PROFESSOR: OK, in that case, let's get going. Today's lecture will be mainly on the blackboard. I have a few slides I want to show. And what we want to talk about is the inflationary universe model.

So I'll start by describing the mechanism of inflation, how it happens. Inflation is based on the physics of scalar fields and gravity. As I think we've said, in present day particle theory, and by present day I mean not yet string theory, all particles are described as fields, quantum excitations of the field. The analogy that most people are at least qualitatively familiar with, is the photon, which is a quantum excitation of the electromagnetic field. But in fact, to describe a relativistic theory of interacting particles, the only way we really know for any kind of particle is to introduce a field and describe the particle as a quantized excitation of the field.

So when we talk about a scalar field, that's the quantum representation of some kind of a scale or particle. And scalar in this case, means spinless, same in all directions. A scalar field then, is just a number defined at each point in space. The only scalar field that we've actually seen in nature so far, is the Higgs field. And indeed, inflation is very much modeled on the Higgs field. Although the field that drives inflation, which is by definition, called the inflation, is probably not the Higgs field of the standard model. Although recently, actually, in the past few years, people have written a number of papers proposing that maybe the Higgs field of the standard model could, in fact, be the field that drives inflation.

So we don't know. It's an open question. But in any case, the field drives inflation is some kind of cousin, at least, of the Higgs field of the standard model and that has many of the same properties. In particular, the properties of a scalar field are pretty much summarized by its potential energy function. Energy density is a function of

the value of the field.

And there are two kinds of potential energy functions that I like to talk about. One is the kind of that is used in New inflationary models. And potential energy versus field value. And it has a plateau with a peak at someplace, which is usually assumed to be phi equals zero and a potential energy function which may or may not be symmetric about phi equals zero but I'll assume it is, just for simplicity.

And the second type, which I'd like to talk about, mainly for comparison-- This is really the one that will be interesting. But one could also imagine a potential energy function which really has a local minimum someplace, which is not the global minimum. And I'll draw it with the local minimum at the origin and the global minima, two of them degenerate, elsewhere. And this, again, is a graph of mi versus phi.

And the reason I'm drawing this is partly for historical interest. This was what was used in the original inflationary model. It is my original paper. It does not work. But we'll want to talk about why it does not work. In both cases we're interested in a state, which can be called a false vacuum, which is a state where the scalar field is just sitting at phi equals zero.

In the case of the second of these potentials, phi equals zero being a local minimum, is classically completely stable. If one had a scalar field in some region of space just sitting in that minimum, there'd be no place where energy could come from that would drive it out of that minimum over the barrier. In the second case, the field is classically stable. But it's still possible for it to quantum mechanically tunnel through the barrier. And that process has been calculated and understood. Originally by Sidney Coleman and collaborators. And an important feature of that tunneling is that does not happen globally. You might think that there would be some probability that suddenly everywhere in the universe the scalar field would tunnel over the barrier and go down to the other side. The probability of that is zero, as you might realize if you thought about a bit more. There's just no way that the scalar field that far over there is going to know the tunnel at the same time as the scalar field far over there.

So the timely happens locally and it really happens in a very small region, which then tunnels over the barrier and the scalar field start rolling down on the other side in this small region. And then that region grows. The scalar field, as it rolls over the barrier, pulls the scalar field nearby. And the region grows with a speed that rapidly approaches the speed of light.

In the New inflationary potential, where we have actually a local maximum here, the situation is classically meta stable, in the sense that the smallest possible fluctuation can start the field rolling down the hill. And in particular, quantum fluctuations will, if nothing else, start the field rolling down the hill in some finite amount of time. We're interested in the case where the amount of time it takes for quantum fluctuations to push the field off the hill is relatively long compared to time scales involved in the early universe. And the key time scale involved in the early universe is the Hubble time. And the Hubble time is just driven by the energy density. So one can calculate the Hubble time.

And one is interested in the case for building inflationary models, where the top of the hill is smooth enough, gentle enough, has a small enough second derivative so that the amount of time it will take for the scalar field to roll off the hill is long compared to the Hubble time of the universe. So in both cases, for short times, the scalar field is just stuck at the origin. And that's what's important, as far as what we want to talk about next.

So the characteristic of this state called the false vacuum is that the scalar field is pinned at a high energy state. Scalar field is pinned at a high energy density. And by pinned, I mean in advance you just can't change quickly. In general, particle physicists use the word vacuum to mean the state of lowest possible energy density. When we call this a false vacuum, we're really using the word false in the sense of the word temporary. These states are temporary vacuums, in that for some period of time, which is long by early universe standards, the energy density can't get any lower. So it's acting like a vacuum.

Now what are the consequences of that? The important consequence of that is that

the pressure has to be large in negative and in fact, equal to the negative of the energy density. And there are two ways we can convince ourselves of that. The first is that if we remember the cosmological equation that we derived somewhere in the middle of course for row dot. We learned that row dot is equal to minus 3 a dot over a where a is the scale factor times row plus the pressure over c squared.

