

8.311: Electromagnetic Theory Final Exam 5/13/04

Time: 9:30-11

Your Name: _____

1. Coaxial transmission line

A circular cylindrical transmission line is made of two coaxial cylinders with the radii a and b , $a < b$. Consider a coaxial cable made of a long cylindrical conducting shell of inner radius a and a wire of radius b on the axis inside. The space between cylinders $a < r < b$ is filled with material of dielectric constant ϵ .

a) The transmission line is used in the operating (TEM) mode, for which both the electric and magnetic field are transverse to the wavevector. Find the E and B fields spatial distribution for this mode and the dispersion relation ω vs. k .

b) Consider the TEM mode in a half-infinite line with a resistor R connected to an end of the line. Write down the wave incident on the resistor end, the reflected wave, and by matching quantities at the resistor determine the ratio of the reflected and incident wave amplitudes E_r/E_i .

c) By calculating Joule dissipation in the resistor and comparing it to the Poynting vector flux for the incident and reflected wave, calculated separately, verify energy balance.

2. EM waves in plasma.

Consider monochromatic waves in plasma, described by the model

$$\epsilon(\omega) = 1 - \omega_p^2/\omega^2, \quad \omega_p^2 = 4\pi n e^2/m \quad (1)$$

a) Find the dispersion relation $\omega(k)$. Show that for ω below the cutoff, $\omega < \omega_p$, waves cannot propagate in plasma.

b) Find the group velocity of waves in the plasma $v_g(k)$

c) Consider a linearly polarized wave of amplitude E_0 with frequency ω normally incident on a halfspace region filled with plasma described by the dielectric constant (1). Find the wavenumber inside the plasma, and the amplitudes of the transmitted and reflected wave. Consider separately $\omega > \omega_p$ and $\omega < \omega_p$.

3. Electromagnetic wave scattering.

Consider a nonrelativistic electron moving in a constant magnetic field \mathbf{B} . The electron is driven by an electromagnetic wave, which makes it move and radiate. The time-averaged radiation power can be expressed as the time-averaged energy flux in the wave times some area, called the scattering cross section. Note that, since both the radiation and the energy flux are proportional to the square of the field amplitude, the cross section is independent of the field intensity, and thus is a characteristic of electron dynamics only

a) Given the EM field amplitude E_0 , find electron motion driven by the field of a circular polarized electromagnetic wave propagating along the field direction. Consider separately the cases of right and left polarization of the wave.

b) Find the scattering cross section for the electron moving as described in part a).