PROFESSOR: Here is the answer, answer. It's easier apparently to write 1 over T. And 1 over T is equal to 1 plus 1 over 4 V0 squared over E times E plus V0 times sine squared of 2k2a.

So the one thing to notice in this formula, it's a little complicated, is that the second term is positive. Because V0 squared is positive, the energy is positive, and sine squared is positive. So if this is positive, the right-hand side is greater than 1. And therefore, the T is less than 1. So this implies T less than or equal to 1.

And there seems to be a possibility of T being equal to 1 exactly if the sine squared of this quantity, or the sine of this quantity, vanishes. So there is a possibility of very interesting saturation, in which the transmission is really equal to 1. So we'll see it.

The other thing you can notice is that, as E goes to 0, this is infinite. And therefore, T is going to 0. No transmission as the energy goes to 0.

As the energy goes to infinity, well, this term goes to 0. And you get transmission, T equals to one. So these are interesting limits.

Now, to appreciate this better, we can write it with unit-free language. So for that, I'll do the following. It's a little rewriting, but it helps a bit.

So think of 2k2 times a, this factor, as the argument of the sine function. Well, it's 2. k2 was defined up there, so it's 2m a squared E plus V0 over h squared. And I put the a inside the square root.

So what do we have here? 2 times the square root of 2m a squared. Let's factor a V0, so that you have 1 plus E over V0. And you have h squared here.

So this is OK. There's clearly two things you can do. First, define a unit-free energy. So the energy is now described by this little E. Without units, that compares the energy of your energy eigenstate to the depth of the potential.

So it should be over V0. So this is nice. You don't have to talk about EVs or some quantity. Just a pure number.

And here, there is another number that is famous. This is the number Z0 squared of a potential well. This is the unit-free number that tells you how deep or profound is your potential, and controls the number of zeros.

So at this moment, this is simply 2 Z0, because the square root is there and takes the Z0 squared out as Z0. Square root of 1 plus e, which is nice. So here, you can divide by V0 squared, numerator and denominator. So you have an E over V0, and a 1 plus an E over V0. So the end result is that 1 over T is now 1 plus 1 over 4e 1 plus e sine squared of 2 Z0 square root of 1 plus e.

So it's ready for numerical calculation, for plotting, and doing all kinds of things with it. But what we want to understand is this phenomenon that you would expect, in general, some reflection and some transmission. But there is a possibility when T is equal to 1, and in particular, when this sine squared function is equal to 0, and that will make T equal to 1, then you have a perfect transmission. So let's see why it is happening, or under what circumstances it happens.

So for what the energies will we have? For what energies? Energies is T equal to 1. It's perfect transmission. No reflection whatsoever. So we need, then, that the argument of this sine function be equal to multiples of pi, 2 Z0 square root of 1 plus e is equal to a multiple of pi.

Now, we would say what the multiple of pi? Well, it could be 0, 1, 2, 3. Not obvious, because the only thing you have here to adjust is the energy. The energy is positive. And that's that little e in here.

So this number n must exceed some number, because this left-hand side never becomes very small. The smallest it can be is 2 Z0. So n must be greater than or equal to 2 Z0 over pi. This is because e, since e is greater than 0.

So the left hand side is a number that is greater than 2 Z0, and the right-hand side must therefore be that way. All right, so this is a possibility. But then, let's calculate those values of the energies. Calculate those en's.

So what do we have? We squared the left hand side for Z0 squared times and 1 plus en is equal to pi squared n squared. And en is equal to minus 1 plus n squared pi squared over 4 Z0 squared.

OK, this is quantitatively nice. But probably still doesn't give us much intuition about what's going on. So let me go back to the total energy. en, remember, was energy divided by V0.

So multiply all terms by V0. E equals minus V0 plus n squared pi squared V0 over 4. Z0 squared, I'm going to go all the way back to conventional language. And, too, 4 times Z0 squared, which is 2ma squared V0 over h bar squared.

So E is minus V0 plus n squared pi squared. The V0s cancel. h squared over 2n times 2a squared. I think I got every term right.

So what does this say? Well, think of the potential. In this region, there's an e here. And there's minus V0 there.

So it says E is minus V0 plus this quantity. So minus V0 plus this quantity, which is n squared pi squared h squared over 2m times 2a squared. So the resonance happens if the energy is a distance above the bottom of the potential, which is equal to this quantity.

And now, you see something that we could have seen maybe some other way. That what's happening here is a little strange at first sight. These are the energy levels of an infinite square well of width, 2a. If you remember, the energy levels of an infinite square well are n squared pi squared h squared over 2m times the width squared. And those are exactly it.

So the energies at which you find the transmission, and the name is going to become obvious in a second, it's called the resonant transition, are those in which the energy coincides with some hypothetical energy of the infinite square well that you would put here. If it is as if you would have put an infinite square well in the middle and look at where are the energies of bound states that are bigger than 0, that might be bouncing the energies here, but those are not relevant, because you only consider energies positive.

So if you find an energy that is positive, that corresponds to a would-be of infinite square well, that's it. That's an energy for which you will have transmission. And in fact, if we think about this from the viewpoint of the wave function, this factor over here, look at this property over here.

So what do we have? The condition was that k2 time 2a, the argument of the sine function would be a multiple of pi. But k2 is 2 pi over the wavelength of the wave that you have in this range, over 2a. It's equal to n pi. So we can cancel the pis and the 2s so that you get 2a over lambda is equal to n over 2.

And what does that say? It says that the de Broglie wavelength that you have in this region is such that it fits into 2a. Let me write it yet in another way. Let me try this a as-- I won't write it like that. Leave it like that.

The wavelength lambda fits into 2a a half-integer number of times. And that's exactly what you have in an infinite square well. If you have a width, well, you could have half a wavelength there for n equals 1, a full thing for n equals 2, 3 halves for n equals 3. You always get half and halves and halves increasing and increasing all the time. Yeah.

So the way I think I wanted to do it, this equation can be written as n is equal to 2a over lambda over 2. That's the same equation. So in this way, you see an integer a number of times is 2a divided by lambda over 2, which is precisely the condition for infinite square well energy eigenstate. So there is no infinite square well anywhere in this problem.

But somehow, when the wavelength of the de Broglie representation of the particle in this region is an exact number of half-waves, there's resonance. And this resonance is such that it allows a wave to go completely through. It's a pretty remarkable phenomenon.

So the infinite square well appears just as a way to think of what are the energies at which you will observe the resonances. But the resonance is simply due to having an exact number of half-waves in this region. So we can do on a little numerical example to show how that works.