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PROFESSOR: OK. Back to cavity QED, back to the fully quantized radiation field, back to vacuum Rabi oscillation. Let me just recapitulate and sort of make the transition from this intense discussion about homework to the intellectually stimulating discussion about atoms and photons.

So in the semiclassical description of the electromagnetic field, photons can only be emitted because we have a Hamiltonian with the semiclassical electric field. So if you don't drive the system with an electric field, you cannot stimulate the emission of photons. But we know this is not what happens. Photons are emitted into empty space, photons are emitted into a vacuum. And for that we needed a quantized description of the electromagnetic field. We did field quantization, and we have now our quantized Hamiltonian.

And on Monday I started to discuss what is sort of the paradigmatic situation, the paradigmatic example, for how you should think about the vacuum and how you should think about emission of photons into the vacuum. And these are the vacuum Rabi oscillation described by the Jaynes-Cummings model. So the situation which I have in mind, or which you should have in mind, is an idealized situation, but it has been realized experimentally. And some of those idealized experiments were recognized with the Nobel Prize research of Haroche and Dave Wineland.

So the situation is we have an atom, but it only talks to one mode of the electromagnetic field, and we make sure that the atom only talks to one mode of the electromagnetic field not by eliminating other modes; they exist. I mean, an atom can emit upwards and downwards. But we surround it with a cavity which has such a small mode volume, it has such a small volume, that the single photon Rabi frequency is huge, and therefore the emission into this one single mode dominates

over the emission into all other modes. So this is a condition that the single photon Rabi frequency has to be larger than gamma.

And, of course, we also have to make sure that the system is idealized so the loss of photons because of losses in the mirror, or finite reflectivity in the mirror, also has to be smaller. So that means for several Rabi periods we have a system which has only two parts, a two-level atom and one single mode of the cavity. So that's the system we have in mind, and we discussed the Hamiltonian.

We saw that the Hilbert space of the atom is excited in ground state, the Hilbert space of the photons is spent by the [? flux ?] states, but what happens is-- so there's an infinite number of states, because of the infinite number of states of the photon field-- but what happens is the Hamiltonian couples only an excited state with n photons to a ground state with $n + 1$ photons. So the whole Hilbert space is segmented now into just pairs of states labeled by the index n .

So after so much work, we are back to a two-level system. And here is our two-level Hamiltonian. And, well, a two-level system does oscillations between the two levels. Rabi oscillations, no surprise. And this is what I want to discuss now.

But the new feature is that these are really, well, these are now really two levels. Each of them is the combined state of the atom and the quantized radiation field. So now we have included in our two-level description the quantum state of the electromagnetic field.

So first you should realize that this Hamiltonian is absolutely identical to $\text{spin } 1/2$ in magnetic fields. And you can recognize by [? comparing ?] this Hamiltonian, this matrix, to the matrices we discussed for $\text{spin } 1/2$ in the magnetic field, that this corresponds to the situation where this $\text{spin } 1/2$ had a transverse field in the x direction which caused a precession from spin up to spin down . And this x component of the field corresponds now to the single photon Rabi frequency times $n + 1$. That's the off diagonal matrix element in this matrix.

The thing which we have to discuss, and I will focus later, is that it depends on n . So

for each pairs of state labeled by n , the photon number, we have a different off diagonal matrix element. But let's discuss first the most important and simplest case.

Let's assume we are on resonance, and we want to assume that we have a vacuum. Then our Hamiltonian is simply this. And when we prepare the system in an initial state, which is an excited state with no photon in the vacuum, then we'll have oscillations to the ground state with one photon. These oscillations are exactly the oscillations we saw on the spin 1/2 system.

We can just map the solution. I'm not really writing anything here. So what we obtain is the famous vacuum Rabi oscillations. where the probability to be in the excited state oscillates with the single photon Rabi frequency ω_1 .

