

Problem Set 2

1. A Lorentz transformation

The origins of two inertial frames, Σ and Σ' , coincide at $t = t' = 0$. The origin of Σ' moves at speed $u = 1.8 \times 10^8 \text{m/sec}$ along the positive z axis in Σ . An event occurs at $x' = 1\text{m}$, $y' = 3\text{m}$, $z' = 2\text{m}$ and $t' = 8 \text{sec}$ as observed in Σ' . Where and when does this event occur in Σ ? [To simplify the algebra, you can take $c = 3 \times 10^8 \text{m/sec}$.]

2. Lorentz transformations are rotations through imaginary angles

French §3, p. 87, Problem 3-9.

3. Spacetime diagrams I

French §3, p. 87, Problem 3-8.

4. A Moving Clock

A clock moves along the x -axis in your reference frame at a speed of $0.80c$ and reads zero as it passes the origin. What time does it read as it passes the 180 meter mark on this axis? How much time has passed in your reference frame?

5. Spacetime diagrams II

A space traveler sets off to explore a star 96 light years from Earth. She accelerates rapidly to $v = 0.96c$, and travels in a straight line. After 100 years (as measured on Earth) she quickly decelerates to $v = 0$ and explores the star for 20 years. Then she returns to earth by quickly reaccelerating to $v = 0.96c$, travelling for 100 years and decelerating as she approaches the Earth. Draw a spacetime diagram showing the space traveler's worldline from the point of view of an observer on the Earth. Measure time in years and distance in light years.

6. Moon contraction

By what amount is the Moon shortened along its diameter (as measured by an observer at rest relative to the Earth) owing to its orbital motion around the Earth? [You can take the orbital velocity of the Moon to be 0.75 km/sec and its radius to be 1722 km .]

7. How fast a ship?

A spaceship is moving at such a speed in the laboratory frame that its measured length is one third its proper length. How fast is the spaceship moving relative to the laboratory frame?

8. Cosmic neutrons?

Cosmic rays are particles from interstellar space that impact on our atmosphere, making extensive showers of secondary particles that can be measured in detectors which observe the ionization they create as they descend through our atmosphere. Most cosmic rays are protons or photons. The showers made by protons and photons

can be distinguished: the ones made by photons contain many more electrons, for example.

A few years ago a puzzling phenomenon was observed: cosmic rays of the “proton type” appeared to be coming from a bizarre object known as Cygnus-X3. The reason this was puzzling was that protons travel on helical paths on their way to Earth in response to interstellar magnetic fields. They would not appear to “point back to” their source.

Scientists looked around for other particles that satisfied three conditions:

- They had to be “strongly interacting”, so they could make “proton type” showers.
- They had to be electrically neutral, so they would not be bent by interstellar magnetic fields.
- They would have to live long enough to make it from their source to Earth.

One obvious candidate is the neutron. It has an average lifetime of 885.7 ± 0.8 sec.

- (a) Assuming the distance to Cygnus-X3 is 10,000 lyr, what is the minimum speed that a neutron must be travelling so that it could make it to Earth in its own proper lifetime?
- (b) Assume that the distance to Cygnus-X3 was not well known at the time. The experimenters could estimate the “ γ -factor” ($\gamma = 1/\sqrt{1 - v^2/c^2}$) of the cosmic rays. They found that $\gamma \lesssim 10^{10}$. Approximately how far away could Cygnus-X3 be if neutrons are the source of the puzzling cosmic rays?

9. Cosmic something else?

This problem refers to the same phenomenon as the previous problem...

Physicists quickly abandoned the idea that neutrons could be the source. They started hypothesizing new, hitherto unknown particles that could be the source of the puzzling events.

- (a) Taking the distance to Cygnus-X3 to be 10,000 lyr, what would be a reasonable estimate of the proper lifetime of a hypothetical new particle (call it an “ H ”, for hypothetical) that would give rise to cosmic ray events with $\gamma \approx 10^{10}$?
- (b) Use Lorentz contraction to describe the journey from the point of view of the H . In particular, how, in its own rest frame, can it cover the distance to Earth in its (short) lifetime?

