. And with that we are pretty much at the equations which you looked at in your homework assignment, which was a classical model for the light force. Because this part is completely classical, and the only quantum aspect which we still have is that we evaluate the dipole moment, not necessarily, it's just a driven harmonic oscillator.

When we take the Optical Bloch Equations, we will also find saturation a two-level system can be saturated, whereas a harmonic oscillator can never be saturated. So let me just repeat, so this is completely classical except for the expectation value for the dipole moment. But for no excitation, of course-- and we've discussed it many, many times-- a two-level system is nothing else for weak excitation, than a weakly excited harmonic oscillator. So then we are completely classical. But we want to discuss also saturation and that's been the Optical Bloch Equations, of course, come in very handy.

OK. Next approximation, we want to know-- approximate-- the dipole moment-- the expectation value of the dipole moment-- with the steady state solution. The steady state solution-- analytic expression-- we just plug it in, and you can have a wonderful discussion about light forces. The question is, are we allowed to do that? Now I hope you'll remember that's actually one reason why I did that when we discussed the master equation. And we discussed an [? atomic ?] [? cavity. ?] I introduced to you the adiabatic elimination of variables. We adiabatically eliminated [? of dipole ?] matrix elements.

Whenever you have something which relaxes sufficiently fast, and you're not interested in this short time scale, you can always say, I replace this quantity by its steady state value. Steady state with regard to the slowly changing parameter. And we want to do exactly the same here. So let me just translate. It's the same idea, exactly the same idea, but let me translate. If an atom moves, we have two aspects, two times [? case. ?] One is the motion of the atom. The atom may change its velocity. The atom may move from an area of strong electric field to an area of weak electric field. But this is really all that is related to the motion, to the change of momentum of a heavy object.

But now inside the atoms, you play with the, you know, Bloch vector, excited state. If the electric field is higher, you have a larger population in the excited state than in the ground state. But this usually adjusts with the damping time of gamma, with a spontaneous emission rate. And that's usually very fast. So you can make the assumption when an atom moves to a changing electric field, at every point in the electric field, it very, very quickly-- the dipole moment-- assumes the steady state solution for the local electric field. And this would now be called an adiabatic elimination of the dynamics of the atom. And we simply replace the expectation value for the dipole moment by the steady state solution of the Optical Bloch Equations.

Well, this is, of course, an approximation. And I want to take this approximation and who you what simple conclusions we can draw from that. But I also want you to know, I mean this is what I focus off in this course, to sort of think about them and get a feel where those approximations break down. So this approximation requires that the internal motion, that this, you know, two-level physics, that the internal density matrix, it has a relaxation rate of gamma. Whereas the external motion, we will actually see that very soon when we talk about molasses and damping, has a characteristic damping time which is one over the recoil energy.

The recoiled energy involves the mass. The heavier the object, the slower is the damping time. It has all the right scaling. And you will actually see in the data today, that we will naturally obtain this as the time scale over which atomic motions is changing. So, is that approximation, this hierarchy of time scales, is that usually fulfilled? It is fulfilled in almost all of the atoms you are working on. For instance, in sodium, the ratio of those two time scales is 400. But if you push it, you will find examples like, helium in the triplets statement-- metastable helium in the triplet state-- where the two time scales are comparable.

Helium is a very light, so therefore, it's faster for a light atom to change its motion by exchanging momentum with electromagnetic field. And at the same time, the transition in helium is narrower, and therefore slower. So for metastable helium, the two time scales are comparable. | if you really want to do quantitative experiments with metastable helium, you may need a more complicated theory. But, of course, if something is sort of simple in one case, the richer situation provides another opportunity for research. And I know, for instance, Hal Metcalf, a good colleague and friend of mine, he focused for a while on studying laser cooling of metastable helium, because he was interested what happens if those simple approximations don't work anymore?

Then the internal motion and the external motion becomes sort of more entangled. You have to treat them together, you cannot separate them by a hierarchy of time scales. Any questions about that?

OK, now we have done, you know, everything which is complicated has been approximated away now. And now we have a very simple result. So I'm pretty much repeating what you have done your homework, but now with expressions which have still the quantum character of the Optical Bloch Equations. So let's assume that we have an external electric field. it's parametrized like that, and at this point, by providing it like this, I leave it open whether we have a standing wave-- if you have a traveling wave, of course, we have a phase factor, kr, here-- if you have a standing wave, we don't have this phase factor. So I have parametrized the electromagnetic field in such a way that we can derive a general expression. And then we will discuss what happens in a traveling wave, and what happens in a standing wave.

I make one more assumption here, namely that the polarization is independent of position. I'm just lazy if I take derivatives of the electric field. I want to take a derivative with the amplitude of the electric field, and I want to take a derivative of the phase of the electromagnetic field. I do not want to take a derivative of the polarization. Of course, what I'm throwing out here, is all of the interesting cooling mechanisms of polarization gradient cooling, when the polarization of the light field spirals around in three dimensions. We'll talk a little bit about that later on, I just want to keep things simple here.

