Welcome back to 8.033!

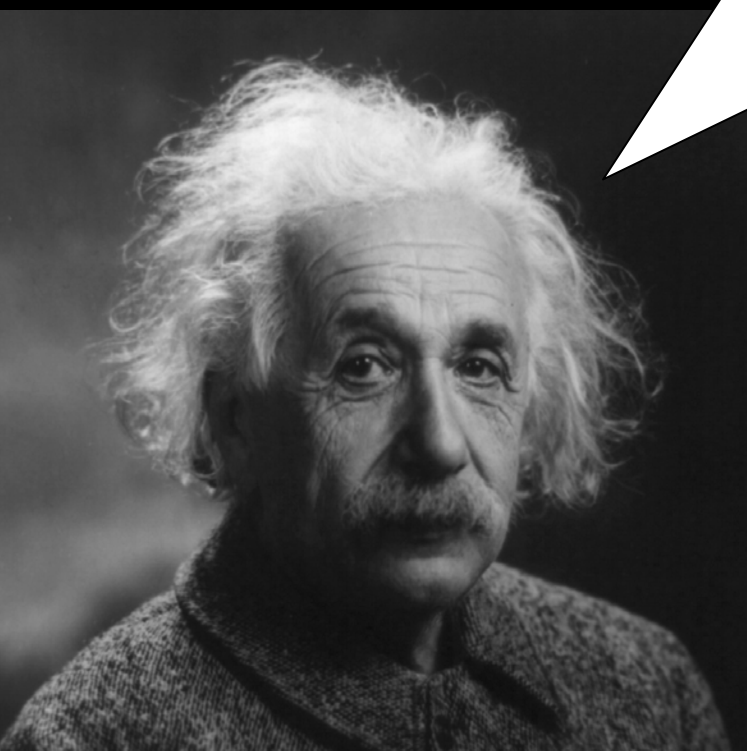


Image courtesy of Wikipedia.

Astrophysical evidence for black holes:

1) Supermassive BH's in centers of most (all?) galaxies:

- existence of quasars, huge jets
- stellar motions => 10^6 - 10^9 solar masses
- orbiting gas disks => size less than 0.4 lightyears (can't be stars)
- devoured star incident => size less than 0.4 A.U.
- X-ray spectra reveal disk extending in to 6-20M!

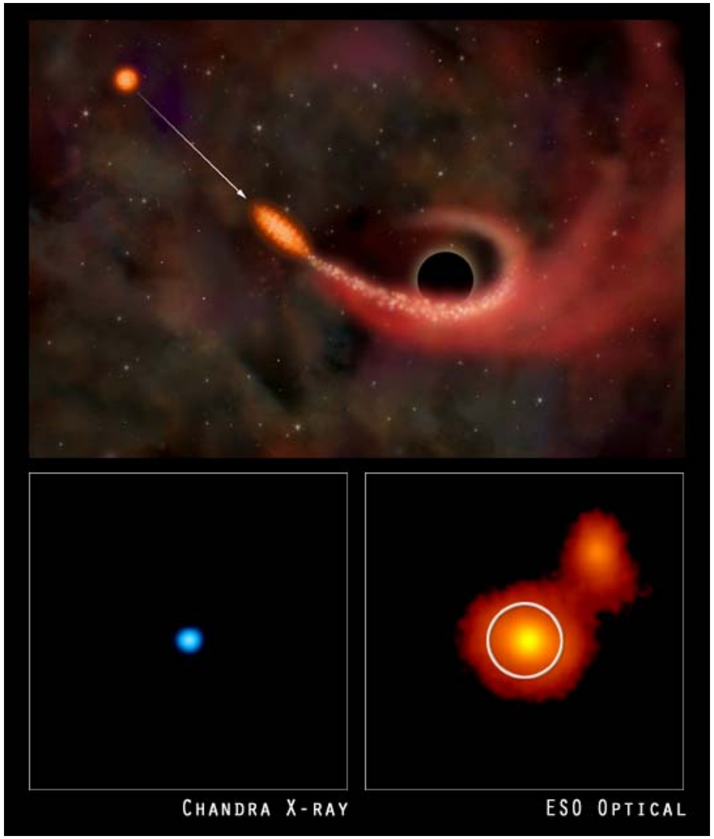


Image courtesy of NASA.

Astrophysical evidence for black holes:

2) Stellar mass BH’s:

- Stars orbiting massive invisible companion

- Maximum neutron star mass is 3 solar masses

- Best example: V404 Cygni
partner mass = 12±2 solar masses.

- Older example: Cygnus X1

- X-ray variability puts upper limit on size

- Appears that no “surface”

Orbits

- For a particle moving in the Schwarzschild metric, the energy E and and angular momentum L are conserved. It's convenient to divide these two by the rest mass of the particle and work with the energy per unit rest enery $\tilde{E} \equiv E/m$ (dimensionless, since $c = 1$) and the angular momentum per unit rest mass, $\tilde{L} \equiv L/m$ (units of length).