Now what we're saying here, is that as the universe expands, the scalar field is just stuck at this false vacuum value. The energy density is stuck at the energy density associated with that value of the field, the potential energy density of the field itself. And therefore, row dot will be zero as the universe expands. And if row dot is zero, we can just read off from this equation what the pressure has to be. Row dot equals zero implies that the pressure is just equal to minus the energy density times c squared, which is another way of saying it's minus-- Excuse me, the mass density times c squared, which is another way of saying it's minus the energy density. I'm using u for energy density and row for mass density. And they just differ by a factor of c squared.

So this is the straightforward equation method of seeing the answer here. But if you want to explain this to your roommates or somebody who is not taking this class, there's also a simple argument based on a thought experiment, which I think is worth keeping in mind. And we've used this argument before, actually, in similar contexts.

If we imagine a piston chamber in our thought experiment. And in our piston chamber, we're going to put false vacuum on the inside. And the false vacuum will have an energy density, I'll call it u sub f. And on the outside. we're going to have a zero energy vacuum. Now we've learned that since 1998, we've known that our vacuum is not a zero energy vacuum. We seem to be seeing a non-zero vacuum energy in our universe.

However, even if that's true, the vacuum energy of our universe is incredibly small compared to the false vacuum energy density that we're talking about in terms of the early universe. So you could still very well approximate it as zero and not worry

about it. So that's what we'll be doing here.

So we'll think of the outside as being either a fictitious vacuum, which by definition, has zero energy density and we can talk about it even if it doesn't exist. Or we could think of it as being the real vacuum in our universe, which has an energy density which is approximately zero on this scale. Now what we want to do is just imagine pulling out that piston. So we have now created an extra region on the interior of the piston chamber. And we're going to be assuming that we've somehow rigged the walls of the chamber so that the false vacuum will be stretched as we pull out the piston. The piston is attached to the false vacuum in some way.

So this entire area inside region is now false vacuum. And therefore, the volume of the false vacuum region has enlarged. And if we call the extra region here delta v, the volume of that region, we now have a situation where the energy has increased by the energy density of the false vacuum times delta v by changing the volume of the chamber.

Now, energy has to be conserved. So this energy has to be equal to the work that was done by whatever force pulled out on this piston. We won't need to specify who was pulling on the piston, but the work done when one pulls out on a piston is just equal to minus p times delta v, the work done by the person pulling on the piston. So the normal case, the pressure would be positive, the piston would be pushing out on the person holding the piston and the interior would be doing work on the person pulling out. And it would be positive if the pressure were positive. The work done on the person, but this is supposed to be the work done on the gas. And that's minus p times delta v.

So if energy is conserved, the work done on the gas has to be equal to the change in energy of the gas. And the change in energy of the gas is that. So conservation of energy implies that delta e equals delta w or use of f times delta v equals minus p times delta v, which of course implies that p is equal to minus use of f, as we said before.

So the point is that if the energy inside the piston is going to increase, the person

pulling out on the piston had better be doing work, had better be doing positive work. And if the pressure inside were positive, the person pulling out on the piston would be doing negative work. The piston would be pushing on him. So for it to make sense here, with the energy in the piston increasing, the person pulling out has to really be pulling against a suction. He has to do work to pull out. And a suction means a negative pressure, if we have zero pressure outside. Pressure inside has to be negative.

So we could reach this conclusion either of two ways and we get the same conclusion. The pressure is just equal to minus the energy density. Yes?

AUDIENCE: I'm a little confused about why the energy's increasing inside. Because why couldn't you just say the energy density decreases with the increased volume?

PROFESSOR: OK the question is why couldn't you just say that the energy density would decrease with the increased volume. That certainly is what will happen if you have normal gas inside. What makes this particular false vacuum odd is the origin of this energy density, which is the potential energy density of the field. So if we were talking about the situation, for example, which is the clearest cut, the only way the energy density here could go down is if the scalar field goes up over the barrier and then comes down over here. And there's no way to drive it there, except to wait for a quantum fluctuation, which is a very slow process.

And similarly, here there's no barrier. So it can't just roll down. But that takes a certain amount of time. And we're assuming that all the things we're talking about here are happening on a time that's fast compared to the amount of time it takes for the scalar field to roll.

So what makes this peculiar false vacuum special is that it cannot lower its energy density quickly. And that's what the word false vacuum implies. And there are states like that. And then those states necessarily have a negative pressure, or pressure that's equal to good accuracy to minus the energy density. Or I say to good accuracy only because the energy density could change a little bit slowly, at least for the top case. But it's limited how much it can change.

OK, now what are the consequences of this cosmologically? Well we've also learned that we could write the second order Friedman equation, which is the equation really tells us what the force of gravity is doing. A double dot is equal minus 4 pi over 3 g times row plus 3 p over c squared.