OK, so now if you take the gradient of the electromagnetic field, finally, we have approximated everything which complicates our life away. And we have two terms. One is the gradient x on the amplitude of the electromagnetic field, or the gradient x on the phase. And since we have taken a derivative, when we take the derivative of the phase, because of the chain rule, the cosine changes into sine. So therefore, the gradient of the electromagnetic field has a cosine in phase and an in quadrature component. And now, for the force, we have to multiply this with the dipole moment. And, of course, we will average over one cycle of the electromagnetic wave.

And, of course, that's what we discussed. There's is a u and v component of the Bloch vector, which gives rise to an in phase and in quadrature component of the dipole moment. And now, if you combine-- if you multiply-- the gradient of the electric field with the dipole moment, and average. The cosine part goes with the cosine part, the sine part goes with the sine part, and the cos terms average out. So therefore, we have the simple result that we have two contributions to the [? old ?] mechanical force of light. And the u component-- the in phase component-- of the dipole moment goes with the amplitude of the gradient. And the in quadrature component goes with the gradient of the phase.

So in other words, when it comes to light forces, the cosine component and the sine component of the dipole moment talk, actually, to two different quantities associated with the electromagnetic field. It's just in phase and in quadrature.

OK. Now I just have to tell you a few definitions and names. This component, which goes with the in phase component, is called the reactive force. Whereas the other one is called the dissipative force. The name, of course, comes that if you have a harmonic oscillator, the component which oscillates in quadrature absorbs directly energy, whereas the in phase component does not absorb energy. I'll leave it for a little bit later in our discussion how you can have a force and not exchange energy. That's a little bit a mystery when we talk about cooling with a reactive force. We will have to scrutinize what happens with energy, because at least in the most basic, the in phase component of the oscillating system does not extract energy from the electromagnetic field.

So maybe just believe that for now, and we will scrutinize it later. Any questions? Am I going too fast? Am I going too slow? Does it mean about right? OK.

So we have now obtained the reactive force, which is also called the dipole force or stimulated force, and the dissipated force, which is also called the spontaneous force. The next thing is purely nomenclature. We want to introduce appropriate vectors which point along the gradient of the phase, and which point along the gradient of the amplitude. And this is done here. So we want to say that the reactive and the dissipative force are written in a very sort of similar way, but the one off them points in the direction of a vector, alpha, the other one in the direction of a vector, beta.

The beta vector is the gradient of the phase. If the phase is kr, the gradient of the phase is simply k, the k vector of the electromagnetic wave. So that will come in handy in just a few seconds. Whereas the reactive force points into the direction of the gradient of the amplitude, but we never want to talk about electromagnetic field amplitude. For us, we always use the electric field in terms of you parametrize the Rabi frequency. The Rabi frequency is really the atomic unit of the electric field. So therefore, this vector which involves now the gradient of the amplitude, becomes a normalized gradient of the Rabi frequency. So these are the two fundamental light forces, the reactive and the dissipative force.

OK. Now we are going to have a quick look at those two forces. So therefore, I'm now discussing the two simple, but already very characteristic cases for those forces. One is we want to look at a plane travelling wave, and then we want to look at a pure standing wave. And the beauty of it is in a plane travelling wave, you've a plain wave, the amplitude of the electric field is constant everywhere, it just oscillates. And therefore, this alpha vector is zero. And in a few minutes, we look at this standing wave, and in the standing wave, the beta vector is zero. So the travelling wave and the standing wave allows us now to look at the two forces separately in two physical situations which are-- as you will see later-- highly relevant for experiments.

Well, I don't think I have to tell anybody in this room-- maybe except some people who take the course for [? Brett's ?] requirement-- that standing waves or optical lattices are common in all labs. And similar, traveling wave beams are used, for instance, for decelerating atomic beams. So we illustrate the two forces now, but we already discussing two experimentally very important geometries.

OK, plain travelling waves. I mean, I just really plug now the electromagnetic field for plain travelling wave, I plug it into the equations above. The important thing is-- and which simplifies things a lot-- is that the alpha vector is zero. The amplitude of the plain wave is-- that's what a plain wave is-- constant. And the beta vector is the gradient of the Rabi frequency is just the k vector of the light. So therefore, the dissipative or spontaneous light force was the Rabi frequency times the v component of the Optical Bloch vector times h bar k.

And this has a very simple interpretation because the steady state solution of the Optical Bloch Equation is nothing else than gamma times the excited state population. And since the excited state scatters, or emits photons at a rate, gamma, this is nothing else than the number of absorbed emitted scattered photons times h bar k. We are in steady state here, so the number of photons which are emitted into all space, into the vacuum has to be the number of photons which has been absorbed into the laser beam. So therefore, the interpretation here is very simple, you have a laser beam. Every time the atom absorbs a photon, it receives a recoil transfer, h bar k.