- In terms of these two constants, the equations of motion become

$$\begin{aligned} \left(\frac{dr}{d\tau}\right)^2 &= \tilde{E}^2 - \tilde{V}(\tilde{L}, r)^2, \\ \frac{d\varphi}{d\tau} &= \frac{\tilde{L}}{r^2}, \end{aligned}$$

where the *effective potential* per unit rest mass is

$$\tilde{V}(\tilde{L}, r)^2 = \left(1 - \frac{2M}{r}\right)\left(1 + \frac{\tilde{L}^2}{r^2}\right)$$

and the proper time τ is related to the t-coordinate (far-away time) by

$$\frac{dt}{d\tau} = \frac{\tilde{E}}{1 - 2M/r} = \gamma_r^2 \tilde{E}.$$

- $\tilde{E} \geq 1$ is a neccessary condition for being able to escape to $r = \infty$ (where $\tilde{V} = 0$).

- To build intuition for Schwarzschild orbits and the effective potential, I highly recommend the interactive simulator at `http://www.fourmilab.ch/gravitation/orbits/`. Note that it crashes and requires reloading if you accidentally fall in.

MIT Course 8.033, Fall 2006, Lecture 23

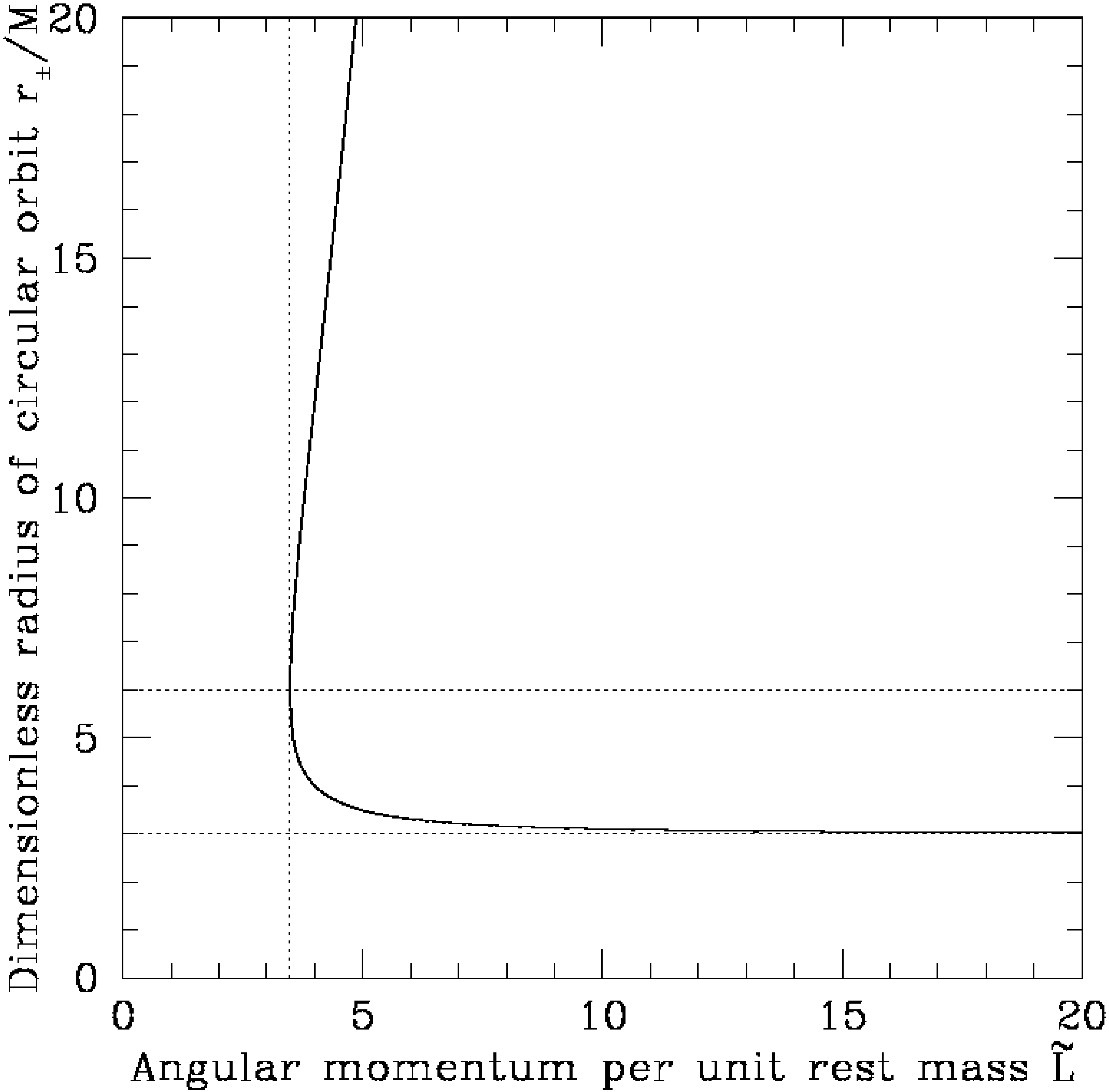
Max Tegmark

TODAY'S TOPICS:

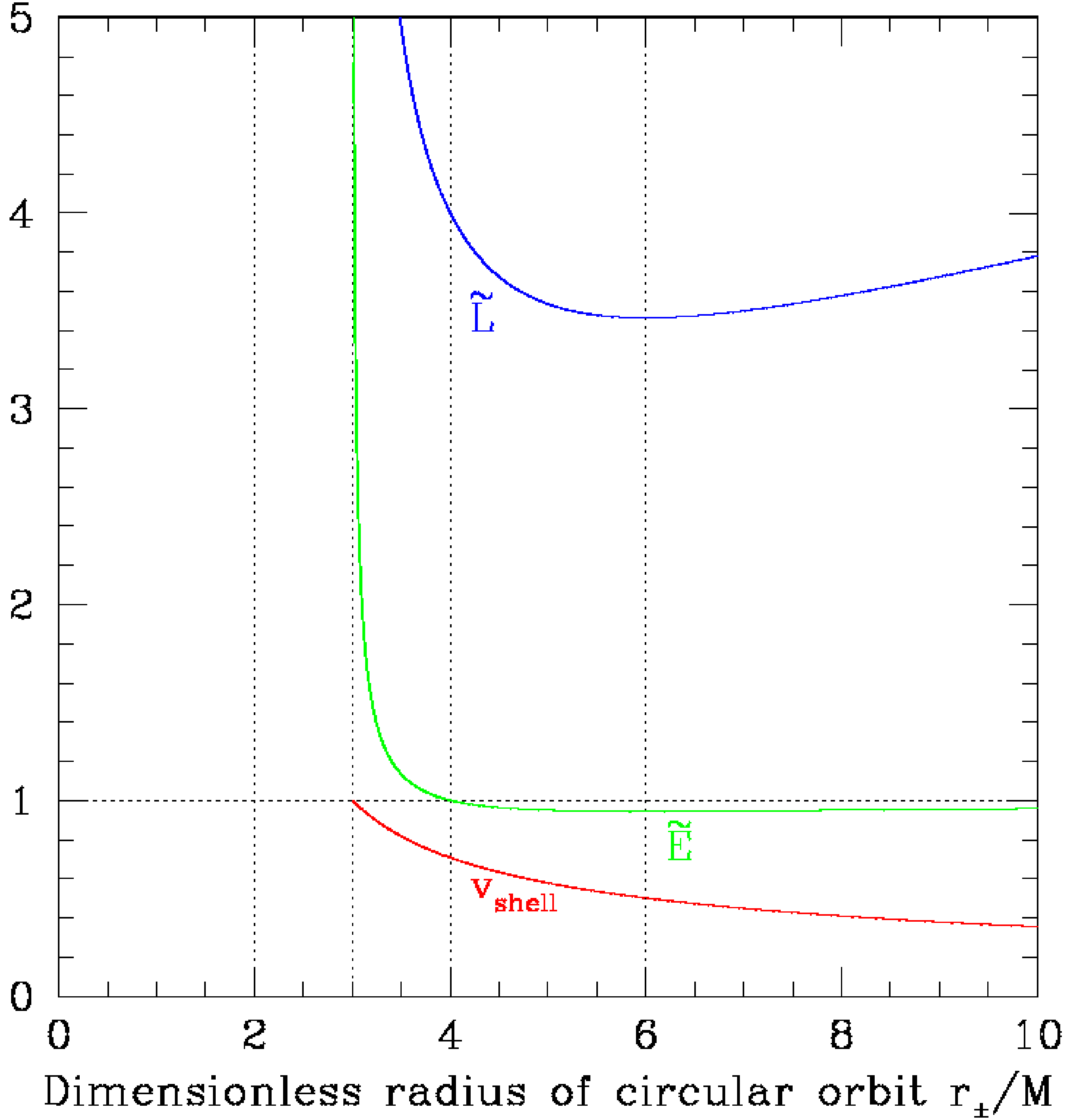
- Applications of the orbital equations:
- Circular orbits
- Near-circular orbits: Mercury perihelion precession
- Radial orbits
- What really happens near the event horizon

