PROFESSOR: All right. So let's get now the state n1. So I want to make a general remark. You have an equation like this. And you want to solve it. It's a vector equation. Operator and a vector equal number and a vector-- a more operator and a vector.

To make sure you have solved it, when you have a vector equation you must make sure that every component-- you can write a vector equation in the form vector equals zero. And then you must make sure that every component of the vector is zero.

What we did here is we found what happens when I look at the component along n0. And I figure out that, whoops, this equation, when I look at the component along n0, tells me what the energy is.

So the rest of the information of this equation arises when I look at it along the components on the other states, not n0 but the k states that we introduced from the beginning, the k0's that run from one to infinity. So what we're going to do is take that original-- this second equation and form k0 h0 minus Em0 m1 1 is equal to k0 Em1 minus delta H n0.

So I now took the same equation and I put it in a problem with k0. And I say, look, k will be different from n, because when we put k equal to n that already we've done. And we've learned all about it. And, in fact, n1 didn't appear. The state that we wanted didn't appear at all. So now we do this with arbitrary k. And we need to figure out what this gives.

So you have to look at these things and try to remember a little of the definitions with both. So h0, we know what it gives from k0. It gives you a number, the energy of that state. So this is another number. So that's great. This simplifies this to ek0 minus En0 times the overlap of k0 with n1. That's the left hand side.

How about the right hand side? All right. Let's see what this is. First term. The En1 is a number, so I must ask myself is what happens when k0 meets n0? Well, those are our original orthonormal states. And we said that k is different from n.

So this term is 0 with an En1. This is a number. And these two states are orthogonal. So this term gives you a 0, not because this number is 0, but because the overlap is 0. And I get here minus k0 delta H n0.

And it's good notation to call this, to save writing, delta Hkn It's a good name for it. It's the matrix k-- the kn-th element of the matrix delta H. And this is a number so I can solve k0 n1 is equal to minus delta Hkn divided by Ek0 minus En0. And this is true for every k different for n.

And here we find, for the first time, our energy denominators. These energy denominators are the things that are going to make life interesting and difficult. And it answers the question already that if you had degenerate states, there would be some k state that have the same energy as this one. And this blows up. And this is unsolvable for this component. So you start getting difficulties if you have degeneracies.

As long as every k state-- all the other states of the spectrum have different energy from En, nevermind if the other states are degenerate. They're not degenerate with the state you care. You care just about one state now, the n-th state. And if that's nondegenerate, all these denominators are non-zero and you're OK.

So here is the solution for this thing. Now I can write the expressions for the state and the energy. So let me do it. So I have this n1 like that. Now you can say the following. Let me do this very deliberately first. n1 is equal to the sum over all k of k0 k0 n1.

This is the resolution of the identity formula. That's the unit operator. You can always do that. And now you know that the state n1 is orthogonal to n0. So this becomes the sum over k different from n, because for k equal to n, these are orthogonal of k0 k0 n1. And that's what we calculated here.

So what did we get? Therefore, the state n1, I can substitute what we had there. It's the sum from k different from n of k0 delta Hnk over Ek0 minus En0. That's n1. I should have a minus sign. The minus sign is there at the state n1.

So the state n1 is a complicated correction. It gets a little component from every other state of the spectrum. And the coefficient depends on the matrix element of your state with the state you're contributing with. So you have the state n and all the other states here. The amount of this state k that enters into the correction is proportional to the matrix element between n and k. If the matrix element is 0, that state does not contribute here. And then there is the energy denominator as well.

So we're getting to the end of this calculation. There's one more thing one can do, which is to find-- so I'm starting to wrap up this, but still an important step what we have to do. I'll get the second order energy correction. What is our second order energy correction?

Our second order energy correction can be found from the formula on that blackboard, En2. We already found the first order energy correction, which I happened to have erased it right now. It was there. En2 is obtained by doing n0 delta H times n1, which we already know. So I must do n0 delta H on that. So look what you get. You get minus the sum over k different from n.

Think of putting the n0 and the delta H, they're all together. It's a [INAUDIBLE] so far. It's a delta H and n0. IT should go into n1. But the only state in n1 is k0. So here we have k0. And then we have delta Hnk over Ek0 minus En0.

OK. A little bit of work. So what is this? This is another matrix element. This is the matrix-- OK. I'm sorry. Here do I have a mistake? Oh, yes, I have kn. I copied it wrong. It's kn. Yes. Yes.

So here I have delta Hnk. but delta Hnk is this. If you complex conjugate-- if you complex conjugate delta Hkn, complex conjugate is k delta H n complex conjugate, which changes the order. n delta H, which is her mission k. And that's delta Hnk.

So delta Hnk is equal to delta Hkn star. And therefore the second order energy correction has a nice formula. En2 is equal to minus the sum over k different from n. Delta Hnk, which is the star of that times this one, so you get delta Hnk absolute value squared divided by Ekn. Ek0 minus En0.

So we've done a lot of work. We've written the perturbation. Here is the answer. So far we have n of lambda equal n0 plus lambda n1. n1 has been calculated. Energy is En0 plus lambda En1. That was calculated what was just delta H in this state plus lambda squared En2, which we have calculated.

So this is as far as we will do for nondegenerate perturbation theory. But we have found rather interesting formulas. And we're going to spend half of its lecture trying to understand them better.