PROFESSOR: So what are we trying to do? We're going to try to write a matter wave.

We have a particle with energy e and momentum p. e is equal to h bar omega. So you can get the omega of the wave. And p is equal to h bar k. You can get the k of the wave.

So de Broglie has told you that's the way to do it. That's the p and the k. But what is the wave? Really need the phase to-- how does the wave look like? So the thing is that I'm going to do an argument based on superposition and very basic ideas of probability to get-- to find the shape of the wave.

And look at this possibility. Suppose we have plane waves-- plane waves in the x plus direction. A particle that is moving in the plus x direction. No need to be more general yet.

So what could the wave be? Well, the wave could be sine of kx minus omega t. Maybe that's the de Broglie wave.

Or maybe the de Broglie wave is cosine of kx minus omega t.

But maybe it's neither one of them. Maybe it is an e to ikx minus i omega t. These things move to the right. The minus sign is there. So with an always an e to the minus i omega t. Or maybe it's the other way around. It's e to the minus ikx plus i omega t. So always an e to the i omega t.

And then you have to change the sine of the first term in order to get a wave that is moving that way.

And now you say, how am I ever going to know which one is it? Maybe it's all of them, a couple of them, none of them.

That's we're going to try to understand.

So the argument is going to be based on superposition and just the rough idea that somehow this has to do with the existence of particles having a wave.

And it's very strange. In some sense, it's very surprising. To me, it was very surprising, this argument, when I first saw it. Because it almost seems that there's no way you're going to be able to decide. These are all waves, so what difference can it make?

But you can decide.

So my first argument is going to be, it's all going to be based on superposition. Use superposition-- --position. Plus a vague notion of probability-- --bility.

So I'm going to try to produce with these waves a state of a particle that has equal probability to be moving to the right or to the left.

I'm going to try to build a wave that has equal probability of doing this thing. So in case 1, I would have to put a sine of kx minus omega t. That's your wave that is moving to the right.

I have to change one sine here. Plus sine of kx.

Say, plus omega t.

And that would be a wave that moves to the right. Just clearly, this is the wave that moves to the left. And roughly speaking, by having equal coefficients here, I get the sense that this would be the only way I could produce a wave that has equal probability to move to the left and a particle that moves to the right.

On the other hand, if I expand this you get twice sine of kx cosine omega t.

The fact is that this is not acceptable. Why it's not acceptable? Because this wave function vanishes for all x at t omega t equal to pi over 2, 3pi over 2, 5pi over 2. At all those times, the wave is identically 0. The particle has disappeared. No probability of a particle. That's pretty bad. That can't be right.

And suddenly, you've proven something very surprising. This sort of wave just can't be a matter particle. Again, in the way we're trying to think of probabilities.

Same argument for 2 for same reason. 2-- So this is no good. No good.

The wave function cannot vanish everywhere at any time. If it vanished everywhere, you have no particle. You have nothing. With 2, you can do the same thing. You have a cosine plus another cosine.

Cosine omega t minus-- kx minus omega t plus cosine of kx plus omega t. That would be 2 cosine kx cosine omega t. It has the same problems.

Let's do case number 3. Case number 3 is based on the philosophy that the wave that we have-- e to the ikx minus i omega t always has an e to the minus i omega t as a phase. So to get a wave that moves in the opposite direction, we have to do minus ikx minus i omega t. Because I cannot change that phase. Always this [INAUDIBLE].

Now, in this case, we can factor the time dependence. You have e to the ikx e to the minus ikx e to the minus i omega t.

And be left with 2 cosine kx e to the minus i omega t.

But that's not bad. This way function never vanishes all over space. Because this is now a phase, and this phase is always non-zero. The e to the minus i omega t is never 0. The exponential of something is never 0, unless that something is real and negative.

And a phase is never 0. So this function never vanishes for all x-- vanishes for all x. So it can vanish at some point for all time. But those would be points where you don't find the particle. The function is nonzero everywhere else.

So this is good. Suddenly, this wave, for some reason, is much better behaved than these things for superposition. Let's do the other wave, the wave number 4.

And wave number 4 is also not problematic.

So case 4, you would do an e to the minus ikx e to the i omega t plus an e to the ikx e to the i omega t. Always the same exponential. This is simply 2 cosine of kx e to the i omega t. And it's also good.

At least didn't get in trouble. We cannot prove it is good at this point. We can only prove that you are not getting in trouble. We are not capable of producing a contradiction, so far.

So actually, 3 and 4 are good. And the obvious question that would come now is whether you can use both of them or either one at the same time. So the next claim is that both cannot be true at the same time.

You cannot use both of them at the same time. So suppose 3 and 4 are good. Both 3 and 4-- and 4 are both good-- both right, even. Then remember that superimposing a state to itself doesn't change the state.

So you can superimpose 3 and 4-- e to the ikx minus i omega t. That's 3. You can add to it 4, which is e to the minus ikx--

minus omega t. I factor a sine. And that's 4.

And that should still represent this same particle moving to the right. But this thing is twice cosine of kx minus omega t. So it would mean that this represents a particle moving to the right.

And we already know that if this represents a particle moving to the right, you get in trouble. So now, we have to make a decision. We have to choose one of them. And it's a matter of convention to choose one of them, but happily, everybody has chosen the same one.

So we are led, finally, to our matter wave. We're going to make a choice.

And here is the choice. Psi of x and t equal to the ikx minus i omega t. The energy part will always have a minus sign.

Is the mother wave or wave function for a particle with p equal hk and e equal h bar omega according to de Broglie. You want to do 3 dimensions, no problem. You put e to the i k vector, x vector, minus i omega t. On p, in this case, is h bar k vector. So it's a plane wave in 3 dimensions.

So that's the beginning of quantum mechanics. You have finally found the wave corresponding to a matter particle. And it will be a deductive process to figure out what equation it satisfies, which will lead us to the Schrodinger equation.