Now, afterwards, the photon is scattered, but the scattering is symmetric, and does not impart the force onto the atom. And this is what we discussed earlier. The quantum part of the electromagnetic field is symmetric. And here, we see sort of what it means visually. Spontaneous emission goes equally probable in opposite direction, and there is no net force, but there is heating as we discussed later.

So we can plug in the solution-- the steady state solution-- from the Optical Bloch Equation. That's our Lorentzian. We have discussed power broadening already when we discussed the Optical Bloch Equation. And, of course, I just remind you if you saturate the system with a strong laser power, what you can obtain is that half of the population is in the excited state, and half of the population is in the ground state. Under those circumstances, do you get the maximum force. And the maximum spontaneous force is the momentum per photon times the maximum rate at which a two-level atom can scatter photons.

So that's, in a nutshell, I think all you have to know about the spontaneous light force as far as the force is concerned. But we still have to discuss the heating associated with a spontaneous force, which are the force fluctuations. Questions?

Well, then let's do a similar discussion with reactive forces. Which is a little bit richer because, well, you will see. So a standing wave is parametrized here. We only have an alpha vector, not a beta vector now, because everything is in phase, there is no phase phi of r. So therefore, there is no dissipative force, the only force is a reactive force. Since the reactive force depends only on u, there is no exchange of energy. So that raises the question, what is really going on it is only the reactive part with u, how can the atom really-- How can the motion change if it cannot exchange energy?

Well, we'll come to that in different pieces. The one part I want to sort of discuss here is that you should-- I want to show you that at the level of our discussion we can already understand that the atom is actually doing a redistribution of energy and momentum. Well, if I only hold onto my standing wave, there is no energy exchange because the atom is always in phase with the standing wave-- the atomic dipole-- which is this one [INAUDIBLE] force. But I can now do what everybody does in the lab. We generate the standing wave as a superposition of two travelling waves.

And if I have a superposition of two traveling waves, there is a point in space where the phases of the traveling waves are the same. Then one phase is advanced in phase, and the other side one of the waves is lagging in phase. And for pedagogical reasons, I pick the point where the two travelling waves are 90 degree phase shifted. OK. Now the situation is that we have our electric field and what is responsible for the reactive light force is the u part of the dipole moment of the Optical Bloch vector, which oscillates in phase.

But nobody prevents me from analyzing it with respect to E1 and E2. And so if I ask now, I say, I have one laser beam. I have another laser beam. And if you want, after the laser beams have crossed, you can put in two photo-diodes, and you can not only ask, what happens to the standing wave, you can also ask, what happens to each traveling wave? Are photons absorbed? Or what happens? And now, what is obvious is if you have an harmonic oscillator, and you have an oscillating dipole moment, if the dipole moment is withing 0 and 180 degrees with a dry field, you absorb power. If it is in the two other quadrants, it emits, or delivers power.

And now you see, that we are in a situation-- which I've peak here-- where one travelling wave loses energy, and the other traveling wave gains energy. And there is a classic experiment, which was done by Bill Phillips a while ago-- he had atoms in an optical lattice, and they were sloshing. And she could really measure that when the atoms were accelerated this way, one of the laser beams gained power, and the other one lost power. So while the atoms where sloshing in momentum space, the power between the two laser beams was distributed back and forth. So this is the character of the dissipative force, that it redistributes momentum and energy between the two traveling waves.

Any questions? Boris?

AUDIENCE: But if there's a net change in the patterns of motion, the energy still has to come from somewhere. Right? Can you make that [INAUDIBLE]

PROFESSOR: This question has bothered in many different iterations. And at some point, I found a very, very easy answer to it. And this is one of your homework assignments for homework number 10. There's a very simple example where you will realize where the energy comes. What happens is, in steady state, if everything is stationary, of course, there can't be an exchange. But, if you're for instance, saying-- and I give you no part of the solution-- if you have atoms, and they are attracted by the dipole force, there's a strong laser beam and the atoms are accelerated in, and you're now asking, where does the energy come from?

Because all of what the atoms do, at least according to what I'm telling you, they redistribute photons of energy, h bar omega, into photons of energy, h bar omega. So where does the energy come from? You have to go higher in the approximation here to understand where the energy comes from. Let me just tell you one thing, this is actually something which I've encountered several times. And it can be really confusing, I've really proven to mathematically the this is false. So in this level of treatment, with these very simple concepts, we correctly obtained the optical force. But if you're now asking, where does the energy go? You have to go deeper.