Epilogue: Follow-up experiments were unable to reproduce the original phenomenon, *ie.* the events went away!

10. String of lights across the desert I

A series of lights is arrayed in a straight line across the desert. Neighboring lights are separated by a distance d . They are set up to flash in sequence with an interval τ between neighboring lights (as measured in the rest frame of the lights). An observer,

O , travels along the same line at a uniform speed v in the same direction of the wave of flashes.

- At what interval do the flashes occur in the rest frame of O ?
- Suppose O travels in the direction opposite the wave. What is the interval in this case?
- For what choices of d , τ , and v do all flashes occur simultaneously in the rest frame of O ?

11. String of lights across the desert II

In the previous problem you were asked for the times at which the flashes occurred in the rest frame of O . Now consider what would be *seen* by the observer O :

Again let the straight line of lights be separated by d and flash in sequence with interval τ . Now compute the interval between the sequential flashes *as seen by* an observer travelling with uniform speed v along the direction of the flashes. [Hint: in this case you must consider not only the Lorentz transform of each flash event, but you must also consider how long it takes a flash to propagate to the observer O .] For what values of d , τ , and v will the flashes *appear to be* simultaneous to O ?

12. Events in two different frames I

Two events, A and B , are observed in two different inertial frames, Σ , and Σ' . Frame Σ' moves along the x axis in Σ . Event A occurs at the spacetime origin in both frames,

$$\begin{aligned}x_A = y_A = z_A = ct_A = 0 \\x'_A = y'_A = z'_A = ct'_A = 0\end{aligned}$$

Event B occurs at

$$x_B = 10, \quad y_B = z_B = 0, \quad ct_B = 6$$

(all distances are in meters).

The two events occur *simultaneously* in frame Σ' .

- Find the velocity of Σ' with respect to Σ .
- What is the space separation of the two events in the frame Σ' ?
- What is the smallest spatial separation between the two events in any inertial frame?

13. Events in two different frames II

Two events, A and B , are observed in two different inertial frame, Σ , and Σ' . Event A occurs at the spacetime origin in both frames,

$$\begin{aligned}x_A = y_A = z_A = ct_A = 0 \\x'_A = y'_A = z'_A = ct'_A = 0\end{aligned}$$

Event B occurs at

$$x_B = 2, \quad y_B = z_B = 0, \quad ct_B = 10$$

(all distances are in meters) as observed in Σ .

The two events occur *at the same point* in frame Σ' .

- Find the velocity of Σ' with respect to Σ .
- What is the time separation of the two events in the frame Σ' ?
- What is the shortest time separation between the two events in any inertial frame?

14. Events in two different frames III

Two events, A and B , are observed in two different inertial frame, Σ , and Σ' . Event A occurs at the spacetime origin in both frames,

$$\begin{aligned} x_A = y_A = z_A = ct_A = 0 \\ x'_A = y'_A = z'_A = ct'_A = 0 \end{aligned}$$

Event B occurs at

$$x_B = 2, \quad y_B = z_B = 0, \quad ct_B = 3$$

in Σ , and at

$$x'_B = 3, \quad y'_B = z'_B = 0$$

(all distances are in meters) in Σ' .

- What time does event B occur in Σ' ?
- What is the relative velocity of Σ' relative to Σ ?

15. The travels of elementary particles reconsidered

Return to Problem 2 on Problem Set 1 and recalculate the speed of the particles given their lifetimes and pathlengths. The problem is reproduced below for your convenience:

Elementary particles have very short average lifetimes (at least measured on our perceptual scale). In experiments at accelerators the particles are produced and then detected at points away from the point of production. If the rules of Newtonian physics were correct for particles travelling at great speeds, the particles would have to travel much faster than the speed of light in order to go so far before they decay.