I think there's a little bit of an ambiguity in language. Is it the single photon Rabi frequency? or is it the vacuum Rabi frequency? Because there's always the question about plus minus one photon because we start in the excited state without photon so you want to say it's a vacuum Rabi frequency. But then you have the ground state with one photon, and this photon is reabsorbed and then you may want to call it the one photon Rabi frequency. So I leave it to you, but it's called vacuum Rabi oscillation and this Rabi frequency is usually referred to as the one photon Rabi frequency because we obtained the Rabi frequency by calculating the electric field of a single photon.

So the Rabi oscillations which we are observing now correspond to the periodic spontaneous emission and re-absorption of the same photon. There's only one photon which is spontaneously emitted and reabsorbed in a completely reversible coherent way, and the time evolution is unitary. So it's a periodic spontaneous emission and re-absorption of the same photon.

This has been experimentally observed. Actually, let me back up. Experiments are done in the microwave regime. The leading groups are, well, in the older days, Dan Kleppner, Herbert [? Weidner, ?] and Serge Haroche. And this involves Rydberg atoms.

Rydberg atoms in superconducting high q cavities. And those Rydberg atoms, because things scale with n and n squared, the principal quantum number, have a fantastically strong coupling to the electromagnetic field. And there is a homework assignment on Rydberg atoms in such cavities.

The other example is in the optical domain. And this really involves the D line of alkali atoms. You drive them on the D line. rubidium and caesium are often used, and the work is enabled by the development of so-called supermirrors which have an extremely high reflectivity, and you can realize an excellent q factor. And the leaders in this field are Jeff Kimble and Gerhard Rempe at the Max Planck Institute.

So let me just discuss an example taken from the optical domain. So the generic situation is that you have two mirrors which define a single mode cavity. Usually, you have a stream of atoms. Traditionally in atomic beams, then in some experiments in slowed atomic beams, more recently in atoms which are falling out of the mode, and only very recently single atoms with the help of other laser beams that are trapped inside the cavity. So they are streamed in such a way that only one or a few atoms are in the mode volume interact with a single mode of the cavity at a given time.

And then you want to figure out what is now happening, and you have the probe laser, you send it through the cavity, and then you record the transmission with a photodiode. Yanosh?

AUDIENCE: What is the mirror made of for the [INAUDIBLE]?

PROFESSOR: The mirror is made of a glass substrate, but then you would [INAUDIBLE] the coating. And the mastering is really to put coatings on which are very pure, but then also I think using ion sputtering, you make sure that the coating is extremely smooth and does not have any surface irregularities which would scatter a tiny fraction of the light. I know there are some people in Ike's group and [INAUDIBLE] group who work with high q mirrors. What is a typical example for the reflectivity? Or the q factor you can reach?

AUDIENCE: 5 [INAUDIBLE] and a finesse of, maybe, 500,000. And they're called superpolished mirrors.

PROFESSOR: So finesse of about a million, and that means the mirrors have 99.9999% reflectivity. And the superpolishing, I think, that was the last step. People had controlled the materials, but then they found ways to make a super polish and avoid these even one part per million scattering by surface roughness.

OK. So if you do that experiment, what would you expect? Well, it's a [INAUDIBLE] experiment so if you would scan the probe laser, and there is nothing in it, what you would expect is you would just expect a transmission peak at the cavity resonance. And if you tune much further, you get the next peak at the free spectral range. Let me just indicate that.

So this is a case for 0 atoms in the cavity. If you put 1 photon in the cavity, you no longer-- sorry, 1 atom in the cavity, you're no longer probing a cavity, you're really probing a system, which is no longer the cavity by itself. It's an atom-photon system. It's a couple system. And we know it's described by our two-by-two Hamiltonian, and this Hamiltonian has two solutions. And the two solutions are split by the one photon Rabi frequency.

So the two eigenvalues of our Hamiltonian are at plus minus omega 1 photon. So therefore, for n equals 1, we have a situation that we have two peaks split by the single photon Rabi frequency. Of course, I have assumed that great care has been spent to make sure that the cavity resonance is right where the atomic resonance is.

So this is now for 1. If you have 10 atoms, remember the two-by-two Hamiltonian looks the same, but it has the square root n plus 1 factor. So neglecting the 1 roughly when we have 10 atoms in the cavity, it's square root 10 larger Rabi frequency. And therefore, we would expect that we have now a splitting of the two modes, which is square root 10 plus 1 larger.

