MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department
Earth, Atmospheric, and Planetary Sciences Department

Astronomy 8.282J-12.402J March 17, 2006

Problem Set 6

Due: Friday, March 24 (in lecture)

Reading: Zeilik & Gregory: Complete Chapter 12, read Chapter 13, and start Chapter 14.

Problem 1

"Magnitudes"

Zeilik & Gregory Chapter 11, problem 11, and **optional** 12, page 233 & 234.

Problem 2

"Planck Distribution"

The following two tables list brightness vs. wavelength for two astrophysical phenomena that can be well approximated by Planck (or blackbody) spectra. The first is a $G-$type star and the other is the cosmic microwave background radiation (left over from the "big bang") that pervades the universe. Carry out the following for both spectra.

a. Plot the spectra (linear brightness vs. linear wavelength for the stellar spectrum; and log brightness vs. log wavelength for the CMB spectrum). We recommend separate plots for the two spectra, each with its own appropriate axes. Draw a smooth curve through the data points.

b. Use the Wien displacement law (equation 8-39 in Z&G) to estimate the temperature T.

c. Superpose on your plot of part (a) the graph of a Planck spectrum, corresponding to temperature T. Plot enough points so that you can draw a smooth *dashed* curve to represent the Planck spectrum. Normalize the overall amplitude of the Planck spectrum so that it gives a reasonable fit to the data (equation 8-37b in Z&G). [To do this more professionally, one would use a computer to carry out a formal "least squares" fit to find the Planck function that best fits the data; the free parameters would be T and an overall scale factor.]

G-Star

λ(Å)	$B(\lambda)$[a]
3000	10.1
3250	23.9
3500	50.6
3750	65.3
4200	111.6
4500	117.0
4700	117.7
5000	114.4
5500	106.1
6000	95.1
7000	68.7
8000	50.2
10000	31.3
11000	25.1
12000	20.7

Cosmic Background

λ(cm)	$B(\lambda)$[b]
50.0	2.7×10^{-7}
21.0	8.0×10^{-6}
12.0	9.1×10^{-5}
8.1	4.0×10^{-4}
6.3	1.1×10^{-3}
3.0	0.020
1.2	0.74
0.91	0.82
0.35	56
0.33	57
0.26	119
0.20	305
0.15	522
0.13	488
0.116	567
0.114	446
0.100	354
0.071	302
0.048	59

[a] *arbitrary units*

[b] *arbitrary units; from Smoot et al. 1988, ApJ, 331, 653*

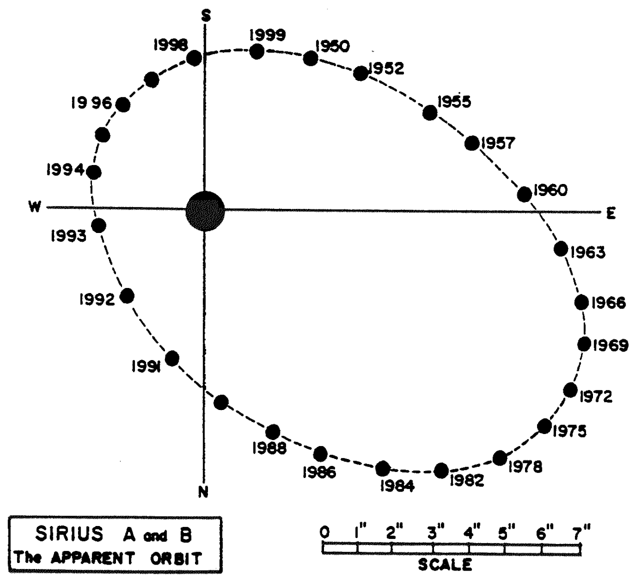
Problem 3

"The Visual and Spectroscopic Binary Sirius A and B"

The bright star Sirius is a binary system consisting of a normal $A-$type star in orbit with a "white dwarf" (star B). It is an especially interesting system in that it is both a visual binary and a spectroscopic binary. Moreover, it is sufficiently nearby that its parallax can be measured with considerable accuracy. A plot of the Sirius orbit is shown below; here the more massive A star is taken to be a fixed reference point, and the distance to the white dwarf represents the actual separation of the two stars at a particular instant in time.