Now for the false vacuum, p is equal to minus row c squared minus the energy density. And that means that this term is negative and three times as big as that term. So for the false vacuum, this quantity, which we normally think of as being positive, becomes negative. I should write factor of a here. And that means that instead of gravity slowing down the expansion the universe for a false vacuum, the expansion is accelerated.

And that's also what we're seeing today with the vacuum energy, which behaves the same way as this false vacuum and produces gravitational repulsion in exactly the same way. So false vacuum implies gravitational repulsion.

OK, this basically is the mechanism of inflation. So we're sort of finished with this chapter. Are any questions before I go on about how this gravitational repulsion arises? Michael.

AUDIENCE: So, for the top vacuum that you've drawn up there, where there's no barrier, you just roll slowly, are we assuming that it takes a long time for it to begin to roll or after it's started rolling that it also takes a very long time to reach the bottom.

PROFESSOR: I guess I'd say both, Begin to roll is not that well defined. Because it may have it an infinitesimal velocity from the time you start discussing it. And then that infinitesimal velocity gets bigger and bigger. But I think what we're saying is that the whole process, however you divide it up, it's going to take a long time compared to the time it takes for the exponential expansion to set in.

OK. So now, I'd like to take this physics and just put a scenario around it. And we'll call the new inflationary scenario because that's what it is. Maybe now I should mention a little bit more about the history here. When I wrote my original paper, I was assuming a potential of something like this, because it seemed generic and

created inflation and I was able to understand that inflation would solve a number of cosmological problems, which are the problems that we've talked about. And we'll come back to talk about how inflation solves them. But

But, one still has to end inflation. In this model, inflation would end only by the tunneling of the scalar field through the barrier, which as I said, happens in small regions which then grow. Those regions are spherical so they're called bubbles. And the whole process really is very much the way water boils. When you boil water, it forms very small bundles initially and the bundles grow and then start colliding with each other and making a big frothy mess.

And it turns out that that's exactly what would happen in the early universe if you had this model. When I first started thinking about it, I hoped that these bundles could collide with each other while they're still small and merge into a uniform, hot region of the new phase, a phase where the scalar field is not there, but there. But that turned out to be the case.

It turned out that the bubble formation process produced horrible inhomogeneities that there did not seem to be anyway to cure. And that then was the downfall of the original inflationary model. But a few years later, Andrei Linde in the Soviet Union, and independently, Albrecht and Steinhardt in the US, proposed what came to be called the new inflationary model, which started with a different assumption about what the underlying potential energy function for the scalar field was. Instead of assuming something like this, which might be called generic in some sense, they instead assumed something that's a little bit more special, a potential energy function with a very flat plateau somewhere, which, well, we normally put in the middle.

And this has the advantage, that the inflation ends, not by bubbled nucleation by tunneling, but instead, by just small fluctuations building up and pushing the scalar field down the hill. And what makes it work, basically is that those small fluctuations have some spatial correlations built into them. So over some small region, which I will calla coherence region, the fluctuations are essentially uniform. And the other

important feature is that once the scalar field starts to roll, it still has some nearly flat hill to roll on. So a significant amount of inflation happens after this homogeneous coherence region forms. So the initial coherence region can be microscopic, but it is then stretched by the inflation that continues as the scalar field rolls down the hill towards the bottom.

So that process of stretching the coherence region after it has already formed is what makes this model workable, while this model was not. So that's the basic story of how new inflation succeeded in allowing inflation to end gracefully, is the phrase that was used. The problems associated with this model came to be called the graceful exit problem. And this is the first solution to the graceful exit problem. They're now other solutions. But they're very similar actually.

So I'll just write here that's it's a modification of the original inflationary model to solve this graceful exit problem problem.

Now I should say a little bit about how inflation starts. But I can only say a little bit about it because the bottom line really is we don't know. We still don't have any real theory of initial conditions for cosmology, whether it's inflationary cosmology or any kind of cosmology. The nice feature of inflation is that it allows a significantly broader set of initial conditions than is required, for example, in the standard cosmological model, where, as we discussed, the needed initial conditions are very precisely specified.

I might say a few things though, about ideas people have had. One idea, which I think sounds very reasonable, is due to Andrei Linde. And it's a vague idea, so it really needs to be more precise before it could really be considered a theory. But this is just the idea that the universe started out with some kind of chaotic random initial conditions. And then the hope is simply that inflation will start somewhere. That somewhere in the initial chaotic distribution there'll be a place where the scalar field will have the right properties, the right configuration to initiate inflation.

There are also models by Vilenkin, Alex Vilenkin of Tufts, and independently, Andrei Linde, who by the way, is at Stanford. They both worked on models where the

universe could begin by a quantum tunneling process, starting from absolutely nothing. I wrote here absolutely nothing and that's more nothing than nothing.

You might think of nothing as just empty space. But from the point of view of general relativity, as you already know enough to understand, empty space is not really nothing. Empty space is really a dynamical system. Empty space can bend and twist and stretch and do all kinds of complicated things.

