Interference :

Superposition of EM waves

$\Rightarrow$ Enhance or Cancel each other

Consider this physical situation :

$$\begin{cases} \vec{E}_1 = A_1 \cos(\omega t - kz + \phi_1)\hat{x} \\ \vec{E}_2 = A_2 \cos(\omega t - kz + \phi_2)\hat{x} \end{cases}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$\vec{B} = \frac{1}{v}\hat{k}\times\vec{E}$$

$$\Rightarrow \vec{E} = \vec{E}_1 + \vec{E}_2$$

Intensity $I = |\vec{S}|$

n: refractive index

$$|\vec{S}| = \frac{1}{\mu_0}|\vec{E}\times\vec{B}| = \frac{n}{\mu_0 c}|\vec{E}^2| = cn\varepsilon_0|\vec{E}^2|$$

* Intensity
= power transfer per unit area
= magnitude of the poynting vector

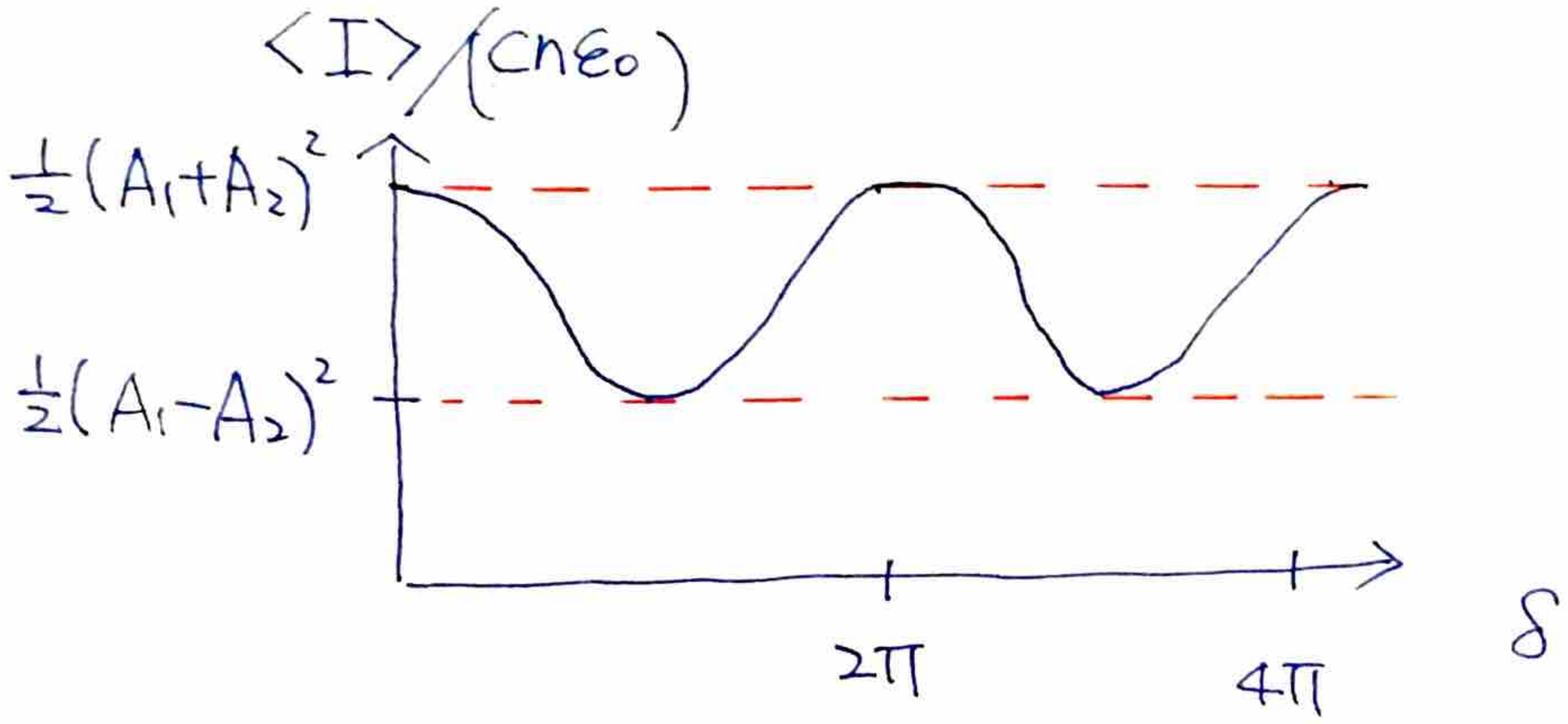
$$|\vec{E}|^2 = A_1^2\cos^2(\omega t - kz + \phi_1) + A_2^2\cos^2(\omega t - kz + \phi_2)$$