Suggested procedure:
a. Determine the orbital period.
b. The orbital shape appears elliptical; however, even a circular orbit will appear eccentric if the orbital plane is tipped at an angle with respect to our line of sight. Present an argument, based on the plot, that the orbit is actually eccentric.
c. Measure the apparent semimajor axis of the binary in arc sec. (The actual semimajor axis can be obtained by *dividing* by a correction factor of 0.95 to take into account the inclination of the orbit.)
d. The parallax of the center of mass of the binary is $\pi = 0.379''$. Use the implied distance to find the physical size of the semimajor axis.
e. Find the sum of the masses, $M_A + M_B$.
f. If the orbits of both stars had been plotted around the center of mass of the binary, the

orbit of the white dwarf would be 2.4 times larger than that of the A star. What is the mass ratio M_A/M_B? What are the values of M_A and M_B?
g. The apparent bolometric magnitudes of Sirius A and B are -2.1 and +8.3, respectively. Compute the luminosity of each star in terms of the solar luminosity, $L_\odot$ (be sure to take into account the fact that the binary is *not* at a distance of 10 pc). The absolute bolometric magnitude of the Sun is +4.6.
h. Use the Stefan-Boltzmann law and the known surface temperatures of the Sun and Sirius A & B (5,800 K, 11,200 K, and 28,500 K, respectively) to compute the radii of Sirius A & B in units of the Sun's radius, $R_\odot$. [Hint: $L = 4\pi R^2 \sigma T^4$]
i. Compute the density (in gm cm^{-3} or kg m^{-3}) of Sirius A and B. [$R_\odot = 7 \times 10^{10}$ cm and $M_\odot = 2 \times 10^{33}$ gm]



Problem 4

"Short Binary Problems from Zeilik & Gregory"

Zeilik & Gregory, Chapter 12, problems 7 and **optional** 6 Page 249.

Problem 5

"Short Questions on Spectral Types"

Zeilik & Gregory, Chapter 13, problems 1, and **optional** 2, Page 268.

Problem 6

"Eclipsing Binary" **optional**

a. Study the three-page supplementary writeup (attached) entitled "Light Curves of Eclipsing Binaries" taken from a book by Shaw and Boothroyd.

b. Study the actual light curve of the eclipsing binary Algol (shown below) and answer the following questions. Assume that the orbit is circular, the orbital inclination angle is 90°, and the two stars have the same radius.

i. Cite a property of the light curve that is consistent with a circular orbit.
ii. How is the shape of the primary eclipse consistent with the fact that the two stars have the same radius?
iii. Find the radius of either star in units of the orbital separation.
iv. What is the difference in depths between the primary and secondary minima?
v. Use the answer to part (iv) to compute the relative luminosities of the two stars.
vi. Is there any evidence for the reflection effect by the less luminous star?

