8.044 SOLUTIONS PRACTICE EXAM #3

1. a) THE HAMILTONIAN SEPARATES INTO A SUM OF INDEPENDENT QUADRATIC TERMS SO THE PROBABILITY DENSITY IS A PRODUCT OF ZERO-MEAN GAUSSIANS.

$$p(\theta_x,\theta_y,L_x,L_y) = (2\pi kT/K)^{-1/2} e^{-\frac{\theta_x^2}{2(kT/K)}} (2\pi kT/K)^{-1/2} e^{-\frac{\theta_y^2}{2(kT/K)}} \times (2\pi IkT)^{-1/2} e^{-\frac{L_x^2}{2IkT}} (2\pi IkT)^{-1/2} e^{-\frac{L_y^2}{2IkT}}$$

b) EXCLUSIVE OF THE QUANTUM CORRECTION

$$p = e^{-\mathcal{H}/kT}/Z_1 \quad \Rightarrow Z_1 = (2\pi)^2 \frac{I}{K}(kT)^2$$

$$Z = \left(\frac{Z_1}{h^2}\right)^N = \underline{\left(\frac{2\pi}{h}\right)^{2N}\left(\frac{I}{K}\right)^N (kT)^{2N}}$$

c) $F = -kT\ln Z = -NkT\ln\left(\frac{Z_1}{h^2}\right)$

$$\mathcal{S} = \left.\frac{\partial F}{\partial A}\right|_T = -NkT\frac{1}{Z_1}\frac{\partial Z_1}{\partial A} = -NkT\frac{1}{Z_1}\frac{\partial Z_1}{\partial K}\frac{\partial K}{\partial A}$$

$$\frac{\partial Z_1}{\partial K} = -\frac{Z_1}{K} \qquad \frac{\partial K}{\partial A} = \gamma\frac{K}{A}$$

$$\underline{\mathcal{S} = \gamma NkT/A}$$

NOTE: THIS IS JUST THE CONTRIBUTION FROM THE MICELLES.

2 a) NO WORK IS DONE, SO $\Delta W=0$. NO HEAT ENTERS THE GAS SO $\Delta Q=0$. THUS $\Delta E=\Delta W+\Delta Q=0$.

INTERNAL ENERGY IS CONSERVED

b) $E(T,V)$ IS A STATE FUNCTION; COMPARE IT BEFORE AND AFTER EXPANSION IN EQUILIBRIUM SITUATIONS.

$$dE = TdS - PdV = \underbrace{T\frac{\partial S}{\partial T}\Big|_V}_{C_V} dT + \left(T\frac{\partial S}{\partial V}\Big|_T - P\right)dV$$

$$C_V = \frac{3}{2}Nk \qquad \frac{\partial S}{\partial V}\Big|_T = \frac{\partial P}{\partial T}\Big|_V = \frac{Nk}{V-bN}$$

WE HAVE USED A MAXWELL RELATION

$$dE = \frac{3}{2}Nk\,dT + a\left(\frac{N}{V}\right)^2 dV$$

$$E = \frac{3}{2}NkT - \frac{aN^2}{V} + \text{CONSTANT}$$

$$\frac{3}{2}NkT_i - \frac{aN^2}{(V_0/3)} = \frac{3}{2}NkT_f - \frac{aN^2}{V_0}$$

EQUATING E BEFORE & AFTER

$$\frac{3}{2}Nk(T_f - T_i) = -\frac{2aN^2}{V_0}$$

$$\underline{T_f = T_i - \frac{4}{3}\frac{a}{k}\left(\frac{N}{V_0}\right)}$$

THE GAS COOLS!

3 a) CARNOT CYCLE $\Rightarrow dS_H = -dS_c \Rightarrow \frac{đQ_H}{T_H} = -\frac{đQ_c}{T_c}$

USE $dQ = C_0 dT$

$$\Delta S_{BODY\,1} = -\Delta S_{BODY\,2}$$

$$\int_{T_H}^{T_F} \frac{C_0 dT}{T} = -\int_{T_c}^{T_F} \frac{C_0 dT}{T} \Rightarrow \ln \frac{T_F}{T_H} = -\ln \frac{T_F}{T_c} = \ln \frac{T_c}{T_F}$$

$$\frac{T_F}{T_H} = \frac{T_c}{T_F} \qquad \underline{T_F = \sqrt{T_H T_c}}$$

b) $-đQ_H - đQ_c = đW_{OUT}$

$$W_{OUT} = -\int_{T_H}^{T_F} C_0 dT - \int_{T_c}^{T_F} C_0 dT = -C_0\left[(T_F - T_H) + (T_F - T_c)\right]$$

$$= C_0 (T_H - 2T_F + T_c) = C_0 (T_H - 2\sqrt{T_H T_c} - T_c)$$

$$\underline{= C_0 \left(\sqrt{T_H} - \sqrt{T_c}\right)^2 > 0}$$

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