It's really no different, in some basic sense, from a big piece of rubber. So nothingness really is intended to mean a state where there's not only no matter present, but also no space and no time, really nothing. One way to think of it, perhaps, is as a closed universe, the limit as the size of the closed universe goes to zero so that there's nothing left.

None of these theories are precise. We don't really know how to precisely formulate them. In this tunneling from nothing, one is talking about tunneling in the context where the structure of space itself changes during the tunneling process. So it's tunneling in the context of general relativity. And we don't really have a successful quantum theory of general relativity. So these ideas are very speculative and quite vague. But they do indicate some possibilities for how the universe might have started.

An idea closely related to this tunneling from nothing is the Hartle and Hawking-- This is Jim Hartle, of UC Santa Barbara, who's also the author of a general relativity textbook now. And Stephen Hawking, who you must know from Cambridge University. They proposed something called the wave function of the universe. From their point of view, it's self contradictory to talk about the universe having an origin because before the origin of the universe, space and time we're not even defined. And therefore, you could not think of there being a time before the universe was created. And therefore universe didn't actually get created. It just is and has some earliest possible time.

And that's what this wave function of the universe formalism reflects. But otherwise, it's pretty similar, really, to the idea of tunneling from nothing. The idea is that the

universe had some kind of a quantum origin, which determined the initial state of the universe.

In any case, for the purpose of inflation, what we really need to assume, and this could be an assumption which follows from any of these theories, we need to assume that the early universe contained at least a patch, and we don't know exactly know how big the patch has to be, but greater than or about equal to the inverse Hubble constant times the speed of light, the Hubble length. And this initial patch also has to be expanding, or else it would just collapse. It really has to be expanding faster than a certain threshold. But I won't try to put that all into the one sentence.

Oh, I didn't say patch of what yet. Whoops. That's where the average value of phi is about zero. And by average, I mean averaging over rapid fluctuations, if there are any. And if one has this, no matter where one got it from, inflation will begin. And once inflation begins, it doesn't matter much how it begins.

To see what happens next, it's easiest to at least pretend, that to a good approximation, you can treat a small region of this patch as if it were homogeneous and behaving like a Robertson-Walker universe of the type we know about. Then we can write the first order Friedman equation, which is a little bit more informative than the second order one. I'm going to leave out the curvature term. We'll argue later that the curvature term becomes small quickly.

But for our first pass, we'll just assume that the universe is described by something as simple as the Friedman Robertson-Walker equation for a flat universe. For row, we're just going to put row sub f. We'll assume that our space is just dominated by this false vacuum energy density. And this can easily be solved. It just says that the first derivative divided by the function itself is a constant. Just take the square root of this equation. And that is an equation which just immediately get solved and gives you an exponential.

So you find that for late times, you just get a of t behaving as a constant times an exponential of time where the exponential time constant, which I'm calling chi. Chi is

just the square root of this coefficient. The square root of 8 pi g over 3 times row sub f. So clearly this is the solution to that equation. And for late times, it is the solution that will dominate.

Actually it is the only solution, the way I've already simplified this. But if we started with the full system of equations, there'd be other solutions with different initial conditions. But this is always what you would be led to, for late times. The exponential expansion would dominate.

So that's what innovation basically is. It's a period of exponential expansion. There are a few features of inflation, which helps to understand why it is so robust. That is, why no matter how it starts, it leads to the same result.

So one feature of inflation I'd like to mention is the cosmological no-hair. Some people call it a theorum and some people call it a conjecture. I think the more precise statement about this theorem is that you can prove it as a theorem perturbatively. That is, if all initial deviations are small, you can really prove it, but people think it's true, even beyond perturbation theory. And in that case it's a conjecture.

But it's basically the statement that if one has a system with p equals minus row c squared and row is greater than zero, if that describes the matter, then essentially any metric that you start with will evolve into this exponentially expanding flat metric. Any system will evolve to locally resemble a flat exponentially expanding space time. And the word locally there is needed to make it true.

If, for example, you start with a closed universe, as just a simple example, which has this kind of matter filling it. It will start to grow exponentially. It will always stay a closed universe. It will never become literally flat. But as it gets bigger and bigger, any piece of it will look flatter and flatter. And it will keep getting bigger and bigger exponentially fast forever. So it will rapidly approach a space which looks like an exponentially expanding flat space.

Now this exponentially expanding flat space time has a name, which is de Sitter

space, named after a Dutch astronomer. It was discovered early on in the history of general relativity. 1917, I think, was the date that de Sitter wrote his paper about de Sitter space. It has some very interesting properties, which De Sitter do not notice all of them. In spite of the fact that I'm describing it as a flat exponentially expanding space time, that's not the only possible description. It turns out that the same space time, by changing what you call equal time services, can be described as either an open or closed Robertson-Walker universe, completely homogeneous in both cases. So it's very weird. But the easy way to think of it for most purposes, is as this flat exponentially expanding picture.

