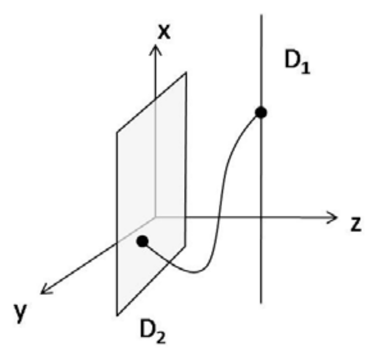
# 8.251 Final Exam Review

## Practice Problems

Problem 1.
Assume standard, space-filling Virasoro brane.
a. Calculate $L^{\perp}_{-6}|0\rangle$ in terms of normal order oscillations.
b. Calculate $L^{\perp}_{-6}\alpha^{I}_{-1}|0\rangle$.

(Hint: $\alpha^I_0|0\rangle = 0, \oslash$ mom. vacuum)

Problem 2.




D2: $z = 0$, D1: $y = 0, z = \pi\sqrt{\alpha'}$, D = 26.
a. Find the $M^2$ formula in terms of $N^{\perp}$
b. Ground states? Describe the lowest mass levels.
c. Generating $f(x) = \sum a(r)x^r$ where $a(r)$ is the number of states of $\alpha' M^2 = r$.

Problem 3.

Consider the Ramond sector of a superstring in $D = 18$.

Naive $\alpha' M^2 = \frac{1}{2}\sum_{n\neq 0}\alpha^I_{-n}\alpha^I_n + nd^I_{-n}d^I_n$.

a. Range of $I$? Write the precise (n.o) $\propto M^2$

b. Write the generating function $f_R(x)$.

**Solutions**

Problem 1.

a.

$$L^{\perp}_{-6} = \frac{1}{2}\sum_{p\in Z}\alpha_{-6-p}\alpha_p$$

$p = 0$:

$$\frac{1}{2}(\alpha_{-6}\cdot\alpha_0+\alpha_{-7}\cdot\alpha_1+\alpha_{-8}\cdot\alpha_2+\ldots+\alpha_{-5}\cdot\alpha_{-1}+\alpha_{-4}\alpha_{-2}+\alpha_{-3}\alpha_3+\alpha_{-2}\alpha_{-4}+\alpha_{-1}\alpha_{-5}+\alpha_0\alpha_{-6}+\alpha_1\alpha_{-7})$$

Since $n > 0$, $\alpha$'s are annihilation operators and kill the vacuum state.

$$= \frac{1}{2}\alpha_{-3}\alpha_{-3} + \alpha_{-2}\alpha_{-4} + \alpha_{-1}\alpha_{-5} + \alpha_0\alpha_{-6} + (\alpha_{-7}\alpha_1 + \alpha_{-8}\alpha_2 + \ldots)$$

$$L^{\perp}_{-6}|0\rangle = (\frac{1}{2}\alpha_{-3}\alpha_{-3} + \alpha_{-2}\alpha_{-4} + \alpha_{-1}\alpha_{-5})|0\rangle$$

b.

$$L^{\perp}_{-6}\alpha^I_{-1}|0\rangle = (\frac{1}{2}\alpha_{-3}\alpha_{-3}+\alpha_{-2}\alpha_{-4}+\alpha_{-1}\alpha_{-5})\alpha^I_{-1}|0\rangle+\alpha_{-7}\alpha_1\alpha^I_{-1}|0\rangle = L^{\perp}_{-6}|0\rangle+\alpha^I_{-7}|0\rangle$$

Alternative method:

$$L^I_{-6}\alpha^I_{-1}|0\rangle = \underbrace{[L_{-6},\alpha^J_1]}_{\alpha^I_{-7}}|0\rangle + \underbrace{\alpha^I_{-1}L^{\perp}_{-6}|0\rangle}_{\text{know}}$$

Problem 2.

a.

| | $x^+$,$x^-$ | $y$ | $z$ | x$x^4x^{25}$ | ... |
|---|---|---|---|---|---|
| D2 | - | - stretched | · | · localized | |
| D1 | - | · | · | · localized | |
| [D2, D1] | NN | ND | DD | DD | DD |

1 ND coordinate; all of the rest are DD

$$\alpha' M^2 = N_a^\perp + \alpha'(TL)^2$$

$$a_{ND} = \frac{1}{48} = -\frac{1}{24} + \frac{1}{16}$$

$$-1 + \frac{1}{16} = -\frac{15}{16} = a$$

$$a = -1 + \frac{1}{16}(p - q)$$

$$\alpha(TL)^2 = \alpha' \left( \frac{\pi\sqrt{\alpha'}}{2\pi\alpha'} \right)^2$$

$$\alpha' M^2 = N^\perp - \frac{15}{16} + \frac{4}{16} = N^\perp - \frac{11}{16}$$

b.

Ground state: $\left|p^+, p^2, \ldots, p^q\right\rangle$ in book, but here $q = 1 \Rightarrow$ GS $= |p^+\rangle$. No momentum in ND (fractional oscillators) or all the DDs. The states live on the D1 brane.

c.

GS has $\alpha' M^2 = -\frac{11}{16}$. Suppose GS $|0\rangle$ and just oscillator $\alpha_{-1}^{(3)}$, then could build $\alpha_{-1}^{(3)} |0\rangle$, $(\alpha_{-1}^{(3)})^2 |0\rangle$, ..., so:

$$f(x) = x^{\frac{-11}{16}} (1 + x + x^2 + \ldots) = x^{-\frac{11}{16}} \left( \frac{1}{1 - x} \right)$$

But we have more!

$$
\begin{aligned}
f(x) &= x^{-\frac{11}{16}} \left(\frac{1}{1-x}\right)^{23} \left(\frac{1}{1-x^2}\right)^{23} \left(\frac{1}{1-x^3}\right)^{23} \cdots \\
&= x^{-\frac{11}{16}} \prod_{n=1}^{\infty} \frac{1}{(1-x^n)^{23}}
\end{aligned}
$$

Now have to account for the fractional oscillators.

$$
f(x) = x^{-\frac{11}{16}} \prod_{n=1}^{\infty} \frac{1}{(1-x^n)^{23}} \frac{1}{(1-x^{n-1/2})}
$$

Plug into Mathematica to get specific answers.