Actually, I didn't-- sorry, I have to collect myself now. I showed you that the Rabi

frequency scales with square root $n + 1$ in the photon field. But you should realize that everything is [? isometric ?] between photons and atoms. It's the complete coupling between photons and atoms. And if you would now look-- but I don't want to do it now-- if you would now look what happens if several atoms are present in the mode volume, you would also get a scaling which is $n + 1$ in the atom number, because the atom coupled coherently.

It is actually an effect of super-radiance, which we'll discussed later. So just take my word. You have the same scaling with the atom number. But I have to give you my word now, because in the experiment this is what people varied.

AUDIENCE: If they had varied power or number of photons instead, like, we couldn't have drawn these same diagrams, right? Because then the top part of the Lorentzian changes. If you're changing the photon number then Lorentzians change.

PROFESSOR: Say again?

AUDIENCE: So, like, right now, yes, we are varying the number of atoms so we can talk about the splitting. But if we were varying the power or the number of photons instead, then each of the Lorentzians, their height would change. How would you draw this observation if you were changing the photon numbers?

PROFESSOR: You know, I don't want to go into line shape. I would probably be a Lorentzian. I mean, all I want to discuss here is that we have a two-by-two Hamiltonian, which is split. And if we have one atom and one photon, it is split by the single photon Rabi frequency. If we have one atom and 10 photons, the atom can of course absorb and emit only one. As I derived on the previous page, we would have now a Rabi splitting, which is square root $n + 1$, n being the number of photons.

But if you would start in an empty cavity with 10 atoms in the excited state, because all the atoms are identical, they would spontaneously emit together, and then you would have 10 atoms in the ground state, and then you would have 10 photons. And so maybe this helps you. If you start with 10 atoms in the excited state, they do everything together. If you have 10 atoms in the ground state with 10 photons, and

now you have 10 photons and it's clear the 10 photons lead to a Rabi frequency, which is proportional to the square root of 10 or to the square root of 11.

So therefore, what you will observe is you will now observe a splitting of the single mode of the cavity which goes by the square root of $n + 1$. I don't want to discuss the line shape and the [? strings, ?] I just want to sort of discuss, in a way, the eigenvalues of the Hamiltonian, and the eigenvalues are the positions of the transmission peaks with a cavity.

And that has been observed. I mentioned the two leaders of the field are Gerhard Rempe and Jeff Kimble. Well, Gerhard Rempe, he did his Ph.D. in the same group at the same time as I did so I know him very well. Then he went and did post doc work with Jeff Kimble, and now is the Director of the Max Planck Institute. He has the world leading group in cavity QED. But this is sort of here the two leaders have a joint paper, which is the first observation of the vacuum Rabi splitting in an optical cavity.

Of course, you can easily observe it if you have a strong atomic beam with many atoms, because then you have a good signal. And secondly, the splitting is large and easily resolved. So what they managed to do is they managed to throttle down the atomic beam that fewer and fewer atoms at the given time were in the cavity. And eventually they came down to the limit of one atom.

That was an historic experiment. Of course, it's not perfect in the sense that you do not see the deep cut between the two peaks simply because, when on average you have one atom in the cavity, sometimes you have two atoms in the cavity, and then you have a peak in the middle. So those experiments in those days were done only with average atom numbers and not with trapped atoms where you know for sure there's exactly one atom in the cavity.

OK. So I don't show you an experiment, but let me just state that this sort of single photon Rabi flopping has been observed. You start with the cavity in the vacuum field, and you sort of see this oscillation to the ground state with one photon. But what I want to discuss now is the situation that we are not starting with an empty

cavity. We are starting with a coherent field. You can also start with a thermal field so there are different experiments you can do.

What would we expect now? So now the initial photon state is not the vacuum state, but the thermal state. If you have a microwave cavity and you heat it up a little bit, you have to cool it down to below 1 Kelvin. People use either helium-free chirostats or dilution refrigerators, but if you warm it up a little bit, you have a few microwave photons in the cavity. Or, that's even more controlled, you can make the cavity ice cold, but then you inject a few photons from your synthesizer into it-- from your microwave generator-- and then you have a weak coherent field. But a thermal state or a coherent state.