OK Next thing I want to point out about de Sitter spaces is that they have what are called event horizons. Now early in the course we talked about horizons and didn't really try to quantify the name. The horizons that we used to talk about are technically called particle horizons. Those are horizons that have to do with the past history of the universe and are related to the fact that since the universe has only a finite past history, or as a cosmological model at least, there's a finite distance that light could have traveled up until this time. And we cannot have any way of seeing anything that's further away than that maximum distance that light could have traveled. That's the particle horizon.

These event horizons are different. They're related really, to the future of the universe, rather than the past. It's a statement that, because of the fact that these universes are exponentially expanding, if two events that happen at a particular time are separated from each other by more than a certain distance, then the light from one will never reach the future evolution of the other.

And one can see that by looking at the total coordinate distance that light could travel between any two times. So I going to let delta r of t1 t2 be equal to the coordinate distance that light travels from t1 to t2. And I'm going to assume that a of t is given by exactly this formula. And I'll write out const because I don't want to write it too many times. I could give it a one variable symbol if I wanted to.

This delta r of t1 t2 is just the integration of the coordinate velocity of light from t1 to

t2 of c divided by a of t dt. The coordinate velocity of light is just c divided by a of t. We've seen that formula before. And this can easily be done by putting in what a of t is. And we get c over the constant that appeared in that formula, whatever it is, times chi, the exponential expansion rate, times e to the minus chi t 1 minus e to the minus chi t 2.

And now, the question we want to ask ourselves is, suppose we let this light ray travel for an arbitrarily long amount of time, which means taking t2 to infinity. And the important feature of this expression is that as t2 goes to infinity, the expression approaches a finite value. The second term just disappears. And you're left with the first term. So no matter how long you wait, anything that started out with a coordinate separation larger than that value, that asymptotic value, will just never be reached by the light pulse that you've sent.

And that's what this event horizon is. And it's easy to see what it actually amounts to numerically. If you want to know how far away the object has to be now, in physical terms, so that its coordinate distance is larger than the maximum we get here, we know how to do that. The maximum value can be written as just the limit as t2 goes to infinity of delta of r1 r2. And we have the expression for it right here. It's just the first piece of this answer. And this is the coordinate distance.

If we want to know the present physical distance of something which is at that coordinate distance, we would just multiply it by the present scale factor. And present here means, t1 and t2 are the arguments here, and we just want to multiply by a of t1 to get the physical distance of an object which is at this boundary, the boundary of what we'll be able to receive a light ray from and what we won't. So this is the event horizon distance, physical distance, and it's just equal to c times chi inverse.

When you multiply by a of t1, you cancel the constant of the denominator and you cancel the e minus chi t1. And you're just left with c times chi inverse. Which is the Hubble length. It's the inverse Hubble constant times the speed of light, which is the Hubble length. So anything that is further away one Hubble length, from us now, if

that object emits a light ray, we will never receive it. And that's called the event horizon.

Now the reason this is important is nothing travels faster than light. And that means that in a de Sitter space, everything is limited in how far it can ever get. And an important implication of that is that if, in our full space, which may not be entirely de Sitter space, if we have a de Sitter region, but junk outside that, which we don't understand, don't know how to predict, could be anything, we would still know, even without knowing what's outside, that whatever's outside can never penetrate into the de Sitter region by more than one event horizon, by more than one Hubble length.

So the interior of the de Sitter region is protected from anything on the outside. And that is a rigorous theorem of general relativity, this protection. And that means that once you have a sizable region of de Sitter space, no matter what's going on outside, it's never going to disappear. It will always be protected by this event horizon.

I should give you now a few sample numbers associated with this scenario. And here I have to say that we don't really know very accurately what are the right numbers to give here. So I think the word sample numbers was well chosen there. What we don't know is what energy scale inflation actually happened at in the history of universe. Turns out that the consequences are pretty much identical for most questions, or all the questions that they have been able so far to investigate observationally, regardless of what energy scale inflation happened.

Inflation was originally invented in the context of Grand Unified Theories. And I think that's still a very plausible context in which inflation might have happened. And the sample numbers I'll give you will be numbers associated with Grand Unified Theories.

And what starts the whole story is the energy scale of Grand Unified Theories, which is about 10 to the 16 GeV billion electron volts. And this number is arrived at by measuring, at accessible energy with accelerators, the interaction strengths of the three fundamental interactions of the standard model of particle physics. The

standard model of particle physics is based on three different gauge groups, su 3, su 2 and u1. Each one of those gauge groups has associated with it an interaction strength. And they can be measured. And that's where we start this discussion.

Then once you measure them at accessible energies, which is like 100 GeV, or something like that, then you can theoretically extrapolate to much higher energies. And what is found is that to good accuracy, the three actually meet at one point. And that is the underlying basis, really, of Grand Unified Theories. That's what allows the possibility that all three interactions are really just a manifestation of one underlying interaction, where the one underlying interaction is made to look like three interactions at lower energies through this process called spontaneous symmetry breaking, which was talked about a little bit in a lecture I gave the time before last, I think, in, probably in Scott's lecture.