Java orbit simulator

CIRCULAR ORBITS



Dimensionless radius of circular orbit $r_\pm/M$
Angular momentum per unit rest mass $\tilde{L}$
0
5
10
15
20



5
4
3
2
1
0
0
2
4
6
8
10
$\tilde{L}$
$\tilde{E}$
v_{shell}
Dimensionless radius of circular orbit $r_{\pm}/M$

Interesting circular orbits:

Photon orbit

Tourist orbit

Last stable orbit

Classical limit

r	v_{shell}	$\tilde{E}$	$\tilde{L}$
$3M$	1	∞	∞
$4M$	$\frac{1}{\sqrt{2}}$	1	$4M$
$6M$	$\frac{1}{2}$	$\sqrt{\frac{8}{9}}$	$\sqrt{12}M$
∞	0	1	∞

GENERAL ORBITS

Perihelion advance: 43 arcseconds/century

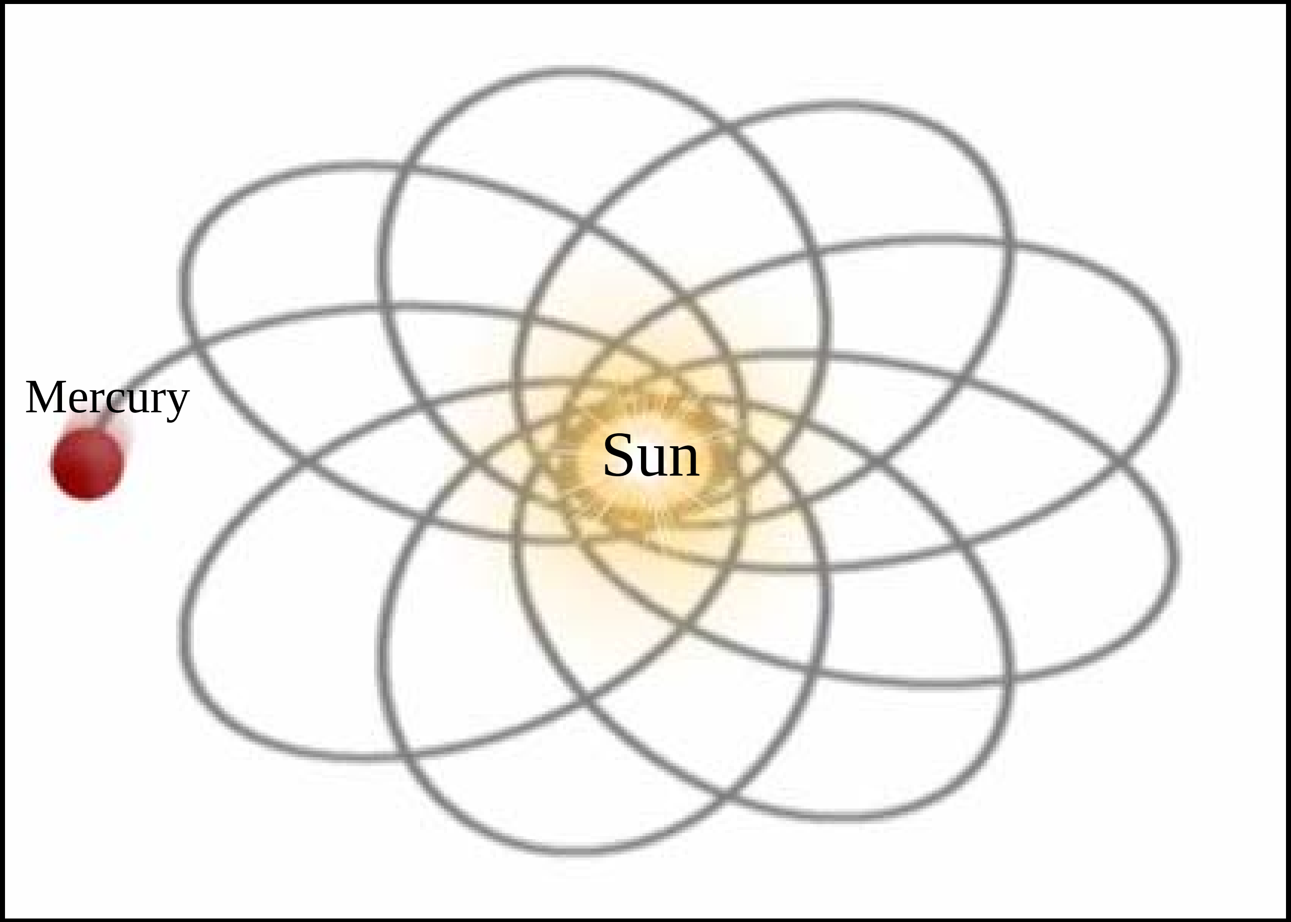


Image courtesy of Wikipedia.

