1. $E(\vec{v},\vec{r}) = \frac{1}{2} m \vec{v}\cdot\vec{v} + \frac{1}{2}\alpha x^2 + \frac{1}{2}\beta (y-y_0)^2$

$p(\vec{v},\vec{r}) \propto e^{E(\vec{v},\vec{r})/kT}$ FACTORS INTO A PRODUCT OF TERMS, EACH OF WHICH CAN BE NORMALIZED BY INSPECTION

$$p(\vec{v},\vec{r}) = \left[\frac{1}{\sqrt{2\pi kT/m}} e^{-\frac{v_x^2}{2kT/m}}\right]\left[\frac{1}{\sqrt{2\pi kT/m}} e^{-\frac{v_y^2}{2kT/m}}\right]\left[\frac{1}{\sqrt{2\pi kT/m}} e^{-\frac{v_z^2}{2kT/m}}\right]$$

$$\times \left[\frac{1}{\sqrt{2\pi kT/\alpha}} e^{-\frac{x^2}{2kT/\alpha}}\right]\left[\frac{1}{\sqrt{2\pi kT/\beta}} e^{-\frac{(y-y_0)^2}{2kT/\beta}}\right]\left[\frac{1}{L}\right]$$

a) $\underline{p(v_y) = \frac{1}{\sqrt{2\pi kT/m}} e^{-\frac{v_y^2}{2kT/m}}}$

b) $\underline{p(y) = \frac{1}{\sqrt{2\pi kT/\beta}} e^{-\frac{(y-y_0)^2}{2kT/\beta}}}$

{BECAUSE THE Z INTEGRAL IS OVER p NOT v.

c) $\underline{Z_1 = (2\pi kT)^{5/2}(\alpha\beta m^3)^{-1/2} L m^3/h^3, \quad Z(N,L,T) = Z_1^N/N!}$

d) $F = -kT\ln Z \qquad \mathcal{F} = \left.\frac{\partial F}{\partial L}\right|_{N,T} = -\frac{kT}{Z} N \frac{Z}{L} = -\frac{NkT}{L}$

$\mathcal{F}$ IS DEFINED AS A TENSION $\Rightarrow$ $\underline{f_z = -\mathcal{F} = \frac{NkT}{L}}$

e) $S = -\left.\frac{\partial F}{\partial T}\right|_{N,L} = k\ln Z + kT\frac{1}{Z}\frac{5}{2}N\frac{Z}{T} = k\ln Z + \frac{5}{2}Nk$

THUS CONSTANT S (ADIABATIC) $\Rightarrow$ CONSTANT Z

$\Rightarrow T_i^{5/2}\beta_i^{-1/2} = T_f^{5/2}\beta_f^{-1/2}$

$\underline{T_f = T_i\,(\beta_f/\beta_i)^{1/5}}$

2. $$V = V_1\, e^{(T/T_1 - P/P_1)} \rightarrow \ln V/V_1 = T/T_1 - P/P_1$$

$$P(T, V) = P_1\left(T/T_1 - \ln(V/V_1)\right)$$

a) $$dS = \left.\frac{\partial S}{\partial T}\right|_V dT + \left.\frac{\partial S}{\partial V}\right|_T dV$$

$$đQ = T\,dS \Rightarrow C_V = T\left.\frac{\partial S}{\partial T}\right|_V \rightarrow \left.\frac{\partial S}{\partial T}\right|_V = C_V/T = DT^2$$

$$\left.\frac{\partial S}{\partial V}\right|_T = \left.\frac{\partial P}{\partial T}\right|_V \left(\begin{array}{c}\text{USING A}\\ \text{MAXWELL RELATION}\end{array}\right) = P_1/T_1$$

$$dS = DT^2\,dT + (P_1/T_1)\,dV$$

$$S = \tfrac{1}{3}DT^3 + f(V), \quad f'(V) = \frac{P_1}{T_1} \Rightarrow f(V) = \frac{P_1 V}{T_1} + \text{CONSTANT}$$

$$\underline{S = \tfrac{1}{3}DT^3 + P_1 V/T_1 + S_0}$$

b) $$C_P = \left.\frac{đQ}{dT}\right|_P = T\left.\frac{\partial S}{\partial T}\right|_P$$ BUT WE HAVE dS ABOVE

$$\left.\frac{\partial S}{\partial T}\right|_P = \underbrace{\left.\frac{\partial S}{\partial T}\right|_V}_{DT^2} + \underbrace{\left.\frac{\partial S}{\partial V}\right|_T}_{P_1/\,,} \underbrace{\left.\frac{\partial V}{\partial T}\right|_P}_{V/T_1}$$

$$\underline{C_P = \underbrace{DT^3}_{C_V} + \frac{P_1 V T}{T_1^2}}$$

c) $$dF = -S\,dT - P\,dV$$

$$\underline{dF = \left(-\tfrac{1}{3}DT^3 - \frac{P_1 V}{T_1} + S_0\right)dT + P_1\left(\ln V/V_1 - T/T_1\right)dV}$$

3. a) $\Delta Q_S = -\Delta Q_L$

$$\int_{T_0}^{1/2} bT^{-2}\,dT = -\int_1^{1/2} aT^3\,dT$$

$$-b\Big[_{T_0}^{1/2} T^{-1} = -\frac{a}{4}\Big[_1^{1/2} T^4$$

$$\left(2 - 1/T_0\right) = \frac{1}{4}\frac{a}{b}\left(\frac{1}{16}-1\right) = -\frac{1}{4}\frac{a}{b}\frac{15}{16} = -\frac{1}{4}\frac{128}{15}\frac{15}{16} = -2$$

$$1/T_0 = 4, \quad \underline{T_0 = 1/4}$$

b) $\Delta S = \Delta S_S + \Delta S_L$ $\qquad dS = \frac{dQ}{T} = \frac{c\,dT}{T}$

$$= \int_{T_0}^{1/2} bT^{-3}\,dT + \int_1^{1/2} aT^2\,dT$$

$$= -\frac{b}{2}\Big[_{1/4}^{1/2} T^{-2} + \frac{a}{3}\Big[_1^{1/2} T^3$$

$$= -\frac{b}{2}\underbrace{(4-16)}_{-12} + \frac{a}{3}\underbrace{\left(\frac{1}{8}-1\right)}_{-7/8} = \underline{6b - \frac{7}{24}a}$$

8.044 Statistical Physics I
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