Lecture 24 - Topics

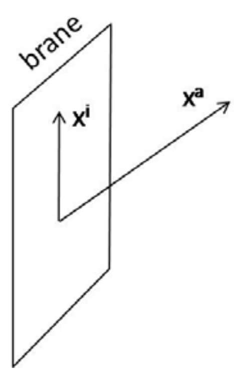
- Dp-brane
- Parallel Dp's

$$\dot{X}^- \pm X^{-\prime} = \frac{1}{2\alpha'}\frac{1}{2p^+}(\dot{X}^I \pm X^{I\prime})^2 \quad (1)$$

$$\dot{X}^I \pm X^{I\prime} = \sqrt{2\alpha'}\sum_{n\in Z}\alpha_n^I exp(-in(\tau \pm \sigma)) \quad (2)$$

$$2p^+p^- = \frac{1}{\alpha'}\Bigg(\frac{1}{2}\alpha_0^I\alpha_0^I + \sum_{n=1}^{\infty}\alpha_{-n}\alpha_n^I + a) \quad (3)$$

Dp-brane



$x^0, x^1, \ldots, x^p$: Coordinates on brane
$x^+, x^-, x^i, i = 1, 2, 3, \ldots, p \Rightarrow (p+1) - 2$ values. NN coordinates.

$x^{p+1}, \ldots, x^d$: Normal to brane
$x^a, a = p+1, \ldots, d \Rightarrow (d-p)$ values. DD coordinates.

Thus:

Equation (1) becomes:

$$\frac{1}{2\alpha'}\frac{1}{2p^+}[(\dot{x}^i \pm x^{i\prime})^2 + (\dot{x}^a \pm x^{a\prime})^2]$$

Equation (2) holds. Equation (3):

$$2p^+p^- = \frac{1}{\alpha'}\left(\frac{1}{2}\alpha_0^I\alpha_0^I + \sum_{n=1}^{\infty}\alpha_{-n}\alpha_n^I + a\right)$$

$X^a(\tau,\sigma)$:

$$X^a(\tau,0) = X^a(\tau,\pi) = \overline{X}^a, \qquad \text{a scalar}$$

$$X^a(\tau,\sigma) = \overline{X}^a + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^a e^{-in\sigma}\sin n\sigma$$

$$X^{a\prime} \pm \dot{X}^a = \sqrt{2\alpha'}\sum_{n\neq 0}\alpha_n^a e^{-in(\tau\pm\sigma)}$$

$$[\alpha_m^a, \alpha_n^b] = m\delta_{m+n,0}\delta^{ab}$$

$$2p^+p^- = \frac{1}{\alpha'}(\alpha' p^i p^i + \sum_{n=1}^{\infty}(\alpha_{-n}^i\alpha_{+n}^i + \alpha_{-n}^a\alpha_n^a) - 1)$$

$$M^2 = \frac{1}{\alpha'}(-1 + N^\perp)$$

$N^\perp = N_{\text{longitudinal}} + N_{\text{transverse}}$
$N_l = \sum_{n=1}^{\infty}\alpha_{-n}^i\alpha_n^i$
$N_t = \sum_{n=1}^{\infty}\alpha_{-n}^a\alpha_n^a$

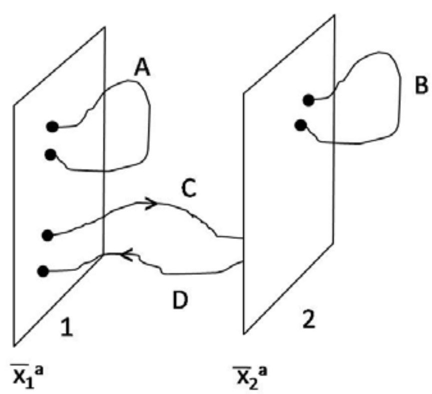
Ground state: $M^2 = -\frac{1}{\alpha'}$, State $\left|p^+, p^i\right\rangle >$ tachyons, gives rise to $\psi(\tau, p^+, p^i)$. $p^a = 0$.

Where do these fields live? Reasonable to say, on the brane. Right number of coordinates $p-1$ dim. space. But where did the $\overline{x}^a$ go?

Next state: $\alpha^i_{-1}\left|p^+, p^i\right\rangle$, $M^2 = 0$.
$(p+1)-2$ of these states. What are they? Photon states!

