Assignment 2 PHYSICS 8.284

Due Wednesday 2006 February 22 11:04am

- Reading: Press, Chapter 2, Chapter 3.5 and Chapter 4.1 and 4.2 through 4.2.5 (`http://www.lanl.gov/DLDSTP/ay45/ay45toc.html`)

1-2. Assume, for the sake of simplicity, that the B filter in the Johnson UBV system has 100% transmission from 3900$\mathring{A}$ to 4900$\mathring{A}$, and 0% transmission outside this range. Assume that the V filter has 100% transmission from 5050$\mathring{A}$ to 5950$\mathring{A}$. Compute (by numerical integration, using Simpson's rule with only one interval if you like) the *fraction* of the total flux emanating in these two idealized passbands from black bodies with each of the following temperatures: a) 10^5 K, b) $35,000$ K, c) 9700 K, d) 6500 K, e) 4700 K, f) 2600 K.

The zero-point of the UBV color system is arbitrary, and was chosen (by Johnson and Morgan) so that an A0 star would have U − B = B − V = 0. Compute B − V colors for the black body temperatures above, choosing your zero point so that a 9700 K black body has B − V = 0.

Suggestion: In doing numerical integrals it's often a good idea to take all the dimensioned constants outside the integral and change variables so that one integrates over a dimensionless variable, e.g. let the variable of integration be $x = h\nu/kT$ or $y = hc/\lambda kT$. The dimensioned constants set the scale of the problem, while the dimensionless integral gives some fraction of that typical scale.

3. In the best circumstances, the noise in an imaging detector is equal to the square root of the number of photons detected. Suppose you observe a star with $m_{AB} = 25.0$ with a 1000$\mathring{A}$ bandpass centered on 5556$\mathring{A}$. Assume that f_λ is constant over the bandpass, and that the detector has an efficiency (not unrealistic) of 50%.

   a) How many photons are obtained in 30 minutes observing with a 2.4 meter telescope (say, for example, the Hubble Space Telescope). If the "signal" is the number of photons detected, compute the signal-to-noise ratio.

   b) Compute the number of photons for the same star observed for 30 minutes with the Magellan I 6.5 meter telescope at Las Campanas.

   c) Suppose the star has a point spread function (its footprint) which is 0.5 arcseconds in diameter Each square arcsecond of the dark sky at Las Campanas has a brightness equal to one star of magnitude $m_{AB} = 21.5$. How many photons are detected from the sky at Las Campanas inside the star's footprint in the same 30 minute exposure.

   d) Our detector detects both photons from the star and photons from the sky. The noise is therefore the square root of the sum of number of photons detected from the star and the sky. The signal is the number detected from the sky. Calculate the signal-to-noise ratio. Which telescope "wins"?

   e) Compute the signal-to-noise ratio obtained in 30 minutes on a night of very good "seeing", when the stellar image has a diameter of 0.3 arcseconds.

4. Recall that the latitude of MIT is roughly $+42°22'.8$, and that the ecliptic is inclined to the Earth's equator at $23°.5$. How long is the Sun above the horizon on the longest and shortest days of the year?

5. Show that the equation we derived in polar coordinates for the orbit of a planet around the Sun,
$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$
takes the following form in Cartesian coordinates, and is therefore an ellipse.
$$(x+ae)^2 + \frac{y^2}{1-e^2} = a^2 \quad .$$

6. We plan to cover the material for this problem on Tuesday February 21. There is, unfortunately, no good treatment of this in a text, but there are "notes on the Kepler problem" on our web page. If you wish, you may hand in this problem with problem set #3. Or you may want to hand it in now as part of problem set #2.
The radial velocity for a star in a binary orbit is given by
$$v_r = \frac{2\pi a}{P}\frac{\sin i}{\sqrt{1-e^2}}[e\cos\phi + \cos(\phi+\theta)]$$
where a is the semimajor axis of the star's orbit (with respect to the center of mass), P is the period, i is the inclination of the orbital plane to the line of sight, θ is the angle made by the star with the position of the star at periastron and ϕ is the angle made by the position at periastron with the line of nodes (where the plane of the orbit crosses the plane of the sky), measured from nodal line to periastron in the direction of motion. This gives you velocity as a function of θ. To compare this with observations, you'd like velocity as a function of time. You can get this using the parametric representation of an ellipse based on the eccentric anomaly, η:
$$t = \frac{P}{2\pi}(\eta - e\sin\eta) \quad ,$$
$$\tan\frac{\theta}{2} = \left(\frac{1+e}{1-e}\right)^{1/2}\tan\frac{\eta}{2} \quad .$$
Consider the case of the binary pulasar, PSR 1913+16, for the discovery of which Taylor and Hulse shared the Nobel Prize. The pulsar (a neutron star) emits pulse of radiation every 59 seconds. The frequency with which these pulses are received is doppler shifted by the star's motion. By analyzing the pulse arrival times, Taylor and collaborators have been able to measure a number of properties of the system with extraordinary accuracy. In particular:
$$P = 27906.980894s,$$

$$e = 0.6171304,$$

$$a_1 \sin i/c = 2.341761s \quad \text{and}$$

$$\sin i = 0.734,$$

where the uncertainties are all in the last decimal place quoted (pulsars may be better clocks than any on Earth). The product of the semimajor axis of the pulsar's orbit, a_1, and $\sin i$ is better known than either individually, hence the odd way of presenting the numbers (drawn from Taylor and Weinberg, 1989, *Astrophysical Journal*, **345**, 434-450). The 8 hour period and masses of the pulsar ($1.44 M_{\odot}$) and its companion ($1.38 M_{\odot}$) strongly suggest that the companion is also a neutron star.

Due to general relativistic effects, the periastron of the orbit precesses 4.2° per year. Hence ϕ changes with time. Plot the pulsar's velocity through one orbit as a function of *time* for each of the following values of ϕ: a) 0°, b) 90°, c) 45°, and d)135°. Feel free to use a computer if it makes this easier.