So what we then have is, OK, we would expect now a Rabi oscillation; however, the frequency for the Rabi flopping is now proportional to $n + 1$. And we have our photon field in the superposition of flux states. So the fact that we have a superposition state implies now that the Rabi oscillations have a different oscillation frequency for the different [? tablets ?] of states labeled by n . And that leads to a dephasing.

So that would mean that if you would look at the probability to be in the excited state-- just think about it. You have a wave function where the atom starts in the excited state, and the photon field is in a superposition. So now you have a two-component wave function which has different parts, and each part has a specific Rabi frequency. So you would have oscillations.

Let's say there is a certain probability that the cavity is in the vacuum, and then that means that there is a component which oscillates at the vacuum Rabi oscillation frequency. But if you have a component in your coherent or thermal state which has two photons in it, then you have Rabi oscillations which are faster. And now you have to superimpose them all. And if you all superimpose them, and you find that very soon there is a damping and maybe a little bit of vigor, but you see at damping of the population in the excited state. Q [? 2 ?] dephasing.

I'm just hesitating. I think I took this plot out of my notes, but I would expect now the

damping should actually lead to a probability to be in the excited state of $1/2$. So let me just try to correct that. So there is a little bit of oscillation, but then there is sort of a damping.

And eventually, if you have only a small number of photon states, then there will be a time where you have sort of at least a partial commensurability. You have maybe five frequencies. You know, square root 5, square root 4, square root 3, square root 2.

But then there is sort of a time where all these different frequencies have done an integer number of oscillations each, and then you get what is called a revival. And if you go to a large photon number, you have square root 100, square root 99, square root 88, the revival will happen at a later and later time and eventually at infinite times if you use a microscopic field. But for small coherent states, or thermal states, which only involve a few photons, you will get a revival phenomenon.

And this has indeed been observed. This was actually the PhD thesis of Gerhard Rempe, and it shows the probability in the excited state. They had previously observed the Rabi oscillations at early times, but now the experiment had to be adjusted, I think by using slower atoms, to observe the longer time. And here, well, 1987 for the first time revivals have been seen.

Let me dwell on that, or first are there any questions about what happens now? Atoms in the cavity to Rabi oscillations? And if the photon field is a superposition of only a few states due to this pseudo commensurability, you find times where you have revivals.

I just worked out something this morning which I think is nice, because it will highlight how you should think about spontaneous emission. So let me discuss. It doesn't really matter, but I want to give you a specific example that we have a coherent state. A lot of you know what a coherent photon state is. For those who don't, it doesn't really matter for what I want to explain, and recover that in [? 8.4.22. ?]

But if you have a laser or if you have a microwave generator, what comes out is a field which has a normalized amplitude of α , but your field is in a superposition state or [? flux ?] states. With these prefactors, I just wanted to give you an example. What I really just mean is that we have a coherent superposition of number states. We have prepared that. So now we have one atom in the excited state, it enters the cavity which has been prepared with the short pulse for a laser or microwave synthesizer in these state α .

And now we want to discuss-- so this is at t equals 0-- and now I want to discuss what happens as a function of time. Well, we know that if you have one atom, n , we have Rabi oscillations between the atoms in the excited state, and we have n photons. Or it has emitted the photon, and then we have n plus 1 photon and the cavity. But now, we have a superposition state, and we have amplitudes α .

So I mean, that's what we get. And this includes everything. It includes everything a two-level atom does in a single mode of a cavity. And this is spontaneous emission, stimulated emission, and reabsorption. But I want to use that now to discuss with you the misconceptions about spontaneous emission. Colin?

AUDIENCE: We're talking about just spontaneous emissions into the cavity?

PROFESSOR: OK. I've singled out a single mode. But what happens is-- and you're just two minutes, 30 seconds, ahead of me-- that we had discussed vacuum Rabi oscillations or Rabi oscillations when we have n photons in the cavity. This was our two-level system, our Hamiltonian, and all we get is Rabi oscillations with the Rabi frequency ω_n . And now we have to sort of do averaging.