GPS

GPS uses a constellation of 24 “NAVSTAR” satellites that are 11,000 miles above the earth's surface.

How GPS receivers calculate your location:

The positioning process:

1. Satellite 1 transmits a signal that contains data on its location in space and the exact time the signal left the satellite.

2. The GPS Receiver collects and interprets this signal and is able to determine the distance from the satellite to the receiver. This creates a circle of possible locations of the receiver.

3. The process is repeated for satellites 2 &3.

4. Your position is where the three circles meet. This process is called trilateration.

5. A fourth satellite is required to obtain the elevation of your current position. Coordinates are displayed on the GPS receiver.

6. More satellites may be used to create a more accurate position.

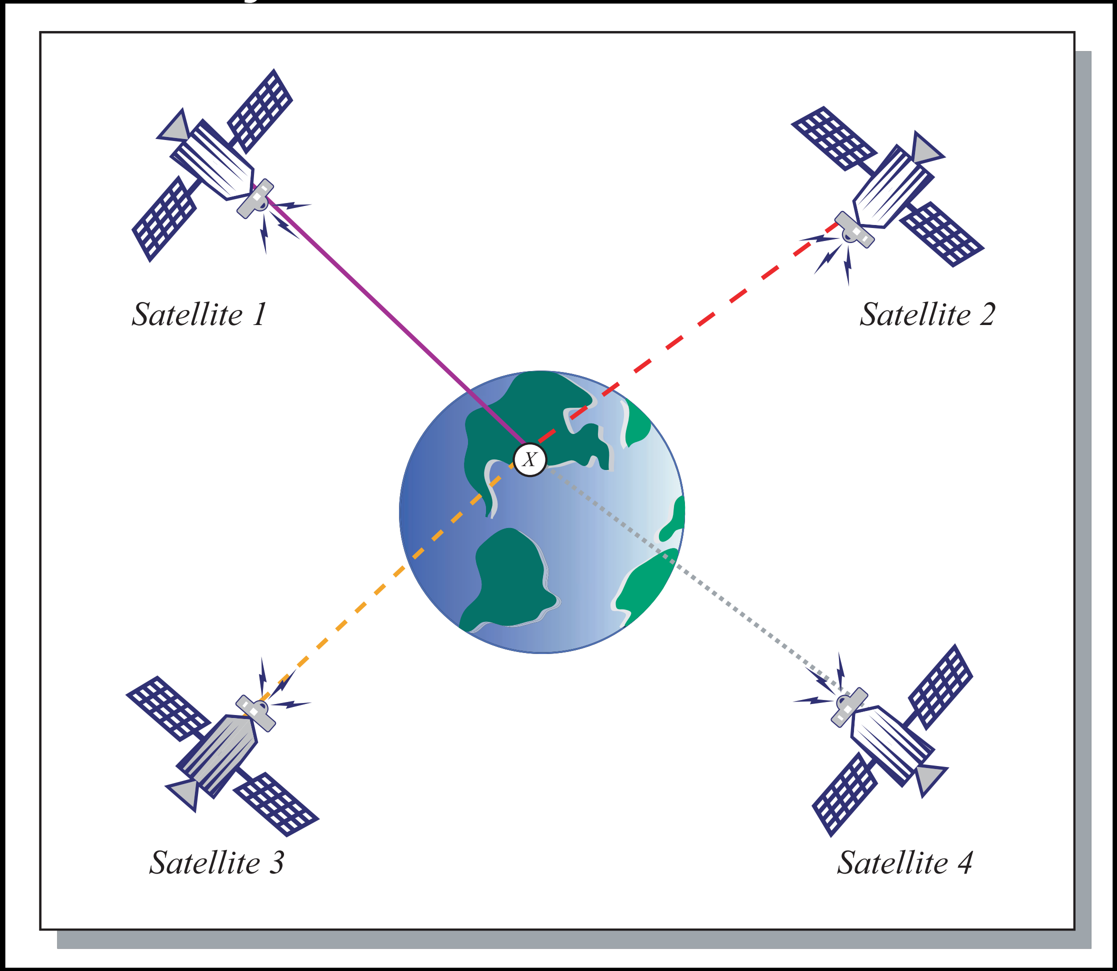


Figure by MIT OCW.