$$+ 2A_1A_2\underbrace{\cos(\omega t - kz + \phi_1)\cos(\omega t - kz + \phi_2)}$$

$$\frac{1}{2}\cos(2\omega t - 2kz + \phi_1 + \phi_2) + \frac{1}{2}\cos(\phi_1 - \phi_2)$$

$$\langle I \rangle = \frac{1}{T}\int_0^T I\,dt \qquad \left(\cos^2 x = \frac{1+\cos 2x}{2}\right)$$

$$= cn\varepsilon_0\left[\frac{A_1^2}{2} + \frac{A_2^2}{2} + 0 + A_1A_2\cos(\underbrace{\phi_1 - \phi_2}_{\delta})\right]$$



If $A_1 = A_2 \Rightarrow$ Completely cancel when

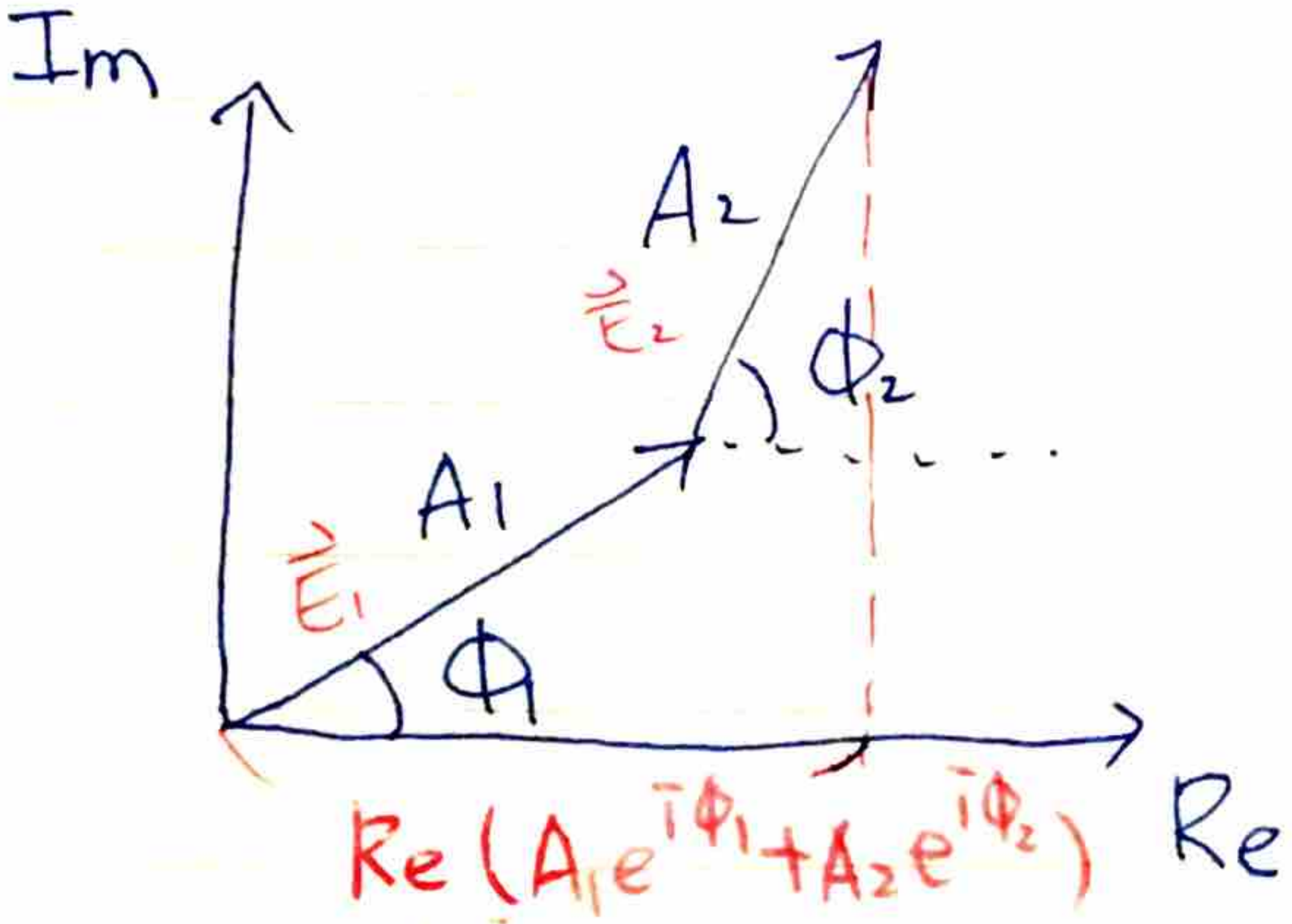
$\delta = \pi, 3\pi, \cdots$!!