I'm now discussing that we have a coherent superposition of number states. Let's say, a pulse of coherent radiation, a coherent state, and this is what we get. You can now, if you want, put in a [? zillion ?] of other modes, have another sum over all the other modes you want. So I'm just doing the first step in discussing with you what will happen, but adding more and more modes will actually not change the structure of the answer and will be, of course, quantitatively a mess but conceptually not more complicated.

So I want you to really look at that and realize where is the spontaneity of spontaneous emission. Where do you see any form of randomness associated with spontaneous emission in this expression? I don't see it. This is a wave function, and this time evolution is unitary. Everything is deterministic, and depending now how we choose our coefficient, there is even a revival. It's not dissipative that a photon is spontaneously emitted, and it's done.

We saw in the single photon Rabi oscillation it can be reabsorbed, we saw in a slightly more complicated situation that there are at least partial revivals, and it now depends how long we wait whether revivals will take place or whether they will be complete revivals or partial revivals. But we don't need a revival in a coherent evolution, the coherent evolution can just go to a complicated wave function and it's still a single coherent wave function fully deterministically obtained from the Hamilton operator. Sometimes it pops into our eyes through a reversible oscillation or through revival, but we don't need that.

So let me write it down but then explain you something. So it's unitary. There is no spontaneity at all. However, eventually we want to retrieve the classical limit. So if we would go to this situation that the average photon number is much smaller than 1, then the fluctuation in the photon field around the mean number are very small. For the coherent state the fluctuations are square root n .

And then, we retrieve the limit of semiclassical Rabi flopping with the Rabi frequency ω_r , which is-- I'm not consistent here with lower and uppercase [INAUDIBLE]. So it's uppercase or lowercase ω_n , and this is square root n times the single photon Rabi frequency. And, of course, for a large number of photons, we can always make the approximation that we do not have to distinguish between n and n plus 1.

So this is the ultimate limit if we would work in the limit of large photon numbers. So the way how you should look at it is the following. This system undergoes a time evolution to a state which is rather complicated. But if you make the number n large, this becomes approximately a state where you have simply-- you know what the

rate of the semiclassical limit? In the semiclassical limit, we have a constant laser beam with constant electric field amplitude, e , and then we have driven Rabi oscillations between ground and excited state.

So therefore, I don't want to show you mathematically, but in the limit of large n , you can approximate this complicated entangled wave function by the product of Rabi oscillations between ground and excited state times a coherent photon field. And the correction between what I just said and this complicated wave function is like 1 over n , because it's sort of a 1 over n approximation where we have neglected terms which the relative importance of them is 1 over n .

So therefore, there are people who will say and who will tell you when we have an interaction of an atom with a coherent state, and let's just think in the number of n being large, that n times out of n plus 1 , we have a coherent state. The atom does Rabi oscillation and what it does is it just emits photon into the coherent field and takes it back, like in semiclassical physics.

But in one case out of n cases, or the rate 1 over n of the rate of the wave function is sort of fuzzy. It's not a coherent state; it's something much more complicated. And if you do not keep track of this complicated nature of the wave function and just do some simple measurement by, let's say, just measuring the phase of the electromagnetic wave by projecting onto a coherent state, then you would find that with the probability of n the system was just staying in a coherent state. And with a probability which is one part out of n , something else has happened, and your detector cannot capture the entanglement of that state. And this last part is what some people associate with spontaneous emission.

I don't know. That's my view where the spontaneity in this process is. It's not a spontaneity in the time evolution. It's more a spontaneity if you do not care to detect this complexity, but map it back to a coherent state. And then with a precision which is 1 over n , you retrieve the semiclassical limit, but the difference between the semiclassical limit and the entangled wave function, this is what some people say is spontaneous because it's not captured by a single picture. I'm actually expecting

some people to disagree with me, but this is sort of my view, what I'm sort of learning from the simple examples I've given to you.

Since Ike is an expert on it, maybe, Ike, can I ask you the question is there actually a simple way to show that if I go to a large n limit that you can sort of really show that n parts out of $n + 1$ is really described fully by the semiclassical limit and there is only a 1 over n fraction where we have to look at the more complicated wave function?