The Distance Calculation

Rate = Speed of Radio Waves (~ Speed of Light) 299,792,459 m/s
Time = amount of time for signal to reach the GPS receiver
Rate * Time = Distance Traveled

RADIAL “ORBITS”

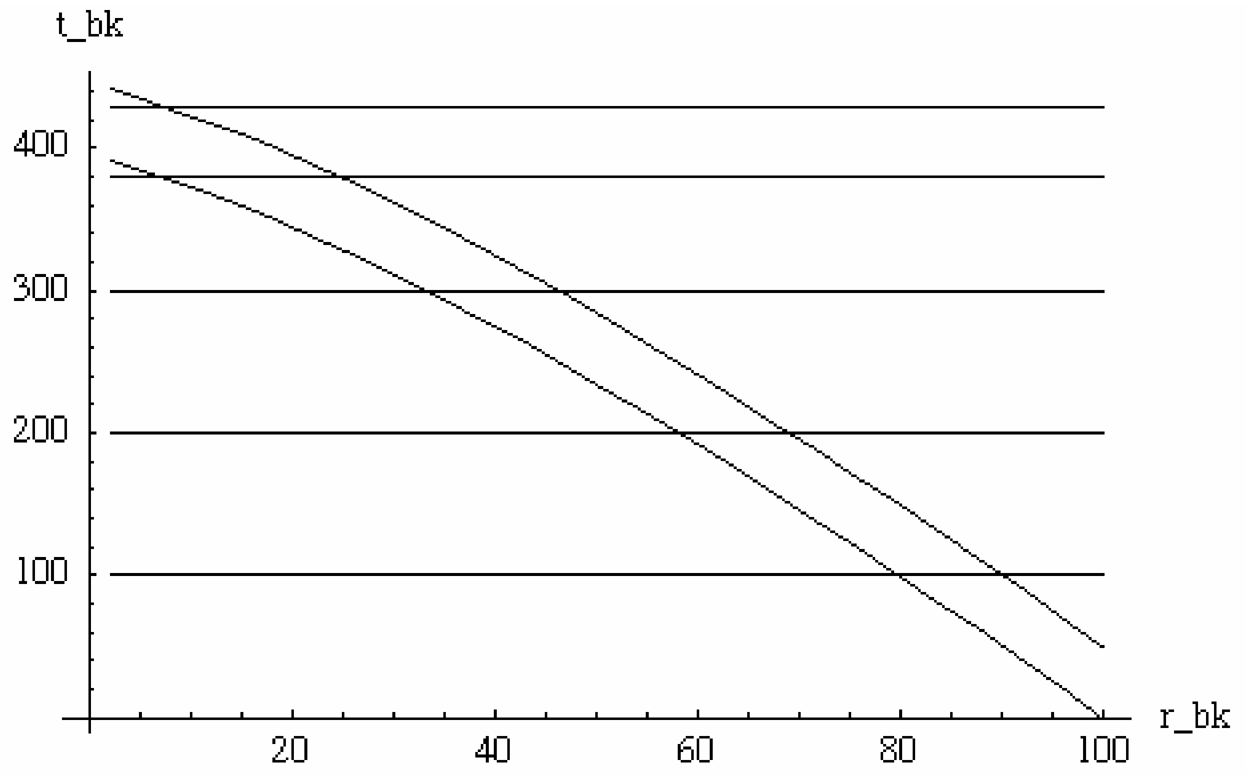
(one way trip)

Spaghetti or Pancake?

Spaghetti or Pancake?