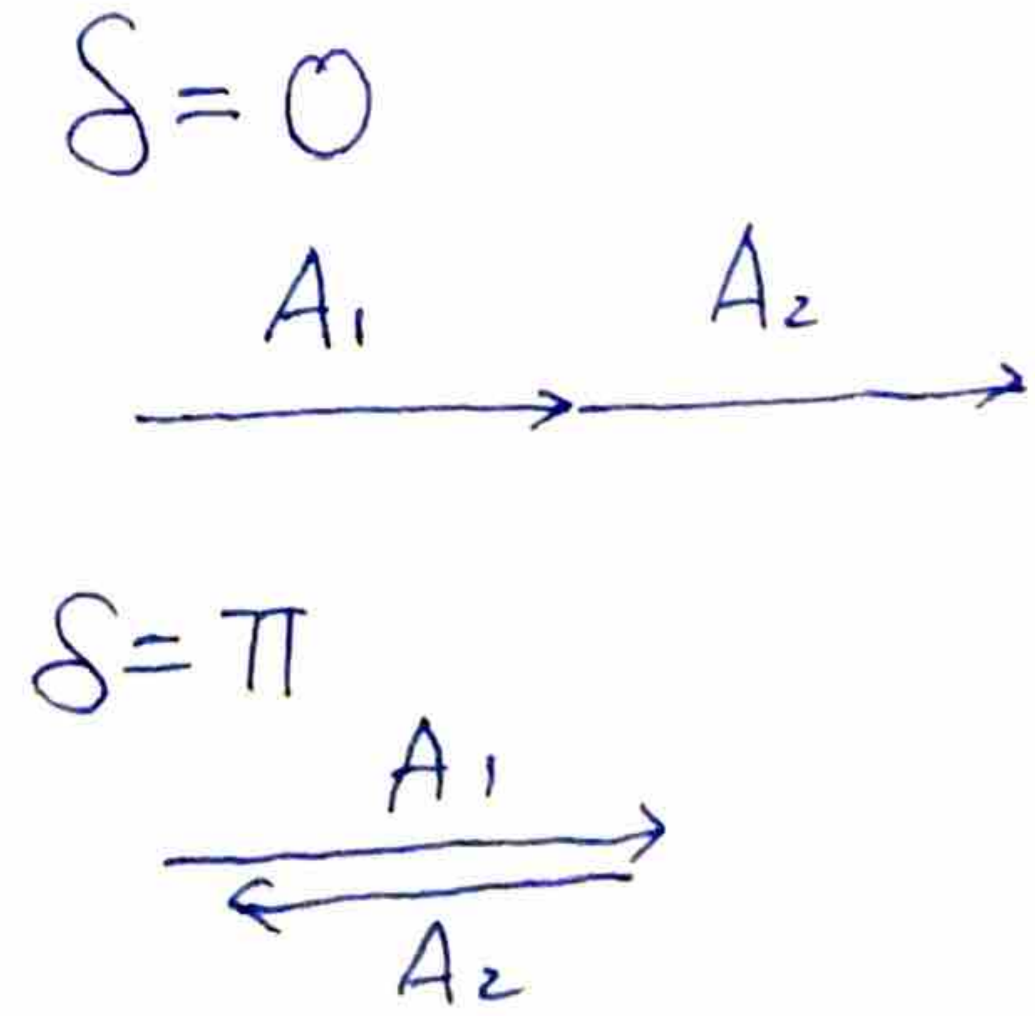
How do we understand this?