AUDIENCE: I don't think there's a simple way to do it, but one can look at the equivalence of a [n and a factor n] state, and they're only different by one photon number.

PROFESSOR: It's sort of clear. I mean, everything is if you approximate n by $n + 1$. If you don't care about the small difference, everything falls into place and is simple. But I was just wondering if one could show sort of in a more direct or more intuitive or maybe more quantitatively what is really the extra part beyond stimulated emission absorption into the coherent state. So what sort of really the nature of what people call the spontaneously emitted photon?

AUDIENCE: I think that I don't the question, because I still argue that it's purely [n unitary n] evolution even--

PROFESSOR: OK.

AUDIENCE: For that system, and therefore, it's purely [INAUDIBLE] and nothing spontaneous is happening at all.

PROFESSOR: OK. All right. Good. OK. Fine. What is next? I think this finishes our discussion on vacuum Rabi oscillations and revivals.

I have now two topics in light atom interaction which you may not find in many textbooks, but it's my experience that they're really relevant. One is very conceptual. It's about the rotating wave approximation. And the other one is just the opposite, very technical. It's not really a new concept, but this is about saturation intensities and cross-section of an atom for absorption. The last things, cross-section for

absorption and saturation intensity, that's what you need when you talk to atoms in the laboratory. These are the quantities in which we think intuitively about light atom interaction.

So it's not involving any concept. I want to spend 20 minutes in introducing for you saturation, saturation parameter, cross-section, what's different between monochromatic light and broadband light. But before I do that, I have a few minutes on the rotating wave approximation. So let's call it rotating wave approximation revisited. Again, rotating wave approximation.

And what I want to discuss can be discussed in the fully quantized picture, but also in the semiclassical picture.

In the fully quantized picture, just a reminder, what we discussed earlier was that when we have the atomic raising and lowering operator and the photonic raising and lowering operator, we got four terms. And two of the terms are co-rotating, two are counter-rotating. But I can get exactly the same number of four terms in this semiclassical picture, and I want you to see both.

But in the quantized picture, it's actually easier, because when you see a and a^\dagger you know immediately one is absorption one is emission. So therefore let me explain to you what I want to tell you about the rotating wave approximation using the semiclassical picture, because then you immediately know how to apply it to the quantized picture.

So what I want to bring in here in addition to what we have discussed about light atom interaction, we had sort of a dipole Hamiltonian, is the fact that we have circular polarized light, left-handed and right-handed light, and I want to sort of use that and combine it with angular momentum selection rules, which as you remember we discussed after our discussion of dipole, quadrupole, and magnetic dipole positions.

So I put now all those parts together and revisit the rotating wave approximation. So what I hope for is it tells you a little bit how selection rules, angular momentum,

circular polarization, and semiclassical field which rotate in one direction how they are all connected. So I have to set up the situation by saying that we use as a quantization axis the direction k , which is either the direction of the propagation of the light beam or, in general, it's orthogonal to the polarization of the electric and magnetic field.

And I can talk about an electric field driving an electric dipole transition. I can talk about a magnetic field driving a magnetic transition. It doesn't really matter. I will use [? Bsc ?] amplitude, but you can also immediately think electric dipole, and this field is linearly polarized. But I want to immediately decompose this field into right-handed and left-handed field. Or a linearly polarized field can be regarded as a superposition of a field which circulates this way plus one which circulates the other way.

And ultimately, the message we will see is that if you have linearly polarized light, we always get counter-rotating term, we always have a [INAUDIBLE] shift and such. But if you use rotating fields or circularly polarized light, selection rules may actually lead to the result that there is no counter-rotating term at all. So this is eventually what I'm aiming for, and this will be the final point of the discussion.

So the field which rotates in the right-handed direction where the rotating field is a superposition of x and y , or i and j . And one the rotates has a cosine ωt and one has sine ωt . I don't need to write down the left-handed part, because there's just a minus sign. Or this will actually become very handy. I do all the discussion for the right-handed part, but I can always obtain the expression for the left-handed part by replacing ω by minus ω .