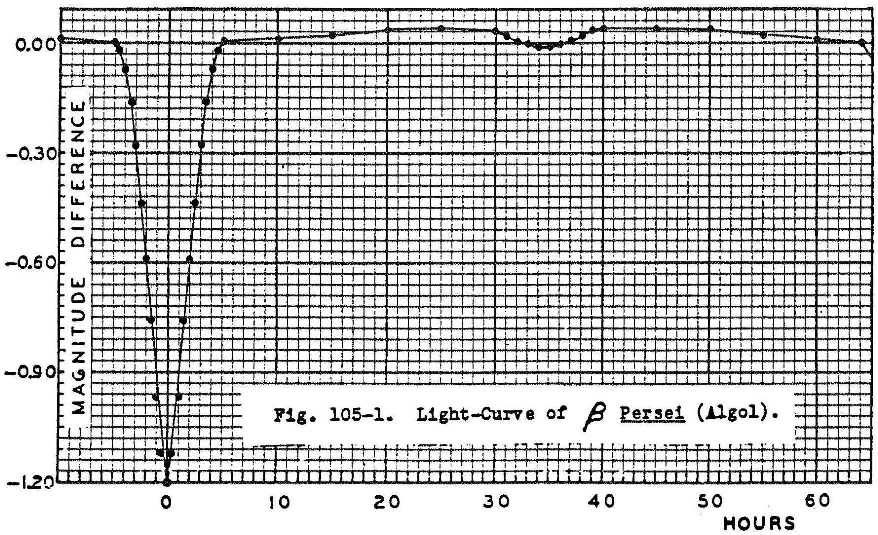


Fig. 105-1. Light-Curve of β Persei (Algol).

Problem 7

"Binary Radio Pulsar" **optional**

The radio pulsar PSR 1913+16 is a neutron star in an eccentric binary orbit with an "unseen" companion star. The orbital period is 7.8 hours. The pulsar comes as close to the center of mass ("periastron" passage) as 3.74×10^{10} cm and recedes to a distance as large as 1.58×10^{11} cm from the center of mass ("apastron"). Measurements of a more subtle general relativistic effect also reveal that the total mass of the binary system is 2.8 solar masses.

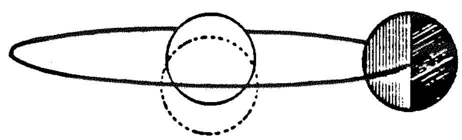
a. What is the semimajor axis of the orbit of the neutron star about the center of mass?
b. What is the eccentricity of the orbit? [Recall that the ratio of closest to furthest approach is $(1 - e)/(1 + e)$]
c. Find the semimajor axis of the binary system as a whole (i.e., total separation).
d. Find the mass of the pulsar and that of its unseen companion.

Light Curves of Eclipsing Variable Stars

That the light emitted by certain stars in not constant with the passage of time has been known for many centuries. Perhaps the most famous instance is that of α Persei or Algol, the Demon star. The light of Algol remains sensibly constant for intervals of nearly sixty hours and then for a period of four and one-half hours undergoes a considerable reduction in intensity, regaining normal brilliance in another four and one-half hours. The reduction in brightness is so marked that the ancients easily observed it with the unaided eye.

Modern research on the so-called variable stars has shown that the characteristics of the light variation may have different explanations in different cases. In the Algol type the variation in light has been shown to be due to the presence of a second star, the companion, which is much less luminous than the primary star. The two stars form a binary system and through chance the line of sight from earth to star lies nearly in the plane of the orbit of the two stars about their common center of gravity. Hence, there occurs periodically an eclipsing of the light of each star and the system is known as eclipsing variable.

Suppose that the following figure represents a pair of eclipsing stars and their relative orbit. Obviously, for an eclipse to occur the plane of the orbit must be nearly in



the observer's line of sight or, in other words, it must be inclined nearly 90° to the plane of the sky at the star. It is also evident from the figure that the characteristics of the light variation will depend on the size and relative luminosities of the two stars as well as on the size, shape and inclination of the orbit. Since the light-variation curves depend on so many factors, general statements regarding them are difficult to make. It is much easier, therefore, to study the results to be expected for a number of typical cases in which certain factors are known or kept constant while others vary.

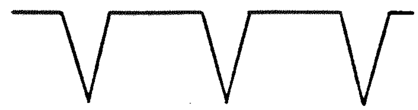
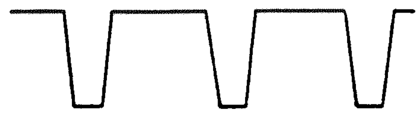
We shall proceed, therefore, by stating the physical characteristics of the system and then deduce the light-cure to be expected. Finally we shall examine several actual light-curves of typical variables and endeavor to deduce the nature of the physical system responsible for them.