Imaginary Plane

$$\vec{E}_1 = \mathrm{Re}\left(A_1 e^{i\phi_1} e^{i(\omega t - kz)}\right)\hat{x}$$

$$\vec{E}_2 = \mathrm{Re}\left(A_2 e^{i\phi_2} e^{i(\omega t - kz)}\right)\hat{x}$$





$\vec{E} = \vec{E}_1 + \vec{E}_2$

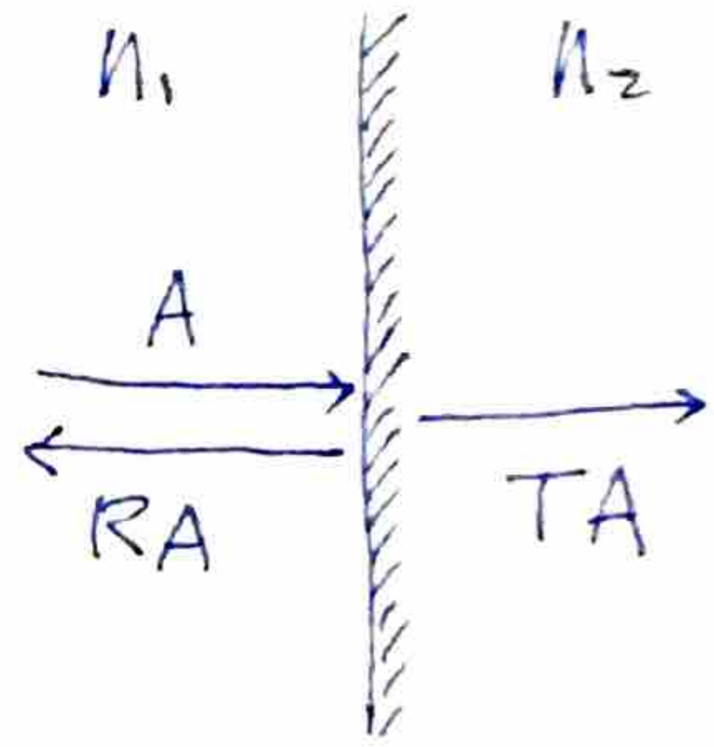
How thick is the soap bubble film?

- (1) 1 mm ~ pinhead ?
- (2) 100 μm ~ human hair ?
- (3) 100 nm ~ diameter of a virus ?

An interesting example : Interference involving dielectrics

Last lecture: We have learned the reflection coefficients (R) and transmission coefficients (T)

$$R = \frac{n_1 - n_2}{n_1 + n_2} \quad , \quad T = \frac{2n_1}{n_1 + n_2}$$



(1) If $R > 0$ (for example $n_1 > n_2$)