Which will mean that some emission process by the right-handed part will be an absorption process by the left-hand part. Be we'll see. You can change angular momentum by plus 1 by absorbing a right-handed photon or-- you'll see. We'll get there. So anyway, those signs will become important.

Let me now take the above expression for the right-handed part and replace cosine ωt and sine ωt by e to the $i \omega t$. So this was the i component, this

was the j component. We divide by 2.

Just to avoid confusion, I want to emphasize I've started with a real field. So I'm not using, as you often do in e and m complex field and the real fields are the real part, I have not started out by adding, you know, imaginary parts to the field. I've started out with a linearly polarized field in the x direction $\cos \omega t$, and I've decomposed it into two real fields. One is right-handed, one is left-handed. Complex numbers only come because I want to use a complex exponential to replace $\cos \omega t$ and $\sin \omega t$.

We are almost done with the decomposition of the field. I just wanted to-- we have now four terms, and I want to [recoup] them. i minus imaginary unit j . i plus imaginary unit j . This is $e^{i \omega t}$. And this is $e^{-i \omega t}$.

So what have we done? Well, we've just started with linearly repolarized light, and I've rewritten the expression twice, and now we are looking only at one of the circular components, and in the end what we have is four terms. Well, that's also what we had in the fully quantized Hamiltonian, and we now want to identify what those four terms. Two will be co-rotating, two will be counter-rotating, but it's very helpful to analyze those terms.

But there are two things we have to look at now. One is we have an $e^{i \omega t}$. And, well, it's probably a sign convention, but trust me, if you put that in the Schrodinger equation, it means that you increase the energy of the atom if you drive it with $e^{i \omega t}$. You take it from a ground state to an excited state, which differs in frequency by ω , and therefore this means you have increased the energy of the system, and this corresponds to absorption.

Whereas this one here means we take an atom from an excited state to a ground state, and this is the situation of stimulated emission. Remember, in selection rules we take our field and we multiply it with a dipole moment, electric, magnetic dipole moment whatever. But now we want to use also the spherical tensor decomposition of those dipole moments.

It's a complicated word, but what it means is those terms are dotted with the dipole moment, and if you do it now component-by-component, we retrieve selection rules because this peaks out the-- let me just write it down-- the x plus y tensor component of the matrix element. And this corresponds to $\Delta m = +1$. We change the angular momentum by one unit, and of course this term is then $\Delta m = -1$.

So we have done the work. What I want to do now is just map those terms into an energy level diagram. I like sort of pictorial representations, and each term becomes now a graphical [INAUDIBLE].

So let us assume we have a system, hydrogen is to p state. But let's say generally we go from a $j = 0$ to $j = 1$ state, which has three components. Now, I have set it up in such a way that-- oops, we need a little bit extra space. I've set it up in such a way that the states here, this is $m = 0$, this is $m = +1$, and this is $m = -1$.

So therefore-- let me just use color coding now-- this one here is $\Delta m = +1$ so this one always moves to the right. It changes angular momentum by 1 so it can always move to the right, whereas the other one, $\Delta m = -1$, moves to the left. Absorption is $e^{-i\omega t}$, always moves up. And stimulated emission moves down.

So with that what happens is this term here transfers one unit of angular momentum and energy. So that would mean this term goes up here. It could go up here, if [? there were a ?] state. The other term-- let me use a green color-- is driving the process in the opposite direction. But now we have to also consider that you can go down here, and you can go down to a virtual state.

A virtual state is just something which has the same wave function as a state, it just has an $e^{-i\omega t}$, which is not-- it's a driven system. You drive it. You [INAUDIBLE] a state. You [INAUDIBLE] a state at the drive frequency, and it just means, in this case, this state has an oscillation $e^{-i\omega t}$, which is very, very different from what a state which is populated would have, and this is what we

call a virtual state.

So in other words, what is possible is we have our three states, plus minus 1 and 0, but this is the spatial wave function including angular momentum. But we can now drive it by plus omega and minus omega, and therefore we can have it as virtual states pretty much at any energy we want. But this process here is not possible, because this would require to go to a state which has m equals 2, which does not exist.