Now this meeting of the three lines is decent in the context of what is literally the standard model of particle physics. But if one modifies the standard model of particle physics by incorporating supersymmetry, a symmetry between fermions and bosons, and that involves adding a lot of extra particles because none of the particles that we know of make up a fermion boson pair. So in a supersymmetric model for every known particle, you introduce a new unknown particle. Which would be it's supersymmetric partner. In that minimal supersymmetric extension of the standard model, the meeting of the lines works much better. So it's a piece of evidence in favor of supersymmetry. In any case, where the lines meet to good approximation in either one of these two discussions, whether it's supersymmetric or not, is it about 10 to 16 GeV. So that becomes the fundamental math scale of the end unified theories. Hold on a second. That' what I'm looking for.

Now once one has this mass scale, one can figure out an appropriate mass density. And that's what we're really interested in, what would be an appropriate mass density for a false vacuum in a grand unified theory. And one can develop that, and we really don't know how to do any better. Because as I've told you, we don't know really know how to calculate vacuum energies anyway.

But as a dimensional analysis answer, we can get the answer because it is really uniquely determined by dimensional analysis up to factors. If one wants to make an energy density out of E gut plus constants of physics, the only way to do that is to take E gut to the fourth power and divide it by h bar cubed c to the fifth. And you can convince yourself at home that that gives you an energy density. And you could even evaluate it numerically, by mass density, excuse me. And this is about equal to 2.3 times 10 to the 81 grams per centimeter cubed. So it's a fantastically high mass density, 10 to the 81 grams per centimeter cubed.

And if one puts this into the formula for chi, the exponential expansion rate, chi turns out to be about 2.8 times 10 to the minus 38 seconds. And c times chi inverse, the Hubble length, it turns out to be about 8 times 10 to the minus 28 centimeters. So all these numbers, off scale by human standards. And that's just a feature of the fact the Grand Unified Theories are off scale by human standards.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Do I have this backwards? No, this is incredibly small. This is 10 to the minus 28.

AUDIENCE: So then it's chi [INAUDIBLE]

PROFESSOR: I'm sorry. Hold on. Yeah, no this--

AUDIENCE: Chi should be [INAUDIBLE]

PROFESSOR: Chi inverse is a time. C times the time is a distance. So I think that's right.

AUDIENCE: So is chi inverse 10 to the [INAUDIBLE]

PROFESSOR: Yeah, if we're in cgs units, Chi inverse by itself would differ by a factor of 10 to the 10. So it would be 10 to the minus 38, Hm. Hold on. This must be chi inverse.

AUDIENCE: Oh, OK.

PROFESSOR: There is an inconsistency here. You are right. Yes, that's chi inverse. This is time. And then this just multiplies by c.

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OK so the way this scenario would work is, we would start with the early universe with some patch or order of magnitude this size. Which I might point out is 14 orders of magnitude smaller than the size of a single proton, which would be about 10 to the minus 13 centimeters. So 15 orders of magnitude, maybe. And then we would need enough inflation, so that at the end of inflation, the patch should be on the order of maybe one to 10 centimaters or more

It has to be at least about this big, but could be much bigger. There's no problem with the being much bigger. Much bigger would just mean there's more inflation than you minimally needed. There's no problem with having too much inflation.

And then it's a matter of checking and a calculation, which I'll tell you the answer of. If we want to go from some size of the end of inflation to the present universe-- And that's really what we're interested in, ultimately, getting the present universe. -- there'd be a further coasting expansion from the end of inflation until now, which can just be calculated by using the idea that a times temperature, scale factor times temperature, is a constant. So the increase in the scale factor is proportional to decrease in the temperature.

And the reheat temperature of this model-- Maybe I didn't describe reheating exactly, I'll describe it quickly in words. At the end of inflation, the scalar field is destabilized by these fluctuations and rolls down the hill, then oscillates about the bottom. And when it oscillates about the bottom, we need to take into account the fact that this field interacts with other fields. And it then gives its energy to the other fields, basically the standard model fields ultimately, heating them up, producing the hot soup of particles that we think of as the starting point for the conventional Big Bang Theory.

So this reheating process at the end of inflation as the inflaton field oscillates about its minimum, reproduces the starting point of the conventional Big Bang Theory. And it produces a temperature which is comparable to the temperature that you started at, which is the temperature scale of the theory. So if it's Grand Unified Theory scales, we would reheat to a temperature of order 10 to the 16 GeV.

And then, to ask what will be the expansion factor between then and now, it would be 10 to the 16 GeV times the Boltzmann constant times 2.7 Kelvin. This is the ratio of the temperature then to the temperature now, both expressed as energies. And then we might want to multiply this by 10 centimeters, if we say that at the end of inflation, the universe was 10 centimeters across. Size at end of inflation. This, I worked out at home, is about 450 times 10 to the 9th light years.

And we would want something like 40 times 10 to the 9th light years to explain the present universe. So this is about 10 times too big. And that's OK. It means that we could get by with one centimeter and 10 centimeters is being a bit generous.