$\Rightarrow$ No flip in amplitude

(2) If $R < 0$ (for example: $n_1 < n_2$) or $V_1 > V_2$

$\Rightarrow$ flip in amplitude

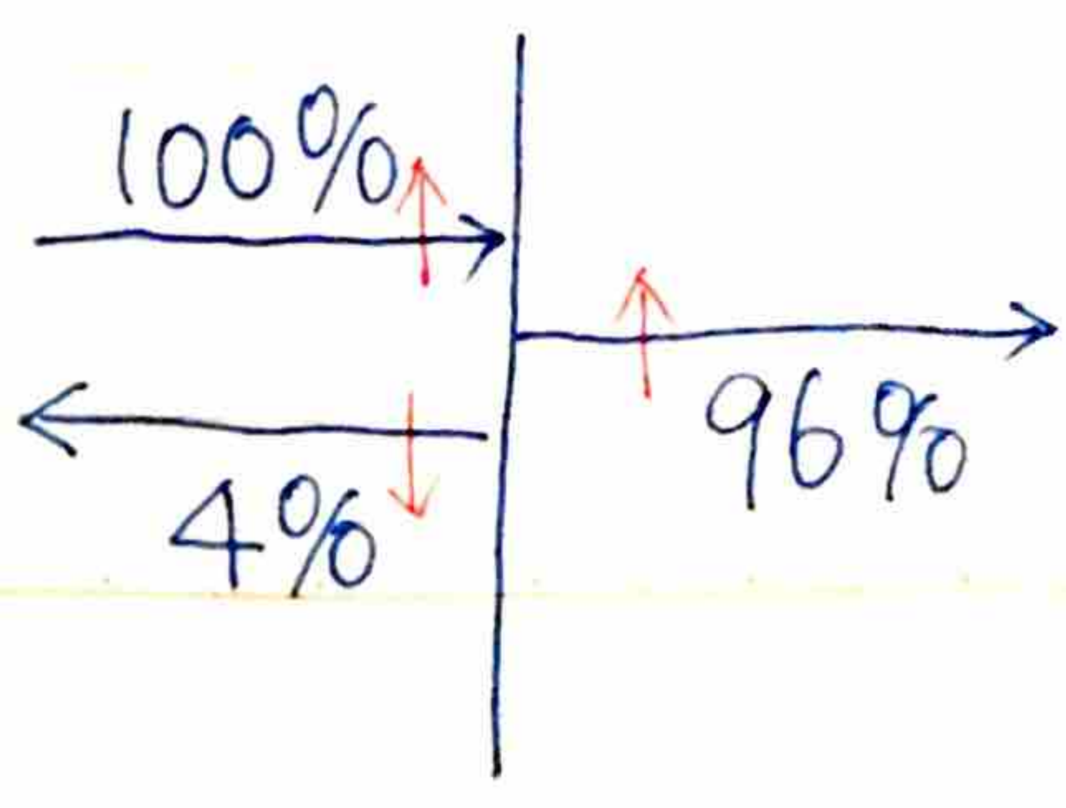
$\Rightarrow$ Introduce a phase difference of π