And, for instance, what happens is if you take a cloud of atoms, and the cloud of atoms moves in an out of a laser beam-- just think of it as center of mass dipole oscillations of a Boson-Einstein condensate in an optical dipole trap-- what happens is when the atoms move in and out, they actually act as a phase modulator for the laser beam. The index of the refraction of your laser beam is changing. So therefore-- and we don't deal with that when he replace the electromagnetic field by a c number-- your laser beam, when the atoms move in will actually show a frequency modulation.

And you will find-- and it's a wonderful homework assignment, I'm pretty proud that we created it, and that it works out so easily-- you will find that the kinetic energy of the atoms is exactly compensated by the energy shift the photons which have passed through the atoms. But I'm giving you a very advanced answer. It's often very difficult if you have a simple picture for the force to figure out what happens to the energy. Actually, let me give you the other example.

When we have the previous example, where we have atoms and we have a laser beam which cools with a radiation pressure. This was our previous example. Where does the energy go? I have a wonderful correct-- within the assumptions-- an exact expression for the force. Where does the energy go? The atoms have a counter-propagating laser beam. They scatter photons with this rate. Every time they scatter a photon, they slow down by h bar k. They lose energy. We have a complete description what happens in the momentum picture and in forces. But where does the energy come? Or where does the energy go? The energy where the atoms have lost?

AUDIENCE: They absorb a slightly high frequency with a Doppler shift. And then slow down and then re-release a photon with slightly lower energy.

PROFESSOR: Exactly. So in that case, it actually depends which way. When the atom loses energy, the energy goes into blue-shifted photons. I'm just saying, in order to address the question, what happens to the energy, we have to bring in our physics, which we didn't even consider to address here. It is now the physics of the spontaneously emitted photons. We said for the force, spontaneous emission vacuum fluctuations can be eliminated. But if you try to understand where does the energy go, we have to go to those terms, and we even find the physics of Doppler shifts, which we didn't put in, which we didn't even assume here. But, of course, it has to be self consistent.

So therefore, you sometimes have to go to very different physics which isn't even covered by your equation to see the flip side of the coin. One side is forces, which are very simple, but the other side of the coin can be more difficult. We'll actually come back to that when we talk about cooling [INAUDIBLE].

Cooling with a stimulated force. How can a stimulated force cool, because it's all u. Well, we'll get there. And it will again be-- the energy-- will be extracted through spontaneous emission from all those side [INAUDIBLE] and all that. So we have to trace down the energy, and we successfully will do that. But energy can be subtle. Forces are simple. Other questions?

Actually, since I'm in chatty mood, when I explained to you where the energy goes in a dipole trap-- that it's a phase modulation for the laser beam-- I have been using optical dipole traps in my laboratory for many, many years, before I really figured out where the energy goes. And I'm almost 99% certain if you go to DAMOP, and ask some of the experts on cooling and trapping, and ask them to figure out this problem, most of the people will not be able to give you this answer. I've actually not found the answer in any standard textbook of laser cooling, until I eventually found it myself and posted and prepared it as a homework assignment.

So many, many people may not be able to give you the correct answer, where does the energy go when atoms slosh around in a dipole trap. Try it out at DAMOP, it may be fun. OK.

So back to the simple physics of forces. We don't need to understand the energy, because we know the forces, but we'll come back to that. So we have the reactive force. The reactive force is written here. What have we done? Well, you remember the alpha vector was the gradient of the Rabi frequency, and we have to multiply with the u component of the steady state solution of the Optical Bloch Equation. Here it is, just put together. It's a nice expression.

There are two things you should know about it. The first one is that you can actually write this force exactly as the gradient of a potential. So there is a dipole potential, and is exactly this dipole potential which is used as a trapping potential when you have dipole traps. Probably 99% of you use dipole traps in the limit where the detuning is large. And then the logarithm of 1 plus x simply becomes x, and you have the simple expression you are using. But in a way, this is remarkable. The dipole force can be derived from a potential, even if the detuning is small, and you have a hell of a lot of spontaneous scattering, which is usually not regarded as being due to conservative potential.

So this expression, that the reactive force is a gradient of a potential, even applies to a situation where gamma spontaneous scattering [INAUDIBLE] And you're not simply in-- and this is what this last term is-- in the perturbative limit of the AC Stark Effect. OK. That's number one you should know about this expression.

The second thing is if you look at this expression, the question is, what is the maximum reactive force? We talked about the maximum spontaneous, or dissipative force, which is h bar k, the momentum of a photon times gamms over 2. What is the maximum force you can get out of the reactive force? Well, of course, if you want a lot of force, you want to use a lot of laser power. Therefore, you want to use a high Rabi frequency. And now, if you want to use your laser power wisely, the question is, how should you pick the detuning? And you realize if you pick the detuning small, there is a prefector which goes to zero, but if you pick your detuning extremely large, the denominator kills you.