So inflations would start with this tiny patch. At the end of inflation the patch would have grown to one or maybe 10 or perhaps more, centimeters in length. And then by coasting up to today it becomes something that's larger than the region that we now observe. And that's basically how the scenario works. Any questions about those numbers or the general pattern of what we're talking about here?

OK. What I want to talk about next, and this will pretty much be where we'll stop, although a few other things we might mention if we have time, I want to talk about how it solves the three cosmological problems that we have discussed of the conventional Big Bang model. And the explanations are actually quite simple. So we can go through them pretty quickly.

First we had the horizons slash homogeneity problem. Remember that was caused, or could be stated as, the problem that the early universe expanded so fast that the different pieces of it did not have time to talk to each other. And, in particular, when the cosmic microwave background was released, points at opposite sides of the universe were separated from each other by about maybe 50 horizon distances, we calculated. And that means there's no way they could have communicated with each other, and therefore no way we could explain how they turned out to have the same temperature at the same time.

In this case, what we've done is, we've inserted into the history of the universe an extra phase of evolution, the inflationary phase. And if we go back to the beginning

of the inflationary phase, we see that that problem is just not there. And if it's not there, it doesn't develop later. At the beginning of the inflationary phase, by assumption, the region that we're starting to talk about was about horizon length in size.

And if we had enough inflation to produce 10 centimeters out of that, that was 10 times more than we needed, it would mean that the entire observed universe would be coming from a region that would be only about a tenth of the size of this Hubble length. So that would therefore be well inside the horizon at that time. And that means that if you allow a little bit of leeway with these numbers by having a little bit of extra inflation, there can be plenty of time for the entire region that's going to become our personally observed region, to come to a uniform temperature by the ordinary processes of thermal equilibrium. Because they're much less than the horizon distance apart.

And then once the uniformity is established, before inflation, when the region that we're talking about is incredibly tiny, inflation takes over and stretches that tiny region so that today, it's large enough to encompass everything that we see. And therefore everything that we see had a causally connected past and had time at the early stages to come to uniform temperature, which is then preserved as the whole thing expands. So that gives a very simple explanation for the homogeneity problem. Basically before inflation the region was tiny.

Second on our list was the flatness problem. And the basis of that problem was the calculation that we did about how omega minus 1 evolves in time. And we discovered that omega minus 1 always grows in magnitude during conventional evolution of the universe. And that therefore, for omega minus 1 to be small today, it would have to be amazingly small in the early universe, as small as 10 to the minus 18 at one second after the Big Bang to be consistent with present measurements of omega minus 1.

The key element there was this unstable equilibrium and the fact omega L minus 1 always grew with time. And that depended on the Friedman equations. During

inflation, the Friedman equations in some schematic sense were the same equations, but the rows that go into it are different. So the equations basically are different.

And if we look at the key equation, the first order Friedman equation, H squared equals 8 pi over 3 G row minus Kc squared over a squared. This was the equation that we used to derive this flatness problem. We could see immediately, if we now think about it, during the inflationary process, things are completely reversed. Omega is driven towards 1, and exponentially fast.

And the way I see that is to just ask what is this equation do during inflation. And during inflation, we just replace row by this constant value row sub f, the energy density of the false vacuum is fixed. And that means that during inflation, this term is fixed. This term is falling off like 1 over a squared. And a is growing exponentially with time. So that means that this term is decreasing relative to that term by a huge factor, by the square of the expansion factor. So in our sample numbers over there, we were talking about an overall expansion from 10 to the minus 27 centimeters to 10. That's expansion by a factor of 10 to the 28.

In that case, during inflation, this term decreases by a factor of 10 to the 56 while this term remains constant. And that means that by the end inflation, this is completely negligible and this equation without this extra term means you have a flat universe. So during inflation, the universe is driven towards flatness, like one over a squared, which is 1 over the square of this exponential expansion factor, so very, very rapidly.

And finally, the third of the problems that we talked about was the monopole problem. We argued, originally Kibble argued, that you'd expect approximately one of these monopoles to form per horizon volume, just because the monopoles are basically twists in the scalar fields. And there's no way the scalar fields can organize themselves on distances larger than the horizon distance. So you'd expect something on order of-- It's a crude argument. --but something on the order of 1 not in the scalar field per horizon volume. And that led to far too many magnetic

monopoles, fantastically too many.

And the formation of one monopole per horizon volume is hard to avoid. I don't know of any way of avoiding it. But what gets us out of the problem here, is that we can easily arrange in our inflationary model for the bulk of the inflation to happen after the monopoles form. And that means the monopoles will be diluted by the exponential expansion that will occur after the monopoles form. The rest of the matter is not diluted, because when inflation takes place it's at a constant energy density so the amount of other stuff that will be produced is not diminished by this extra expansion. But the monopolies, which will produce first, will be thinned out by the expansion.