Release two particles from r=100M a short time appart

Book Keeper Time after release

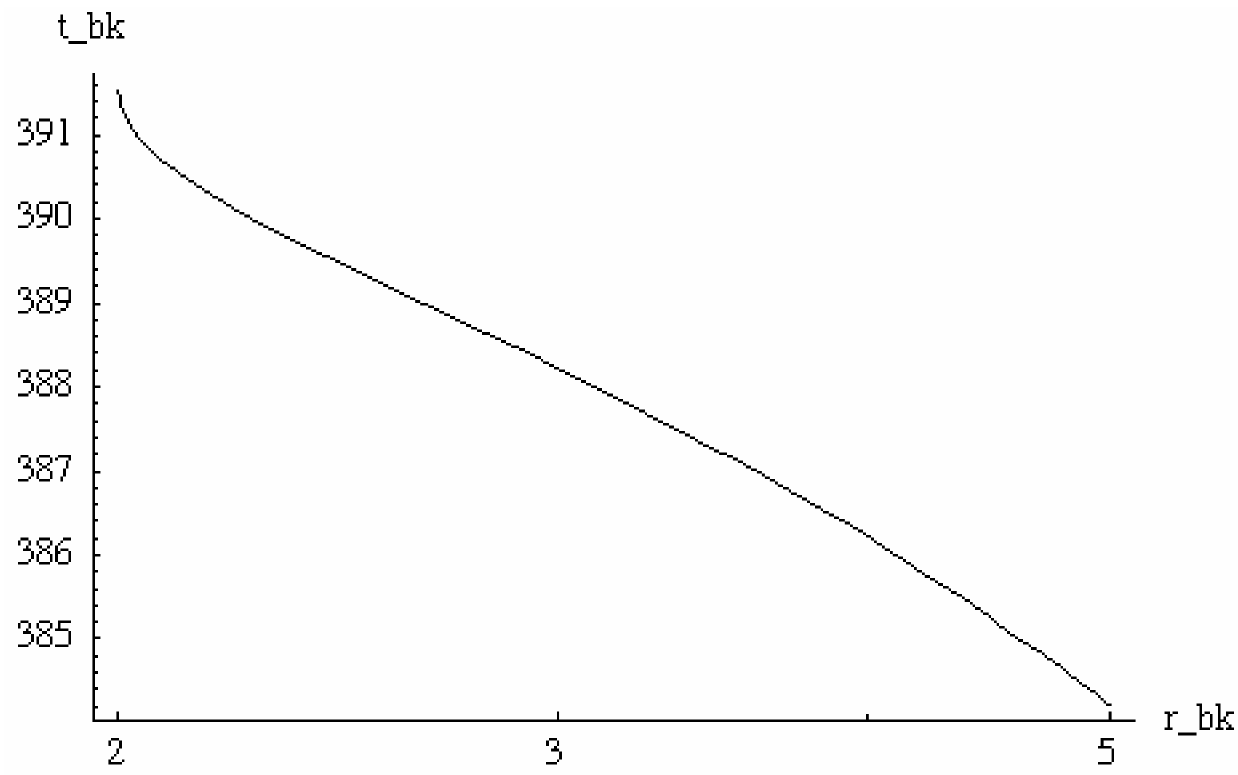


Book Keeper Radius r

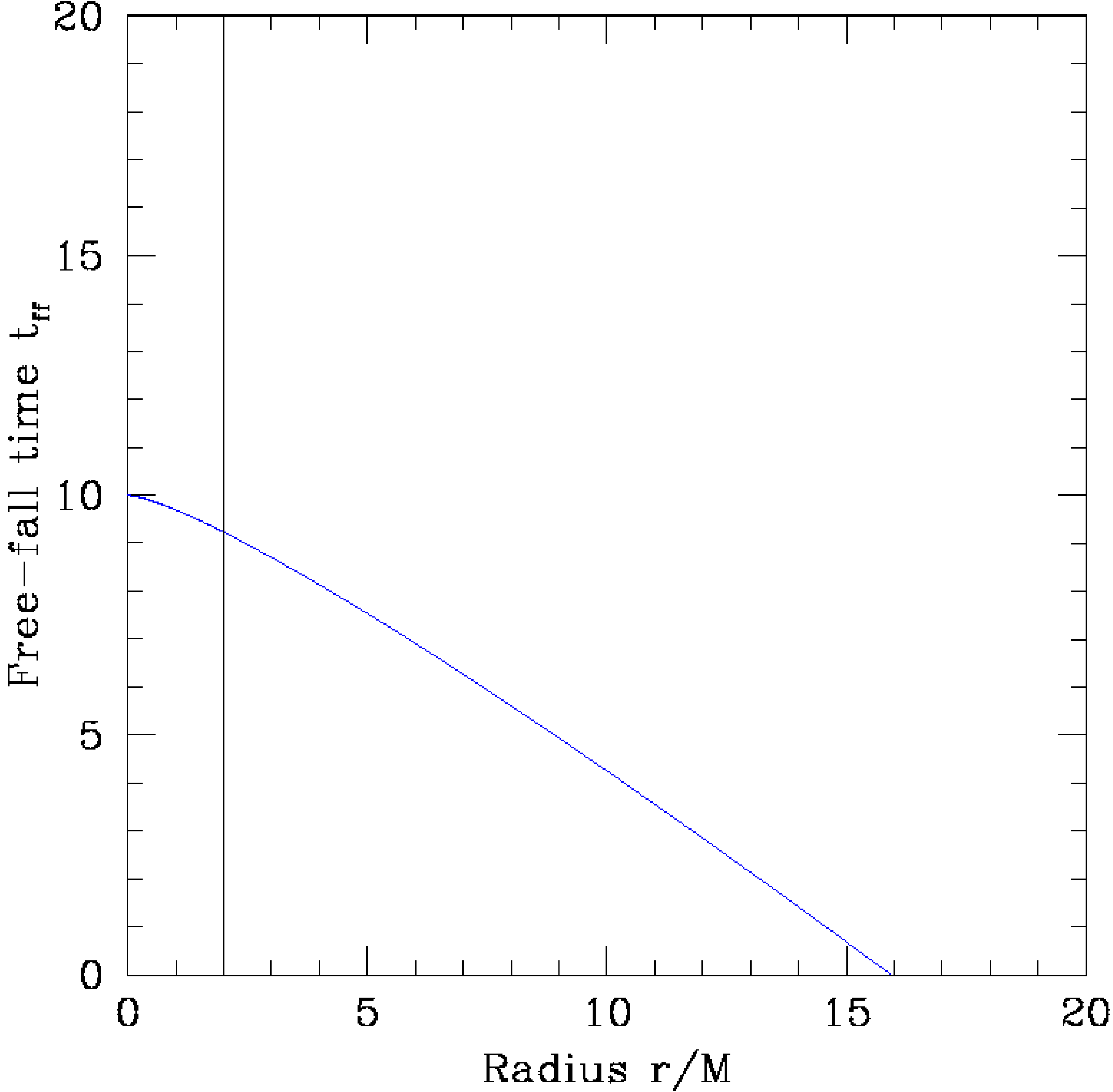
Spaghetti or Pancake?