Also, $\alpha^a_{-1}\left|p^+, p^i\right\rangle$ lives on brane but has x^a index. Nothing to do with spacetime. Inert scalar states. $d-p$ massless scalars. Physical interpretation: Represent possible excitations casting 0 energy and 0 momentum, displacing the brane. (See QFT theory of Goldstone).

Parallel D_p's



Know everything about strings A and B.
New problem: strings C and D. (note $C \neq D$ since orientation matters)
$A = [1,1]$, $B = [2,2]$, $C = [1,2]$, $D = [2,1]$.

In general, $[i,j]$ with $\sigma = 0 \in D_i$, $\sigma = \pi \in D_j$. (Before, always talking about same D-branes, so there was an implicity $[1,1]$ always.)

$$X^a(\tau,\sigma) = \overline{X}^a_i + \frac{1}{\pi}(\overline{X}^a_2 - \overline{X}^a_1)\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha^a_n e^{-in\sigma}\sin n\sigma$$

$$X^{a\prime} = \sqrt{2\alpha'}\sum_{n\neq 0}\alpha^a_n e^{-in\tau}\cos(n\sigma)$$

$$\boxed{\sqrt{2\alpha'}\alpha_0^a = \frac{1}{\pi}(\bar{X}_2^a - \bar{X}_1^a)}$$

Alternatively, write as:

$$\frac{\alpha_0^a}{\sqrt{2\alpha'}} = \frac{1}{2\pi\alpha_0'}(\overline{X}_2^a - \overline{X}_1^a)$$

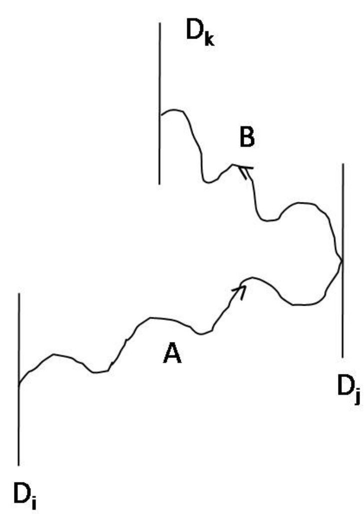
$$2p^+p^- = \frac{1}{\alpha'}(\alpha^i p^i p^i + \frac{1}{2}\alpha_0^a\alpha_0^a + N^\perp - 1)$$
$$M^2 = \frac{1}{\alpha'}(N^\perp - 1) + \frac{1}{2\alpha'}\alpha_0^a\alpha_0^a$$

$\frac{1}{\alpha'}(N^\perp - 1)$: Contribution of Quantum Oscillation
$\frac{1}{2\alpha'}\alpha_0^a\alpha_0^a = (T_0(X_2^a - X_1^a))^2$: Contribution of Tension×Length

Now M^2 quantized for any sector, but can choose sectors.
Ground states: $\left|p^+, p^i[1,2]\right\rangle$, $\left|p^+, p^i[1,1]\right\rangle$ (we know how to handle this one)

State $\alpha_{-1}^i \left|p^+, p^i[1,2]\right\rangle$ D-branes separate, so now not massless ⇒ photons
State $\alpha_{-1}^a \left|p^+, p^i[1,2]\right\rangle$ same mass. Always a scalar state.

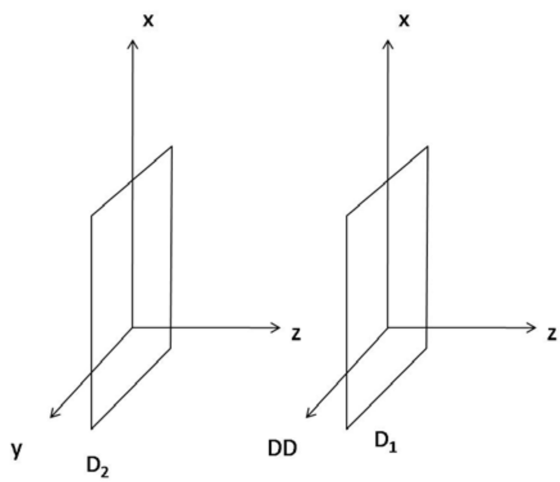
Suppose have 3 branes:



String $A = [i, j]$
String $B = [j, k]$

Can interact to form one string between D_i and D_k: $[i, j] \times [j, k] = [i, k]$. (Then D_j doesn't notice the string anymore). Theory of interacting gauge fields.

D_p brane parallel to D_q



Coordinates have split into common D, common N and split ND.

Branes / Coordinates	x	y	z
D2	–	–	•
D1	–	•	•
Sector [D2,D1]	NN	ND	DD