# Problems 3

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These problems are simply directed toward calculating the QCD $\beta$ function at one loop using Feynman graphs. By doing it in several different ways, you will check gauge independence and universality, as well as getting practice in a sort of calculation that is central to many – I think it's fair to say most – of the deeper applications of quantum field theory.

In this problem "Fermi gauge" means the sort of gauge in which the gluon propagator is proportional to

$$g_{\mu\nu} - \alpha \frac{k_\mu k_\nu}{k^2}$$

1. Gluon wave function renormalization

   a. Calculate the logarithmically divergent part of the wave function renormalization for gluons, due to gluons, in an arbitrary Fermi gauge, using the brutal cutoff procedure described in class. You will have to include the Fadeev-Popov ghost contribution, of course. Pay attention to the issue of longitudinal versus transverse polarizations!

   b. How is the answer modified by charged fermions and scalars?

   c. Give arguments that the quadratic and linear divergences can be ignored, as can the renormalization of longitudinal polarizations, in calculating the $\beta$ function.

2. Beta function from gluons

   a. Calculate the log divergent parts of the "proper" gluon 3-point function – the part that matches the tree graph structure, including glue, fermions, and scalars. Use this, together with earlier results, to calculate the $\beta$ function. Check that the $\alpha$ dependence cancels.

   b. Do the same thing using the 4-point function.

3. Beta function universality

   a. Calculate the $\beta$ function from renormalization of the gluon-fermion coupling.

   b. Calculate the $\beta$ function from renormalization of the gluon-scalar coupling.

   c. Calculate the $\beta$ function from renormalization of the gluon-ghost coupling.

4. Looking back

   Now that you've done all this, and (presumably) acquired some faith in the framework, design a shortcut. If you take it for granted that you can work in an arbitrary gauge and calculate charge renormalization using any appearance of the charge, what is the most efficient way to calculate? Can you find a way to do it on one not-too-cluttered page?