And well, if you analyze it and or stare at this expression for more than a second, you realize that the optimum detuning is when the detuning is on the order of the Rabi frequency. And if you plug that in, you find that under those optimum conditions, the reactive force can be written again as the momentum of a photon-- it must be, the momentum of the photon is the unit of force, the quantum of force, which has to appear-- but not the frequency is not gamma, but it is the Rabi frequency. So the picture you should sort of have is that under those situations, just assume the standing wave consists of two traveling waves, and the atom is Rabi flopping.

But it is Rabi flopping in a way that it goes up by taking a photon from one laser beam, and then it goes down by emitting the photon into the other laser beam. So during each Rabi flux cycle, it exchanges a momentum of 2 h bar k with the electromagnetic field. So therefore, the reactive force-- the stimulated force-- never saturates if you increase the Rabi frequency-- depending, of course, on your detuning, but if you choose the detuning correctly-- you can get a force which is just going further and further. Questions? Yes, Jen?

AUDIENCE: Is this the same concept as the Raman constants, except that the frequencies are just the same? Or is it just a--

PROFESSOR: It is a Raman process. Actually, everything is a Raman process here. Because in laser cooling, in everything which involves the mechanical-- the motion, the external degree freedom of the atoms-- we have an atom in one momentum state. We go-- light scattering always has to involve the excited state-- and then we go down to a different momentum state. So therefore, laser cooling is nothing else than Raman processes in the external degree of freedom of the atom. You may be used to [INAUDIBLE] Raman process more when you start in one hyperfine state, or for molecules in one vibration rotation state, and you go to another one, but, well, for vibration, and rotational states, and hyperfine states of it, Raman is used all the time. I usually used it also for the external motion, because I think it clearly brings out common features between all those different Raman processes.

So you can actually say that the reactive force is a stimulated Raman process between two momentum states, where both legs are stimulated by laser beams. In a standing wave, it would be-- the two legs-- would be stimulated by the two travelling wave components of the standing wave. Whereas the dissipative force is spontaneous Raman process, where one leg is driven by the laser, and the other one comes with spontaneous emission. Nicky?

AUDIENCE: [INAUDIBLE] when the atoms slow down, the [INAUDIBLE]

PROFESSOR: So the spontaneously emitted photon?

AUDIENCE: Yeah, I'm just thinking of actually the stimulated force. Because the atom must be stimulated. [INAUDIBLE] I'm confused how when the energy of the emitted photon must be slightly different due to the phase modulation? [INAUDIBLE] stimulated emission can be added to the frequency? [INAUDIBLE]

PROFESSOR: I mean we had the discussion after [INAUDIBLE] question, if you go up and down, and you are stimulated, you go up in a stimulated way, you go down in a stimulated way, and both laser fields which stimulate the transition have the same frequency, omega, you would actually see that there is no energy exchange. So at our current level of description we have described where the force comes from, but we don't understand yet where the energy goes, or where the energy comes from. And this is really more sophisticated, and I don't want to sort of continue the discussion.

We will encounter one situation, and this is in the [INAUDIBLE] atom picture, where we will discuss in a week or so, sisyphus cooling. And we will find out that there is symmetries in the mono-triplet. So again, we will find the missing energy in spontaneous emission. But if you don't have spontaneous emission, if you just have atoms moving in an optical lattice without spontaneous emission, it is really what I said, the phase modulation. And I would probably ask you at this point to do the homework assignment, and maybe have a discussion afterwards, because then you know exactly what I'm talking about. Does it at least-- You can take that as a preliminary answer.

So these are the two limiting cases. One is spontaneous emission, and the other one is phase modulation at a certain frequency. One can be sustained in steady state, you can spontaneously emit in sort of an atom goes to a standing wave, and this is sustainable. Whereas the phase modulation thing is actually a transient. It's an oscillation which is added in.

AUDIENCE: [INAUDIBLE] You're adding the force to the other one [INAUDIBLE]

PROFESSOR: Yeah, actually--

AUDIENCE: [INAUDIBLE]

PROFESSOR: If that helps you, but it's the following. I would sometimes say you know, I like sort of intuitive explanations of quantum physics. Let's assume I'm an atom. I'm in [INAUDIBLE] standing wave. And you don't know it yet, but I do sisyphus cooling. By just exchanging photons, a stimulated force, I'm actually cooling. And you would say, how is it possible? Because all the photon exchanges involve the same photon, the same energy. How can I get rid of energy? And I think what really happens is the following, the atoms can lose momentum without energy, but due to Heisenberg's Uncertainty Relationship this is possible. So the atom is first taking care of its momentum-- is losing momentum-- by stimulated force, and eventually, before it's too late, it has to do some spontaneous emission where it is paying back it debts in energy to Heisenberg.

So and of course, after a certain time, everything is OK. The force was provided by exchanging photons of identical frequency. And the energy is provided by an occasional spontaneous emission event, using one of the [INAUDIBLE]. So everything is there. Everything is a perfect picture, but you have to sort of, in this description, allow a certain uncertainty that the force is the exchange of identical photons, and the energy balance is reconciled in spontaneous emission. And on any longer time scale there are enough events that the balance is perfectly matched.

