

PROFESSOR: And let me I assume, for example, that I'll put the state alpha beta in. Alpha and beta. What do I get out? So you have this state, alpha beta. What do you get out? Well, state comes in and is acted by beam splitter 1. So you must put the beam splitter, 1 matrix.

And then it comes the mirrors. And lets assume mirrors do nothing. In fact, mirrors-- the two mirrors would multiply by minus 1, which will have no effect. So lets ignore mirrors.

And then you get to beam splitter 2 and you must multiply by the matrix of beam squared 2. And that's the output. And that output is a two-component vector. That gives you the amplitude up and the amplitude down.

So I should put BS2 here, BS1 over here, alpha beta. The numbers move away, $\frac{1}{\sqrt{2}}$ over square root of 2 and $\frac{1}{\sqrt{2}}$ over square root of 2. Commute in matrix multiplication. Then you multiply these two matrices. You get 0, 2, minus 2, and 0, alpha beta. And you put the 2 in so you get beta minus alpha.

So here is the rule. If you have alpha and beta, you get, here, beta and minus alpha here, or a beta minus alpha photon at the end. Good.

So let's do our first kind of experiment. Our first experiment is to have the beam splitters here. D0-- detector D0 and detector D1 over there. And let's send in a photon over here only-- 1 or input 0, 1.

Well this photon, 0, 1, splits here. You act with BS1-- the matrix BS1. You get two things. You act with the matrix BS2, and it gives you this. But we have the rule already. If you have an alpha beta, out comes a beta minus alpha. So it should have as 1, here, and minus 0, here, which is 0. So you get a 1, 0.

So what is really happening? What's really happening is that your photon that came in divided in two, recombined, and, actually, there was a very interesting interference here. From the top beam came some amplitude and gave some reflected and some transmitted. From the bottom beam, there was some transmission and some reflection. The transmission from the top and reflection from the bottom interfered, to give 0. And this, too, the reflection from the top and transmission from the bottom, were coherent and added up to 1. And every single photon ends up in D0.

If you would put the beam-- well, Mach and Zehnder were working in the late 1800s, 1890s. And they would shine light. They had no ability to manipulate photons. But they could put those beam splitters and they could get this interference effect, where everything goes to D0. So far, so good.

Now let me do a slightly different experiment. I will now put the same thing, a BS1 and a beam going in, mirror, mirror, BS2 here. But now, I will put a block of concrete here on the way. I'll put it like this. So that if there is any photon that wants to come in this direction, it will be absorbed. Photon could still go like this, but nothing would go through here. And here, of course, there might be D0 and D1. And here are the mirrors, M and M.

Now the bottom mirror is of no use anymore because there is a big block of concrete that will stop any photon from getting there. And we are asked, again, what happens? What do the detectors see? And this time, we still have a 01.

Now I would be tempted to use this formula, but this formula was right under the wrong assumption-- that there was no block here. So I cannot use that formula. And certainly, things are going to be different.

So I have to calculate things. And we're doing a quantum mechanical calculation. Well, up to here, before it reaches here, I can do my usual calculation. Certainly, we have BS1 acting on the state, 01, and this is $\frac{1}{\sqrt{2}}$, I think, minus $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$. Yup, that B is $\frac{1}{\sqrt{2}}$, acting on 01. And that gives me $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$. So, yes, here I have one over square root of 2 amplitude. And here I also have $\frac{1}{\sqrt{2}}$ amplitude.

OK. Now that's the end of this amplitude. It doesn't follow. But on the other hand, in this branch, the mirror doesn't change the amplitude, doesn't absorb. So you still have $\frac{1}{\sqrt{2}}$ here. And now you're reaching BS2.

Now what is the input for BS2? The input is a $\frac{1}{\sqrt{2}}$ in the top beam, and nothing in the lower beam because nothing is reaching BS2 from below. This is blocked. So yes, there was some times when something reached from below, but nothing here.

So to figure out the amplitudes, here, I must do BS2 acting on $\frac{1}{\sqrt{2}}$, 0. Because $\frac{1}{\sqrt{2}}$ is coming in, but nothing is coming in from below. And, therefore, I get $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$, minus $\frac{1}{\sqrt{2}}$, 0. This

time, I get $1/2$ and $1/2$. OK, we must trust the math. $1/2$ here and $1/2$ there, so $1/2$ a column vector, $1/2, 1/2$.

OK, let me maybe tabulate this result, which is somewhat strange, really. So what is strange about it is the following.

In the first case, where the interferometer was totally clear, nothing in the middle, everything went to D2. And nothing went into D1. But now, you do something that should block some photons. You block some photons in the lower path, and yet, now you seem to be able to get something into D1. There is an amplitude to get into the D1. So by blocking a source, you're getting more somewhere. It's somewhat counterintuitive. You will see by the end of the lecture in 10 minutes, that it's not just somewhat counterintuitive, it's tremendously counterintuitive.

Let's summarize the result here-- the outcome in the blocked lower branch case and the probability for those events. So photon at the block-- the photon can end in three places. It can end on the block. It can end on the D0. Or it can end on D1. So photon at the block-- well, the amplitude to be here is one over square root of 2. The probability should be $1/2$. Photon at D0, probability amplitude, $1/2$, probability, $1/4$ -- photon at D1, probability, $1/4$.

You could put another table here-- outcome all open, probability. And in this case, there's just photon at D0. That's 1. And photon at D1 was 0.