For example: air $n_1 = 1$, soap solution $n_2 = 1.5$

$$I = \frac{cn\varepsilon_0 E^2}{2}$$

$$R = \frac{1 - 1.5}{1 + 1.5} = -0.2 \qquad T = \frac{2}{2.5} = 0.8$$

$$I_R = 0.04\, I_0 \qquad I_T = 1.5 \cdot 0.8^2\, I_0 = 0.96\, I_0$$

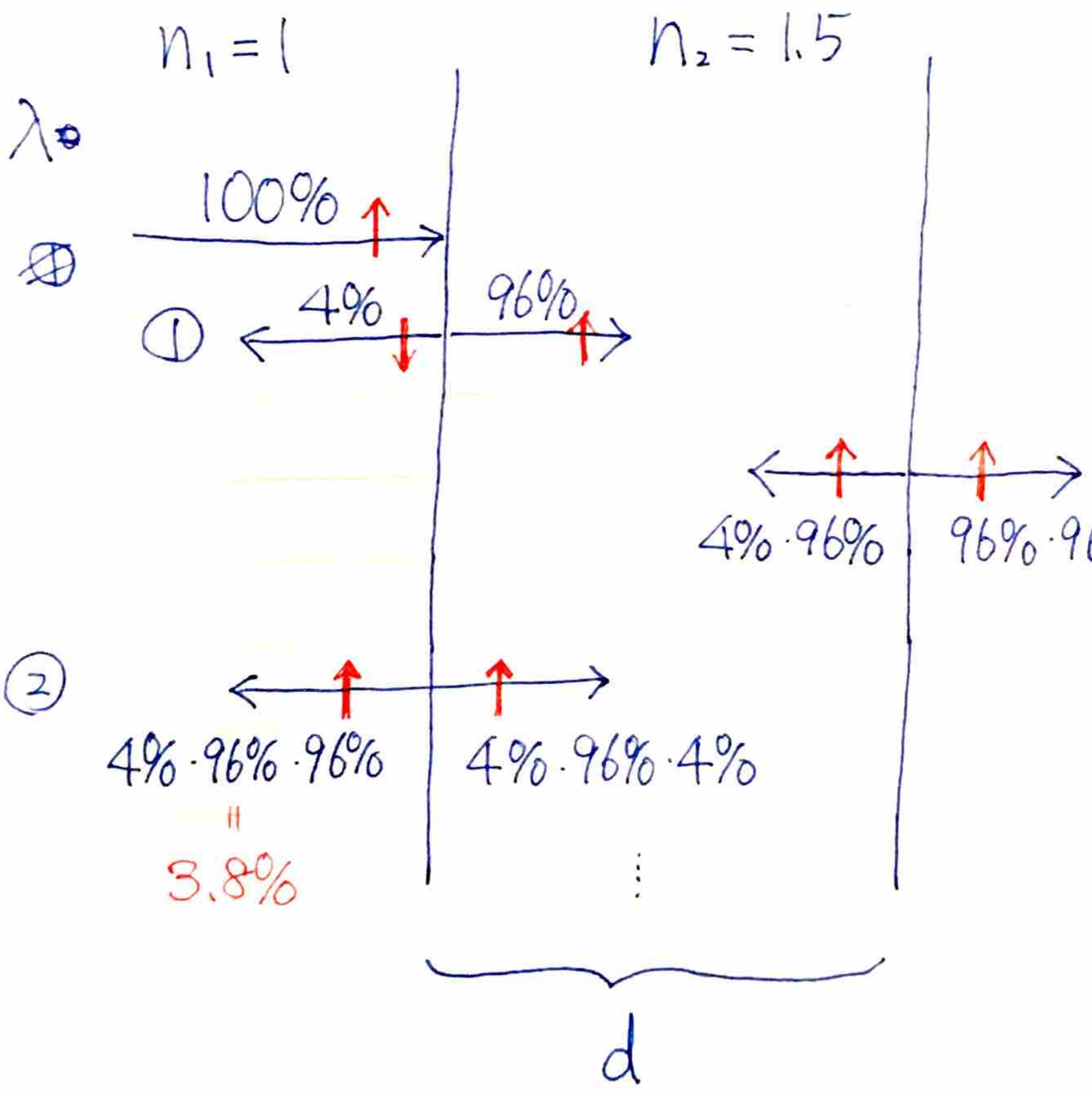


↓ flip!

Now we are in position to understand the soap bubble. Why is it colorful? (slide)

⇒ Translate this physical situation into mathematics.

Consider a thin layer of soap water:
(Simplify it by considering normal incidence)



$$R = \frac{n_2 - n_3}{n_2 + n_3} = +0.2$$

We are looking at the interference between ① and ②!

Question: What is the thickness d that is needed to have constructive interference?

Phase difference : ① − ②
δ

Change in amplitude sign (↑ vs ↓)

$$\delta = \frac{2d}{\lambda / n_2} \cdot 2\pi + \pi$$

⇑ From optical pathlength difference between ① and ②

Constructive Interference : $\delta = 2N\pi$

Destructive Interference : $\delta = (2N+1)\pi$

(1) $d \to 0$ (very very thin)

⇒ Destructive Interference.

(2) Constructive Interference :

$$d = \frac{(2N-1)\lambda}{4n_2}$$

(3) Destructive Interference :

$$d = \frac{2N\lambda}{4n_2} = \frac{N\lambda}{2n_2}$$

(4) If we fix the d and change λ

$$\lambda_{max} = \frac{4dn_2}{2N-1}$$

Thin layer:

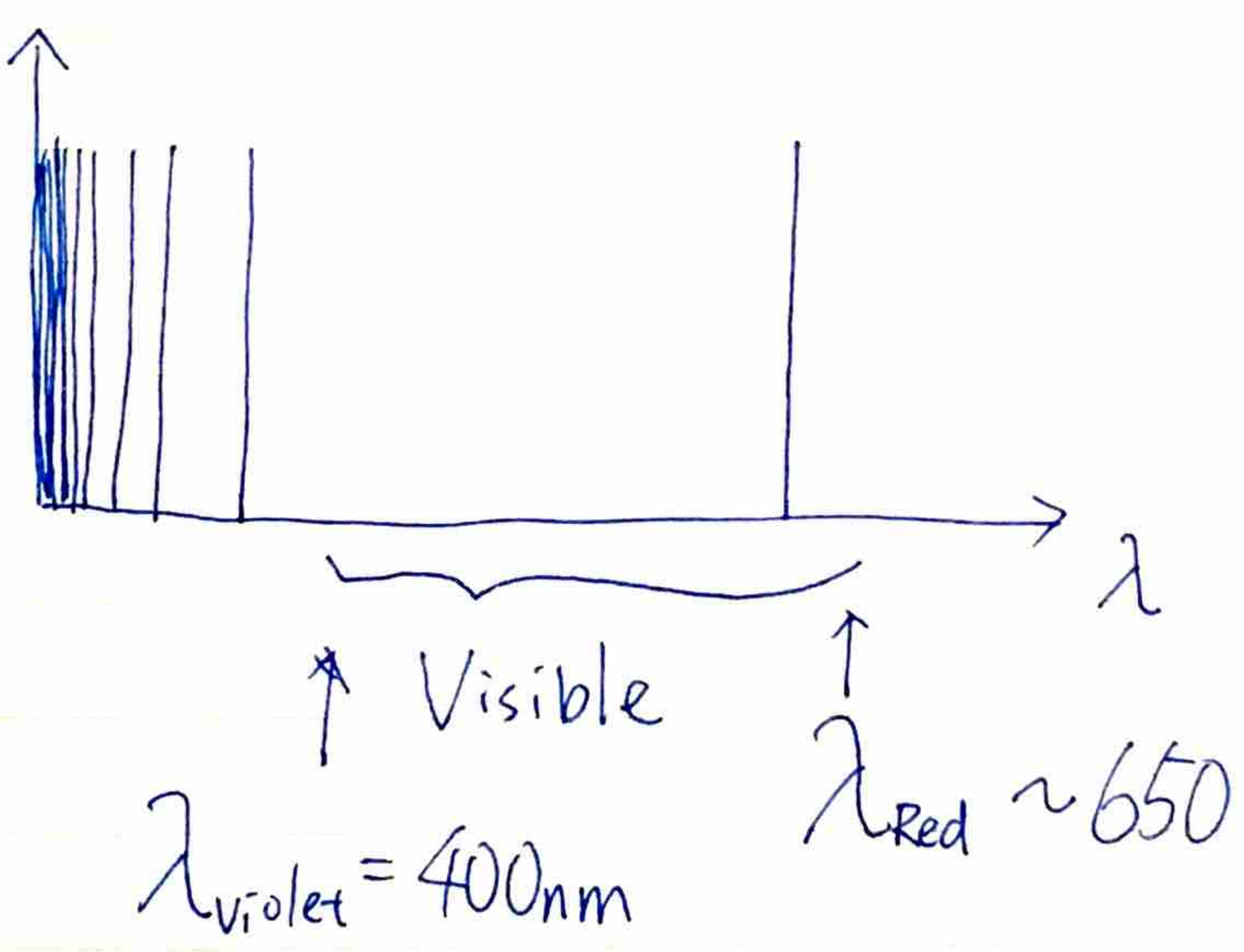
if $d \sim 100\ nm$

$$\lambda_{max} = \frac{4 \cdot 100nm \cdot 1.5}{2N-1} = \frac{600\ nm}{2N-1}$$

$$= 600nm,\quad 200\ nm,\quad 120\ nm \ldots$$

$N=1 \quad N=2 \quad N=3$

Thin



$\Rightarrow$ We see color in the soap bubble !!