So now what I've shown here is if we would stick in the m equals 0 state, I've shown you the four terms, two are co-rotating and two are counter-rotating. If you neglect this virtual state which has a detuning of about 2ω , or 2 resonance frequency of the atom, this is the rotating wave approximation. One term is responsible for absorption; the other term is responsible for a stimulated emission. But if I don't make the rotating wave approximation, I have those two extra terms.

So this is only the right-handed light, and I want to sort of play a little bit with this concept. If I would take the left-handed light, I would add sort of four more arrows. Two more here and two more here. But let's just keep the situation as simple as possible. But I really sort of like that you write down right-handed, left-handed side, decompose it into its components, and each component is now in this diagram connected to an arrow where one direction is angular momentum, the other one is energy.

So let me now talk about other energy diagrams. And this will lead to the answer. Well, can we create a situation where we have only two terms, which would be the simplest two-level system, can be directly realized without any rotating wave approximation a two-level system?

So if we had two levels, which have only m equals 0 and m equals 1. So this would be the situation I just discussed with those two levels. So the only way how I can fit in this arrow is this one, and the diagonally downward arrow is that. So in this case, rotating wave approximation is not an approximation, it is exact. But some purists will actually say, hey, you can never realize that when you have an n equals plus 1

state. Then you always have an m equals minus 1 state. And then you have a virtual state down there, and then you get two more terms, which are the counter-rotating terms which I just showed above.

So whole I would say if you have a neutron star which makes an infinitely high magnetic field, you can have a huge [INAUDIBLE] splitting between m equals plus 1 and m equals minus 1 and completely move one of the angular momentum states out of the picture. But, of course, in the rotating wave approximation we are neglecting off resonant terms at 2ω , ω being an electronic excitation energy, so I'm really talking about Zeeman shifts here to eliminate the other state which may be comparable to electronic energies.

So in principle, I can say this is my Hilbert space, and in this Hilbert space no rotating wave approximations is needed. But it's maybe an artificial Hilbert space. When I had a discussion with other people, we came up with the possibility of some forbidden transition. If you go from a doublet s to a doublet s state so all you have is a spin system which has $1/2$ angular momentum plus $1/2$ minus $1/2$. And then you realize that the only way how you can fit in the orange arrow is in this way, and the green arrow in this way.

So here you would have a situation where the rotating wave approximation is exact. But, of course, it's not an electric dipole transition; it's some sort of weaker conversation, which may be forbidden.

I need two more minutes. I have discussed the case where we have quantized along a direction, I called it the k direction, and the polarization of the electromagnetic field was [i and j ?] was perpendicular to it. So let me now discuss a case where we quantize along the polarization of the electromagnetic field, and you remember from our discussion on selection roles that this is π light. So in this case, our magnetic or electric field is polarized along the i direction, and the real cosine ωt gets decomposed into $e^{i\omega t}$ plus $e^{-i\omega t}$. And we know already one term is absorption one is emission.

And now, if I take my j equals 0 to j equals 1 system, π light has a selection role of

$\Delta m = 0$. So now I have an arrow, which I want to be orange, which goes up. And a green arrow-- this is a great program. The only thing is you have to be very careful when you change color and press carefully. That's why sometimes the colors are not doing what I want. But here is green.

But now, of course, with linearly polarized light we can always go down to a virtual state. We have now four terms. Two are rotating, two are counter-rotating. So therefore the quick conclusion of the last ten minutes is that there is the possibility that counter-rotating terms can be 0 for σ^+ σ^- light due to angular momentum selection rules.

But what we have also learned is if you have the $m + 1$ state in there's an $m - 1$ state, if you have circularly polarized light and we drive a transition between two m states, the counter-rotating term does not come from the same set of two states, $m = 1$. It involves $m = -1$. So it's the other state which is maybe degenerate or only slightly [? split ?] by a magnetic field which is responsible for the counter-rotating terms. Anyway we have talked so much about rotating wave approximation and those terms, I just wanted to show you how it is modified if you use degeneracy p states and angular momentum.

Any question? OK.