But if you would take the position, no, this is not possible. You know, the atom can only-- I mean, it's sort of in diagrams. It's not that one diagram which scatters a photon has to conserve energy. You have a little bit of time. You have Heisenberg's uncertainty time to make sure that another diagram jumps in, and reconciles energy conservation. That's at least my way of looking into it, but it's a very maybe, my personal interpretation of how all these diagrams and photon scattering events work together.

OK. Let's do something simpler now. So I've explained to you the dissipative force, the reactive force, and in the next unit I want to show you simply applications of the spontaneous force. In every experiment on cold atoms, the spontaneous force is center stage. It is necessary to slow down atomic beams. It provides molasses, which is the colling of atoms to micro-Kelvin temperature, and the spontaneous force is also responsible for the Magneto-optical trap. So what I want to do here is, again, by showing you the relevant equations, how the spontaneous force, which we have just discussed, how these spontaneous force leads to those three applications. So this is one of the most experimental sections of this course, because this equation has it all in it. And I just want to show you how this equation can be applied to three different important experimental geometries.

OK. So this is our equation. It has the momentum transfer per photon. It has the maximum scatter rate, gamma over two. And then, here, it has the Lorentzian line shape. And the important thing is when we talk about molasses and beam slowing is that the detuning is the laser detuning and the Doppler detuning. So the velocity dependence enters now the spontaneous force through the detuning in the Lorentzian denominator, and it enters through the Doppler effect.

So you can pretty much say it like this, if you have a bunch of laser beam, and slow and cool your atoms, how can the lasers do the job? Well, they measure the velocity of the atoms through the Doppler shift. And if is the Doppler shift which tells the laser beams what to do, so to speak. And this is how laser cooling works. So just as an experimentalist, you should actually know what the scale is. If somebody asked you, how strong is the spontaneous scattering force? Well, a way to connect it with real time units-- with real life units-- is, what is the maximum deceleration? Well, this is 10 to the 5 G. You can ask, is 10 to the 5 G a lot? Or not? Well, for an astronaut, it would be a lot. It would. No living organism can sustain 10 to the 5 G.

But I'm really surprised when I did this calculation. When you compare it to the electric force on an ionized atom, the same force, which is provided by the spontaneous light force, would be provided by an electric field of one millivolt per centimeter. So it's not easy if you have a stainless steel chamber to avoid petch effects, which create those electric fields. And if you've a battery with 9 volt, a centimeter apart-- just a 9 volt battery-- would accelerate an ion four orders of magnitude faster than your beloved strong spontaneous light force. So in that sense, 10 to the 5 G is a lot in the macroscopic world, but if you would look at microscopic forces-- which are often electric forces-- it's absolutely tiny.

And sometimes I say, the fact that you can do 100 kilovolt per centimeter, that you can make electric forces which are seven, eight, or nine, orders of magnitude stronger, that's actually the reason why ion traps were invented before trapping of neutral particles. So some developments in the field of trapping particles-- and eventually laser cooling them-- it first started with ion traps, and then it proceeded to neutral atoms. And the reason is the ion trappers have forces at their disposal which are eight or nine orders of magnitude stronger.

OK. Optical molasses. I know some of you will hate it, because I've just explained to you that in a standing wave, we don't have any spontaneous light force, all we have is the reactive force, the stimulated force, because it is the u, the in phase oscillation of the atomic dipole operator which is responsible for everything. But now I'm just, you know, wearing another hat, and I tell you that there is a limit where the total force can be regarded with some of the two forces. So therefore, I'm pretending now that if you have near resonant light in a standing wave, that the force in the standing wave-- which you know is a purely reactive force-- can now be written as the sum of the two propagating waves. And, of course, each propagating wave does a purely dissipative force.

That is, what I'm telling you can be mathematically proven. That the stimulated force in a standing wave is equal to the sum of the two dissipative forces or each travelling wave. I just keep that in mind whenever you think you can rigorously distinguish between the dissipative force and the stimulated force. Keep in mind that a standing wave-- which has only a stimulated force, as I rigorously proved to you-- can alternatively be described as the sum of two spontaneous light forces, each of which provided by one of the travelling waves. Well, it sort of makes sense in a perturbative limit. You can just take one beam, you can take the other beam, and the combined effect of the two laser beams is higher order. So the fact that in some low intensity limit this has to be valid, is pretty clear.

Anyway, but just a warning, if you really want to use a stimulated force here, you're in big trouble. It's much, much harder to get this result out of the stimulated force. Because to get a stimulated force with velocity dependence requires you to take solutions of the Optical Bloch Equations, which are not steady state, but non-adiabatic. So just a warning. You can do it, if you want with a stimulated force, but I would strongly advise you to first use the simpler formalism. You have to go to very different approximation schemes and much more technical complexity if you want to get it out of the stimulated force.