Thick layer :

if $d \sim 100\,\mu m$

$\lambda_{max} = 600\,\mu m$ (N=1)

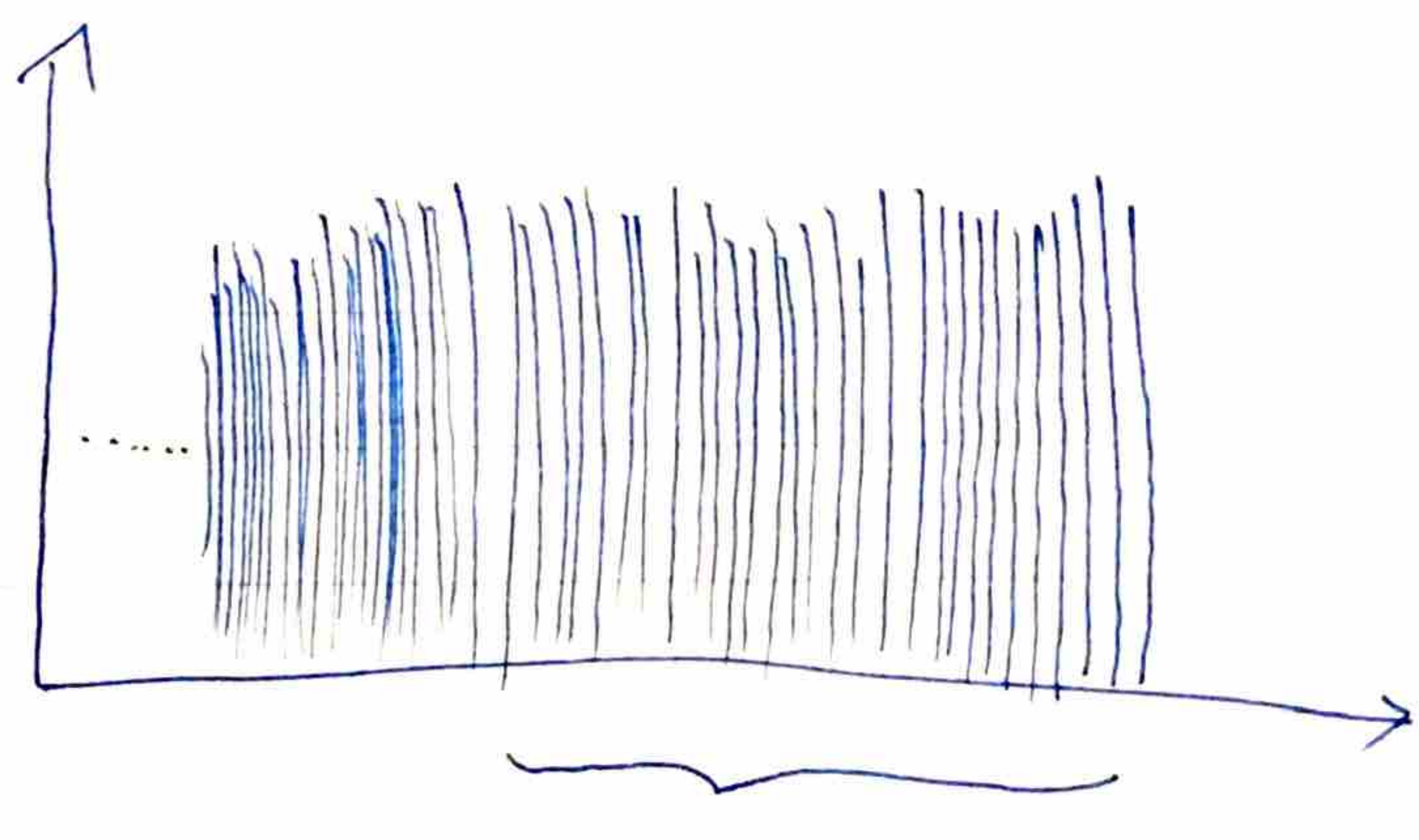
⋮

600.6 nm (N=500)
599.4 nm (N=501)
598.2 nm (N=502)

⋮

A lot of wave lengths in the range of visible light has constructive interference

⇒ White in our brain!



DEMO

We learned (1) need $d \sim 100$ nm ⇒ colorful soap bubble

(2) No color if thick

Break

(3) Color disappear (or bubble become transparent) when $d \to 0$