Before beginning a detailed study of the light-curves, some general remarks concerning them may be appropriate. A light-curve is a curve showing the variation of the light of a system with time. The magnitude or luminosity of the system is always plotted as the ordinate and time as the abscissa. Sometimes the difference in magnitude which the star undergoes is used rather than the actual magnitude. Again the phase or time from a characteristic point in the light-variation is used rather than the actual date and time of observation. Examples of these methods of plotting will be given later.

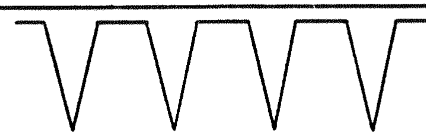
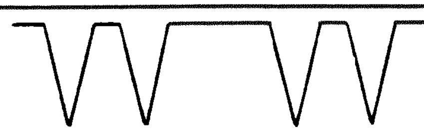
A. Light-Curves of Specified Star Systems

The following postulated circumstances and light-curves should be carefully studied until the student fully appreciates that the given light-curve would result. Variations on the postulated system are considered in the form of questions. The instructor may make additional queries.

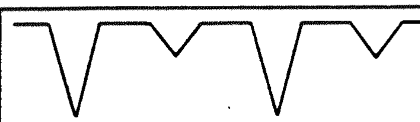
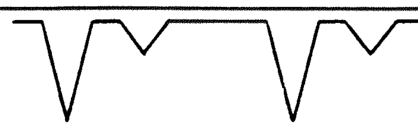
Case I. Two spherical stars, one uniformly luminous and the other completely dark, are postulated.

(a) Partial Eclipse. Orbit circular or elliptical with the major axis coinciding with the line of sight.	
(b) Annular Eclipse. Orbit circular or elliptical with the major axis coinciding with the line of sight.	

Case II. Two spherical stars of equal size, both uniformly luminous, are postulated.

(a) Partial Eclipse. Orbit circular or elliptical with the major axis coinciding with the line of sight.	
(b) Partial Eclipse. Orbit is elliptical and the major axis is not in the line of sight.	

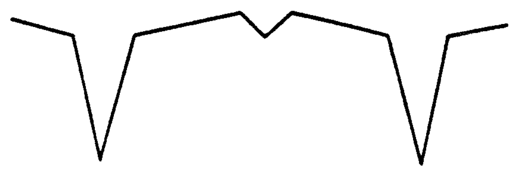
Case III. Two spherical stars, uniformly luminous but of unequal luminosity, are postulated.

(a) Partial Eclipse. Orbit circular or elliptical with the major axis in the line of sight.	
(b) Partial Eclipse. Orbit elliptical with the major axis making an angle with the line of sight.	

B. Interaction Effects

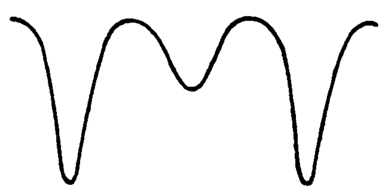
In many cases the light-curves of eclipsing variables differ markedly from the theoretical curves shown above. These differences are due to various interactions of one star upon the other.

Reflection Effect. The reflection effect arises from the heating of the fainter star by radiation from the brighter star so that the side toward the latter is hotter than the opposite side. As a result the light-curve is higher just before and after the secondary minimum than before and after the primary minimum. The light-curve would appear as follows rather than as in Cases III(a) or (b).



Limb Darkening. Very often the stellar disks are not uniformly luminous but show a limb darkening similar to the sun. As a result of limb darkening the changes of light at the beginning and end of an eclipse are slower. Also the curve for an annular eclipse is not flat-bottomed since more light is cut off when the companion obscures the bright center thanwhen it obscures a region near the limb.

Ellipticity of the components. If the components of an eclipsing system are very close together, the tidal forces may be very great and the stars may be drawn into elongated forms. At the time of eclipse they appear endwise to us and halfway between eclipses they present the broader sides toward us. Due to the increased areas at the latter time the luminosity will be greater. The ellipticity effect causes the light-curve to be bowed up between minima with the maximum of light intensity halfway between them. The following light-curve shows the effects of both limb darkening and ellipticity of the components.



Light-Curve of Beta Lyrae