OK. So with that assumption, you'll remember we had the dissipative force of each laser beam. Now we have two laser beams. We summed them up. Because the two laser beams come from different directions, we have a minus sign here. And for the same reason, we have plus and minus sign in the Doppler effect. So what we have is the following, each force of each laser beam has this Lorentzian envelope, shown in black, but the other force of the other laser beam has the opposite sign. And the two have opposite Doppler shifts. So if I add up the two black forces, I get the red force. And the red force is anti-symmetric with respect to velocity. So therefore, in the limit of small velocities I get a force which is minus alpha times v. And this is the force of friction.

And in a whimsical way, people called this arrangement of two traveling waves Optical Molasses because the atom literally gets stuck in this configuration like a tiny ball bearing you throw into honey. So this is optical molasses. Actually, I think some of the Europeans had to learn the word molasses for it. In the US, everybody knows what molasses is. Well, in Europe, we know what honey is, but molasses is not a standard staple. But anyway, it's optical molasses.

OK. Very simple result. And, again, it's not easy to get it out of the stimulated force. But it's possible. I will actually tell you how we get cooling out of the stimulated force later in some limit.

AUDIENCE: Is there [INAUDIBLE] why this assumption of dissipative forces is easier [INAUDIBLE] and the stimulated force will be made difficult [INAUDIBLE]?

PROFESSOR: I'm not sure if there is a simple answer why it's easier. Well, it can't be easier than that.

AUDIENCE: I mean, there has to be something wrong with our assumptions of stimulated forces so that we cannot get [INAUDIBLE]

PROFESSOR: Yeah. I think what happens is what I briefly said. To the best of my knowledge, but I'm not 100% certain. We have the simply expression because we have used the steady state solution of the Optical Bloch Equation. In other words, we have a laser beam, and it's just the power of the laser which tells us how much light scattering happens. And we use the steady state solution. And we sort of have folded that into the force for one laser beam. And then we have the same package for the other laser beam. And we have never considered that the two laser beam may have some cross talk. And this is indeed valid for all low laser power.

However, if you want to get cooling out off the stimulated force, I will show some of you later, you can no longer use the steady state solution because it would miss the effect here. You wouldn't get any cooling out of it. In order to get something which is dissipative, out of a reactive force-- reactive force is by definition not dissipative-- you need a dissipation mechanism. And the dissipation mechanism is that you're not quite steady state, there is a relaxation time, a time lag. The atom is not instantaneously in its steady state solution. It needs a little bit of time to adjust. And it is these time lag of the atom which eventually gives rise to an alpha coefficient in the stimulated force.

AUDIENCE: So then physically, you are, during cooling experiments, you can take care that you are actually not in the steady state limit?

PROFESSOR: What I'm telling you is, for the spontaneous force, we get in leading order the effect by assuming the steady state limit, but if you want to get it out of the stimulated force, as far as I know, you don't get it with the steady state limit.

AUDIENCE: OK.

PROFESSOR: If you use different approaches-- I mean this is why you want to pick your approach. Sometimes you get something already in lower order. Sometimes you get it in higher order. And I think one favorite example is you solve this problem about [INAUDIBLE] scattering and Thompson scattering, pick your Hamiltonian, d dot e, or p minus a. In one case, you have to work harder than in the other case. And I think here it's similar.

Anyway, let's just put in-- I think that's, yeah, we can do in the next few minutes. So we now want to put in the effect that the force fluctuates. And that means we have heating. Before I do that, let me just tell you what the solution is for force equals minus alpha times v. Well, it means we extract energy out of the system at a rate-- Well, energy per unit time is force times velocity. Force times velocity is now minus alpha times v square. But v square is the energy, the kinetic energy. In other words, this equation tells us that the atomic motion, the kinetic energy, is exponentially [? damned. ?] And if there were nothing else, you just need two laser beams-- optical molasses-- and you would go not just to micro-Kelvin, but to nano- and pico- Kelvin temperatures. It's an exponential decay to absolute zero.

However, we don't each reach nano- and pico- Kelvin temperatures in laser cooling because there are other processes. And what is important here is spontaneous emission. OK. So the way you treat spontaneous emission is the following, every time an atom emits spontaneously, there is a random momentum kick of h bar k. If you have n photons scattered because the momentum kicks going random direction, they only add up in a random box. You get square root N. Or if you ask, what is the average of p square due to spontaneously emission? It is the momentum of the photon squared times the number of scattering events.

So therefore, in the form of a differential equation, the heating rate, or the derivative of-- the TEMPO derivative-- of p square goes now with the number of photons per unit time, which is the scattering rate. OK. If you would stop here, and that's what many people do who explain heating in this situation, you would miss half of the heating, because what you have treated here is only the photon transfer and spontaneous emission. However, there is also fluctuation and absorption.

