

## MITOCW | L1.3 Calculating the energy corrections

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**PROFESSOR:** The left hand side here is 0 because  $H_0$  and  $N_0$  by taking the eigenstate equation is equal to  $E_{n0}$  times  $N_0$ . So this really evaluates to  $E_{n0}$  and therefore the two cancel, so this thing is 0.

The right hand side, this is a number,  $E_{n1}$ . It's a number. It's in an expectation value.  $N_0$  has a unit expectation value, so this is  $E_{n1}$ . That's the number.

And then I have an operator here. So this is minus  $N_0 \Delta H N_0$ . So we get a very famous and important equation that  $E_{n1}$  is equal to  $N_0 \Delta H N_0$ .

The first correction to the energy is obtained by finding the expectation value of the perturbation in the unperturbed state. So you should be happy to hear that. It says that to find the first correction in the energy, you don't need to know what happens to the state.

I didn't need to know what  $N_1$  was, how the state changes, to know how the energy changes. The energy changed before, and I just don't need it. I just need the original state and the perturbation. So it's a very nice result, and it has a simple generalization.

The simple generalization is that I'm going to use another blackboard, but let's look at it. The simple generalization comes from doing the same exact thing with the order  $k$  equation. So let's do that.

I put an  $N_0$  to the left of this equation. Now what do we get? From this term, we get 0 for the same reason. There's an  $N_0$  now here, and the  $H_0$  is killed by this term, gives you 0.

And then we're going to continue with this and see what happens with this various terms. Let's look at it. So I'll put the  $N_0$ -- 0 on the left hand side was equal to 0. And on the right hand side, what do we get?

Well, let me do a couple of terms.  $N_0 E_{n1}$  minus  $\Delta H$  and  $k$  minus 1 plus all these other terms.

So look what happens here. Let's do the next term, for example.  $E_{n2}$ ,  $N_0$ , and  $k$  minus 2. Well, from here, this is a number, so I have here this goes out the overlap of  $N_0$  with  $n$  minus 1. But we said that all the higher corrections have no component along  $N_0$ . So this thing will give you 0.

On the other hand, here is an operator, so there's nothing I can say. So I'll write it minus  $N_0$   $\Delta H$  and  $k - 1$ .

And then what else? Well, you have  $N_0$  then  $k - 2$  here. That's 0 because this is a higher state and all the terms give you 0 until you get here where the  $N_0$  with the  $N_0$  give you 1. So the only term that survives is the last one, and we get  $\epsilon_{nk}$ .

So this gives you the result that  $\epsilon_{nk}$  is equal to  $N_0 \Delta H$  and  $k - 1$ .

I box it because it's another nice formula. It tells you that the  $k$ th order energy is given if you know the  $k - 1$  state. If you have figured out the  $k - 1$  correction to the state, then you know the energy of the  $k$ th correction.

This formula certainly works when  $k$  is equal to 1 in which case it reproduces the formula we had on the blackboard to the right. When  $k$  is equal to 1, you get the expectation value of  $\Delta H$  around zero.