Assuming that the particle in question lives for the average lifetime of that species, compute its average speed using Newtonian mechanics:

- A “ π -meson” has a lifetime of 2.6×10^{-8} sec. It is observed 1 kilometers away from its point of production.
- A “Lambda” hyperon has a lifetime of 2.6×10^{-10} sec. It is observed 30 meters away from its point of production.
- A “ Λ_B baryon” has lifetime 1.23×10^{-12} sec. It is observed 3 millimeters away from its point of production.

16. Invariance of the interval

- (a) Show that the square of the spacetime interval (Δs^2) associated with two events (assumed to occur in the $x - x'$ axis) is invariant under a Lorentz transformation. That is, show that

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2$$

- (b) For each of the following pairs of events, say whether
- i. there is a frame where they are simultaneous, or
 - ii. there is a frame where they occur at the same place, or
 - iii. neither

$$\begin{array}{ll} \text{I} & x_A = 2, y_A = 1, z_A = 0, \quad ct_A = 2 \\ & x_B = 1, y_B = 2, z_B = 0, \quad ct_B = 1 \end{array}$$

$$\begin{array}{ll} \text{II} & x_A = 6, y_A = 7, z_A = 2, \quad ct_A = 8 \\ & x_B = 3, y_B = 3, z_B = 2, \quad ct_B = 3 \end{array}$$

$$\begin{array}{ll} \text{III} & x_A = 2, y_A = 4, z_A = 1, \quad ct_A = 2 \\ & x_B = 3, y_B = 3, z_B = 2, \quad ct_B = 4 \end{array}$$

17. The expanding universe

Adapted from Resnick and Halliday, Basic Concepts in Relativity (MacMillan, New York, 1992).

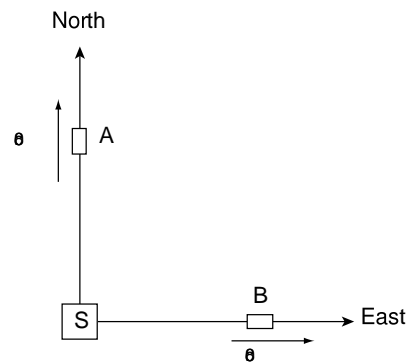
- (a) Galaxy A is reported to be receding from us with a speed of $0.4c$. Galaxy B , located in precisely the opposite direction, is also found to be receding from us at this same speed. What recessional speed would an observer on Galaxy A find
- i) for our galaxy?
 - ii) for Galaxy B ?
- (b) It is concluded from measurements of the red shift of the emitted light that quasar Q_1 is moving away from us at a speed of $0.75c$. Quasar Q_2 , which lies in the same direction in space, but is closer to us, is moving away from us at speed $0.50c$. What velocity for Q_2 would be measured by an observer on Q_1 ?

18. Perpendicular velocities

Adapted from Resnick and Halliday, *Basic Concepts in Relativity* (MacMillan, New York, 1992).

A and B are trains on perpendicular tracks (see the figure), shown radiating from stations S . Both trains travel at a speed $v = 0.8c$ in the rest frame of the station.

- Find \vec{V}_{AB} , the velocity of train B with respect to train A .
- Find \vec{V}_{BA} , the velocity of train A with respect to train B .
- Comment on the fact that these two relative velocities do not point in opposite directions.

**19. Transforming angles I**

A particle moves with speed u in the $x - y$ plane, making an angle θ with respect to the x -axis in frame Σ . The origin of Σ moves to the right (along the positive x' axis) in the frame Σ' with speed v . What speed u' and angle θ' will the particle appear to have to an observer in Σ' ?

20. Transforming angles II

A right triangular plate is at rest in the frame Σ . Its legs are placed on the x and y axes and its hypotenuse makes an angle θ with respect to the x -axis. The origin of Σ moves to the right (along the positive x' axis) in the frame Σ' with speed v . What are the angles of the triangle as measured in Σ' ?