I mean just look at two atoms in the [INAUDIBLE]. Kind of me and another atom. And we both scatter photons. On average, in the same laser beam, we absorb in photons and get in momentum kicks. But there is Poissonian statistics how many photons I absorb, and how many photons my twin brother absorbs. So therefore, due to the randomness in absorption, or the fluctuations in absorption. There is another square root n variance in the recoil kicks which comes from the absorption process. And it so happens, for exactly that reason, that the heating-- the derivative of kinetic energy, or the difference of momentum squared due to absorption is exactly the same as in emission.

Well, if you now make different assumptions, spontaneous emission has a dipole pattern. And you can sort of factor on the order of unity, depending what the pattern of the spontaneous emission is, whether you make a 1D or 2D model of spontaneous emission that the photons can only go in one dimension, in two dimension, or three dimensions. So you have to get other prefectors, but the picture is that without going into numerical factors on the order of unity, you have fluctuations in the absorption, you have a randomness in spontaneous emission, and they both equally contribute.

So that means now the following, that we have heating rate. The heating rate we just talked about. The increase in p square. Well, if you divide by 2 times the mass, it's increasing kinetic energy. So the increase in kinetic energy is given by this expression. And it is common if you have a heating process to introduce a momentum diffusion coefficient in that way. It's just the definition. It shows you how p square increases per unit time. And now we can get the cooling limit for spontaneous emission, namely by saying, in steady state when we have-- due to photon scattering-- the same amount of heating and the same amount of cooling, the temperature will have asymptotically reached a steady state value.

The heating rate was parametrized by momentum diffusion coefficient, d over m. So this is sort of independent of the velocity of the atom. Whereas you'll remember the cooling rate was proportional to the energy because it was an exponential approach to zero energy. So therefore, when we have the heating equals the cooling, we have the energy of the atoms there, and therefore, we find that the energy of the atoms in steady state is given by, actually, the ratio of heating versus cooling. It's a simple expression. The energy or the limiting temperature for molasses is the ratio of heating versus cooling. Heating is described by momentum diffusion coefficient and alpha is described by the damping force.

I want you, actually, to keep this expression in mind. A limit in temperature in laser cooling is always-- actually, in other processes in laser cooling-- is the ratio of heating over cooling. And momentum diffusion coefficient due to heating, over a friction force due to cooling. Because we will later find polarization gradient cooling, cooling in blue molasses, we will find other cooling schemes, where we will calculate-- with the appropriate model-- the heating and the cooling. And I don't have to repeat this part. The moment I calculate the heating, the momentum diffusion, and I calculate the friction, I know what the limiting temperatures is.

Again, when I said, the energy is kt over 2, you know, of course, kinetic energy is kt over 2 times the number of degrees of freedom. I assumed 1D, here. So in everything I've said on this page, there are numerical factors which may change whether you assume one or two or three dimension.

OK. I think that's the last thing I want to tell you. And then on Wednesday we do Zeeman's slowing and Magneto-optical trapping.

We had an expression for alpha. Remember we had this Lorentz profile, for the other Lorentz profile, and alpha was just the slope. I mean, everything is known. I just didn't bother to calculate it. But it just involves derivative of Lorentzian. And now we can ask, what is the lowest temperature we can reach? Well, you can now analyze your expression for alpha for a two-level system solved by the Optical Bloch Equation. And you find that you will have the most favorable conditions for low temperature-- you get the minimum possible temperature-- in the limit of low laser power.

And your detuning-- your optimum detuning-- is half a line width away. And then you'll find this famous result, that if you cool a two-level atom, the limiting temperature is simply given by the spontaneous emission rate, or the line width of your transition, gamma. And this is the famous result for the Doppler limit. For sodium atoms, it's 250 micro-Kelvin, for rubidium and cesium, because of the heavier mass, it's lower, is 10's of micro-Kelvin, 50 or 100 micro-Kelvin. So this is a famous limit.

And the physics of it is the following, the narrower the line width is. Then you can say the better you can cool particle down to zero temperature-- and I think, Jenny, your idea of the Raman process comes now in very handy-- I can have an atom, and I can have a Raman process where I scatter photons and go to higher energy or I scatter photons and got to lower energy. And the difference between the two processes is actually, the one that whether I cool or whether I heat comes from the Doppler effect. So the more I can discriminate between through the Doppler effect, the better I can cool. But the Doppler effect-- the Doppler shift, kv-- how well I can resolve it, depends on the width of the atomic transition.

The narrower the atomic transition, the more the Doppler effect can steer the laser cooling-- the Raman process-- towards lower velocity, not towards higher velocity. So that's the natural result that the-- or an expected result-- that the natural line width appears here.

OK. I've talked a few extra minutes. Is there any question? Then I see you on

Wednesday.