8.286 Lecture 17
November 12, 2013

BLACK-BODY RADIATION AND THE EARLY HISTORY OF THE UNIVERSE, PART 3

Summary of Lecture 16: Dynamics of a Flat Radiation-dominated Universe

$$H^2 = \frac{8\pi G}{3}\rho \ , \quad \rho \propto 1/a^4 \qquad \Longrightarrow \qquad \left(\frac{\dot a}{a}\right)^2 = \frac{\text{const}}{a^4} \ .$$

Then

$$a\, da = \sqrt{\text{const}}\, dt \qquad \Longrightarrow \qquad \frac{1}{2}a^2 = \sqrt{\text{const}}\, t + \text{const}' \ .$$

So, setting our clocks so that $\text{const}' = 0$,

$$a(t) \propto \sqrt{t} \qquad \text{(flat radiation-dominated)} \ .$$

$$H(t) = \frac{\dot a}{a} = \frac{1}{2t} \qquad \text{(flat radiation-dominated)} \ .$$

$$\ell_{p,\text{horizon}}(t) = a(t)\int_0^t \frac{c}{a(t')}dt'$$

$$= 2ct \qquad \text{(flat radiation-dominated)} \ .$$

$$H^2 = \frac{8\pi G}{3}\rho \qquad \Longrightarrow \qquad \rho = \frac{3}{32\pi G t^2} \ .$$

Summary of Lecture 16: Black-Body Radiation

Black-body radiation is a gas of massless particles at temperature T.

Energy Density: $u = \rho c^2 = g\dfrac{\pi^2}{30}\dfrac{(kT)^4}{(\hbar c)^3}$,

where $g = 2$ for photons. (There are two spin states, or polarizations, for photons: either left-circularly polarized and right-circularly polarized, or x-polarized and y-polarized.)

Pressure: $p = \dfrac{1}{3}u$.

Number density: $$n = g^* \frac{\zeta(3)}{\pi^2} \frac{(kT)^3}{(\hbar c)^3} \ ,$$

where $\zeta(3)$ is the Riemann zeta function with argument 3,

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots \approx 1.202 \ ,$$

and $g^* = 2$ for photons.

Entropy density: $$s = g \frac{2\pi^2}{45} \frac{k^4 T^3}{(\hbar c)^3} \ .$$

Entropy is a measure of "disorder," in the sense that it measures the number of microscopic quantum states that contribute to a given macroscopic state. The second law of thermodynamics says that entropy never decreases. If the system stays close to thermal equilibrium, then entropy is essentially conserved. In the early universe, entropy is essentially conserved for all processes except for inflation.

8.286 The Early Universe
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