So the basic idea here is that the volume goes by a factor of the order, using our sample numbers, it's linearly growth by a factor of 10 to the 28. Volumes go like cubes of linear distances. So 10 to the 28 cubed is 10 to the 84, I think. Probably right. And that means that we can dilute these monopoles by a fantastic factor and make everything work, if we just arrange for the monopoles to be produced before the exponential expansion sets in.

OK. Finally, and I think this is probably the last thing we will talk about, another problem that we could have talked about and we'll talk about the solution of it now, even though we never really talked about as a problem, is the small scale nonuniformities of the universe. And if we look out around the universe we don't see a uniform mass distribution, we see stars and stars collected in galaxies and galaxies collected in clusters and clusters collected in super clusters, a very complicated array of structure in matter. Those are all nonuniformities. And we think we understand how they evolve from early times because we also see in the cosmic microwave background radiation, small fluctuations, which we can now actually measure to very high degree of accuracy.

Those small fluctuations provide the seeds for the structure in the universe that happens later because of the fact that the universe is gravitationally unstable. So at very early times what we're seeing directly in the CMB, these nonuniformities were

only at the level of one part in 100,000. But nonetheless, in regions where there was slightly more mass density, that pulls in slightly more matter, producing still stronger gravitational field pulling in more matter and that amplifies the fluctuations. And that affect, we believe, is enough to account for all the structure that we see in the universe as originating from these tiny ripples on the cosmic microwave background.

But that still leaves the question of where do these tiny ripples come from. And in conventional cosmology, one really had no idea where they come from. One knew they had to be there even before they were seen because we had to account for the structure in the universe and how it evolved. When they finally were seen, it was seen just right. Everything fit together.

In inflationary cosmology, the exponential expansion tends to smooth everything out. And for a while, those of us working on it were very worried that inflation would produce a perfectly smooth universe and we'd have no way of accounting for the small fluctuations that were needed to explain the existence of stars and galaxies. But then it was realized that quantum theory can come to our rescue.

Classically, inflation would smooth everything out and produce a uniform mass density everywhere. But quantum mechanically, because quantum mechanical theories are fundamentally probabilistic, the classical prediction of a uniform density turns into a quantum mechanical prediction of an almost uniform density, but with some places being slightly higher than that uniform density, other places being slightly lower. And qualitatively, that's exactly what we see in the cosmic microwave background radiation. And furthermore, we can do it quantitatively. One can actually calculate the effects of these quantum fluctuations.

And that's what I want to show you now. The actual data on that, which is just gorgeous. Shown here is the Planck seven year data. Shown here is the Planck seven year data, where what's being plotted is the amplitude of the fluctuations versus the angular wave length. One is seeing these as a pattern on the sky. So the wavelength you see as an angle, not as a distance.

And long wave lengths are at the left. Short waive lengths are at the right. It's really done as a multiple expansion, if you know what that means. And those numbers are showing on the top. And the data points are shown as these black bars with their appropriate errors. And the red line is the theoretical prediction due to inflation, putting in the amount of dark matter that we need to fit the data that we also measure from the supernovae.

And it's absolutely gorgeous. So I have a little Eureka guy to show you how happy I was when I saw this graph. And with the help of Max Tegmark, we've also put on this graph what other theories of cosmology would give.

So if we had an open universe, where omega was just 0.2 or 0.3, as many people believed before 1998, we would have gotten this yellow line. If we had inflation without dark energy, making omega equal what out of matter, out of ordinary matter, we would get this greenish line, which also doesn't fit the data at all. And there's also something called cosmic strings that we haven't talked about. It was for a time, thought to be a possible source of the fluctuations in the universe. But once this data came in, that became completely ruled out.

Now this is not quite the latest data. The latest data come from the Planck satellite. And it was released last March. And I don't have that plotted on the same scale, but this is the latest data which, as you see, fits even more gorgeously than the data from WMAP. The more accurately it gets measured, the better it fits the theoretical expectations.

Now I should mention for truth in advertising, that this data is to some extent fit to the model. It's actually a six parameter fit. But of those six parameters, I don't have time to talk about them in detail, but four of them are pretty much determined by other features. Two of them are just fit to the data. And one of them is something that changes the shape a little bit. It's the opacity of the space between us and the surface of last scattering.

An important parameter that's fit that you should know about, is the height of the curve. The height of the curve can, in principle, be predicted by inflation if you knew

the full details of this potential energy function that I've erased for the scalar field. But we don't. We just have some qualitative idea about what it might look like. So the height of the curve is fit to the data.

But nonetheless, the location of all these peaks and everything really just come out of the theory and it's just gorgeous. And it works wonderfully. So the bottom line is I think inflation does look like a very good explanation for the very early universe. It's kind of bizarre since it talks about times like 10 to the minus 35 seconds after the Big Bang which seemed like a totally incredible extrapolation from physics that we know. But nonetheless, marvelously, it produces data that agrees fantastically with what astronomers are now measuring.

So we'll stop there. I want to thank you all for being in the class. It's really been a fun class to teach. I have very much enjoyed all of your questions and enjoyed getting to know you and hope to continue to see you around. Thank you.