MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics 8.224 Exploring Black Holes
General Relativity and Astrophysics
Spring 2003

ASSIGNMENT: WEEK 3

INTRODUCTION

Every electronics store carries hand-held receivers for the global positioning system (GPS). For a couple hundred dollars or less you can locate yourself within ten meters or so anywhere on Earth. These devices depend crucially on the accuracy of atomic clocks, since distance is measured by the travel time of radio waves. Light and radio waves move approximately one foot per nanosecond. In Project A, The Global Positioning System, you will use general relativity to show that orbiting clocks drift tens of thousands of nanoseconds per day compared with clocks on the surface of Earth. If this relative drift is not corrected, the GPS is useless. When the first GPS satellite was put into orbit, it had a "general relativity on-off switch" set to the "off" position, because the US military was not sure that academic advisors were right about general relativity. The orbiting clocks drifted away from the Earth-clock readings at the rate predicted by the academics. So finally the military threw the general relativity switch to the "on" position. The rest is history. One example: By employing various tricks, geologists use the GPS to measure drift of the Earth's continents in millimeters per year.

READING

EBH Chapter 2: Curving, page 2-25 to the end
AND
Project A. The Global Positioning System
AND
Thorne Chapter 2: The Warping of Space and Time
AND
Notes: "Gravity, Metrics, and Coordinates"

EVENING SEMINAR

Beginning of week #3

RECITATION

At least one of the following questions will appear on the section quiz this week:

1. The clock in the hand-held GPS receiver is much less accurate than the GPS atomic clocks in orbit. How can the GPS system operate with such poor receiver clock accuracy?

2. Observers on Earth's surface detect timing pulses from orbiting clocks to be increased in rate compared to that measured at the satellite.Is this increased rate due to orbiting clocks running fast or to the or blue shift of radio signals from the orbiting clocks as these signals move downward? Explain your answer.

3. Ions in the ionosphere create small random added delays in signal transmission between the orbiting satellite and GPS receivers on Earth. In dense fog an airline pilot wants to measure his location with respect to an airfield. What technical advice can you give the airline and airport authorities that will greatly reduce the error due to transmission delays in the ionosphere?

4. State Einstein's principle of relativity in a form satisfied by his general relativity.

5. In teaching relativity we still use arguments presented by Einstein prior to 1912, but we use few of his post-1912 arguments in our classrooms. What is the reason for this difference, according to Thorne?

PROJECTS

Beginning of week #4 is the deadline for requesting approval from instructor for a project topic not on the original list. Approval is tentative; an approved topic must later attract enough class members to form a team.

PROBLEM SET

Solve the following exercises in EBH PLUS additional exercises given below. Turn in byend of week #3.

Problem 1. Astronaut Stretching According to Newton. Exercise 6, starting on page 2-46 in EBH. This problem provides practice in using geometric units.

Problem 2. Area of Black Hole Horizon (Almost!) Never Decreases, Exercise 8, page 2-48 of EBH. MODIFY parts A through C: One black hole has a mass M_0 and the other a mass nM_0, where n is an arbitrary integer. Set $n = 2$ in the following ADDED Parts D and E..

D. Assume that the mass lost in the analysis of part B is escapes as gravitational radiation. What is the mass-equivalent of the energy of that gravitational radiation?

E. The Caltech-MIT Laser Interferometer Gravitational-Wave Observatory, LIGO, has begun its search for gravitational wave sources. LIGO consists of two huge detectors near Hanford, Washington and Livingston, Louisiana. At present, LIGO has its greatest sensitivity for gravitational waves oscillating with a frequency of about $f = 100$ Hz. LIGO directly measures a dimensionless quantity h known as the strain. Currently LIGO could detect gravitational wave sources with $h > 10^{-21}$. The energy flux in watts per square meter carried by a gravitational wave is $S = [c^3/(16\pi G)](dh/dt)^2$. What is the distance in light years to the coalescing system of this exercise such that LIGO can barely detect the process? Assume (a) The value of M in the statement of the exercise is $M = 10$ times the mass of our Sun (b) the frequency of emitted gravitational radiation is equal to the frequency of greatest sensitivity of LIGO, (c) the coalescence process takes one second, (d) the gravitational radiation is emitted isotropically (i.e. the same in all directions), and (e) the coalescing system is at rest with respect to Earth.

Problem 3. GPS Synchronization Discrepancy after One Day due to Difference in Altitude. QUERY 4, page A-4 of EBH.

Problem 4. GPS Numerical Clock Rate Difference. QUERY 9, page A-6.

Problem 5. Metrics in Transformed Coordinates.
Derive the metrics for the three new coordinate systems of Fig. 1 in the handout "Coordinates and Proper Time", starting from the metric $d\tau^2 = dt^2 - dx^2$.

Problem 6. Distance between Two Points on a Sphere. (See the handout "Coordinates and Proper Time".) Two points on a sphere of radius 1 have coordinates (θ_1, φ_1) and (θ_2, φ_2).

(a) What is the distance between them, measured along a great circle arc on the surface of the sphere? Do *not* assume that the two points are close to each other.

(b) Now suppose that the two points are very close: $\theta_2 = \theta_1 + d\theta$, $\varphi_2 = \varphi_1 + d\varphi$. Show that the distance between the two points reproduces the metric for a sphere given in the notes.

Problem 7. Static Spherical Metric.
Starting from equation (9) of the *second* handout "Gravity, Metrics, and Coordinates", put primes on the variables: $(r', t', \Phi', \Lambda')$. Then find expressions for the coordinate transformations $t = t(t', r')$ and $r = r(r')$ to a new set of coordinates giving the more general form, equation (8). For a given spacetime, is equation (9) unique? That is, are there any coordinate transformations that leave the metric in the same form as equation (9)?

Problem 8. Schwarzschild Metric.
This problem is based on equations (15) of the notes "Gravity, Metrics, and Coordinates".

(a) Assuming that the pressure vanishes everywhere, solve equations (15) exactly. Your answer for $e^{-2\Lambda(r)}$ may depend on the enclosed mass $M(r)$ defined in equation (14) of the notes. Your answer for $\Phi(r)$ should be written as a definite integral, assuming boundary condition $\Phi(r) \longrightarrow 0$ as $r \longrightarrow$ infinity.

(b) Show that if the mass is all concentrated at $r = 0$ you obtain the Schwarzschild metric.