Release two particles from r=30M a short time appart

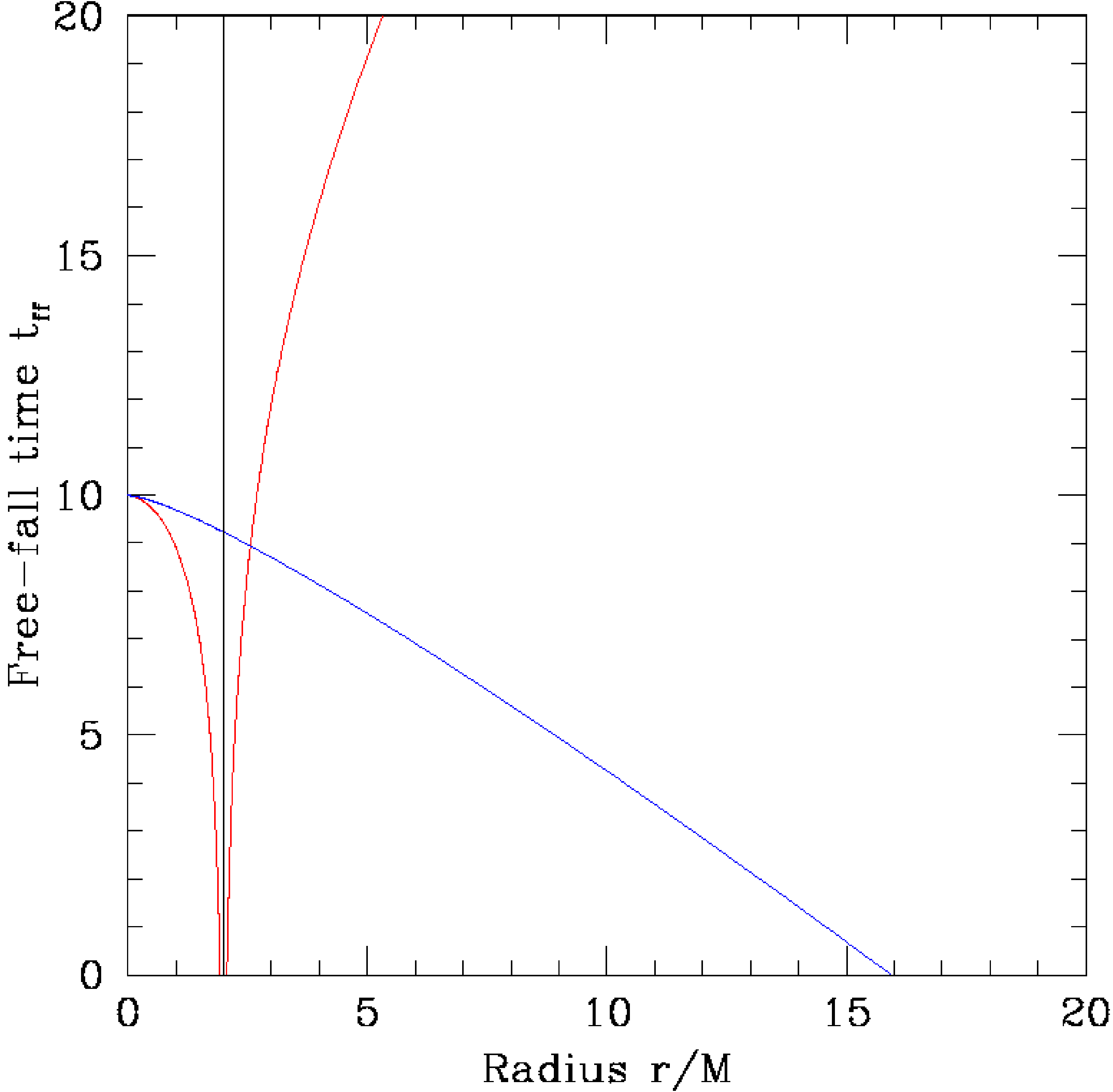
Book Keeper Time after release



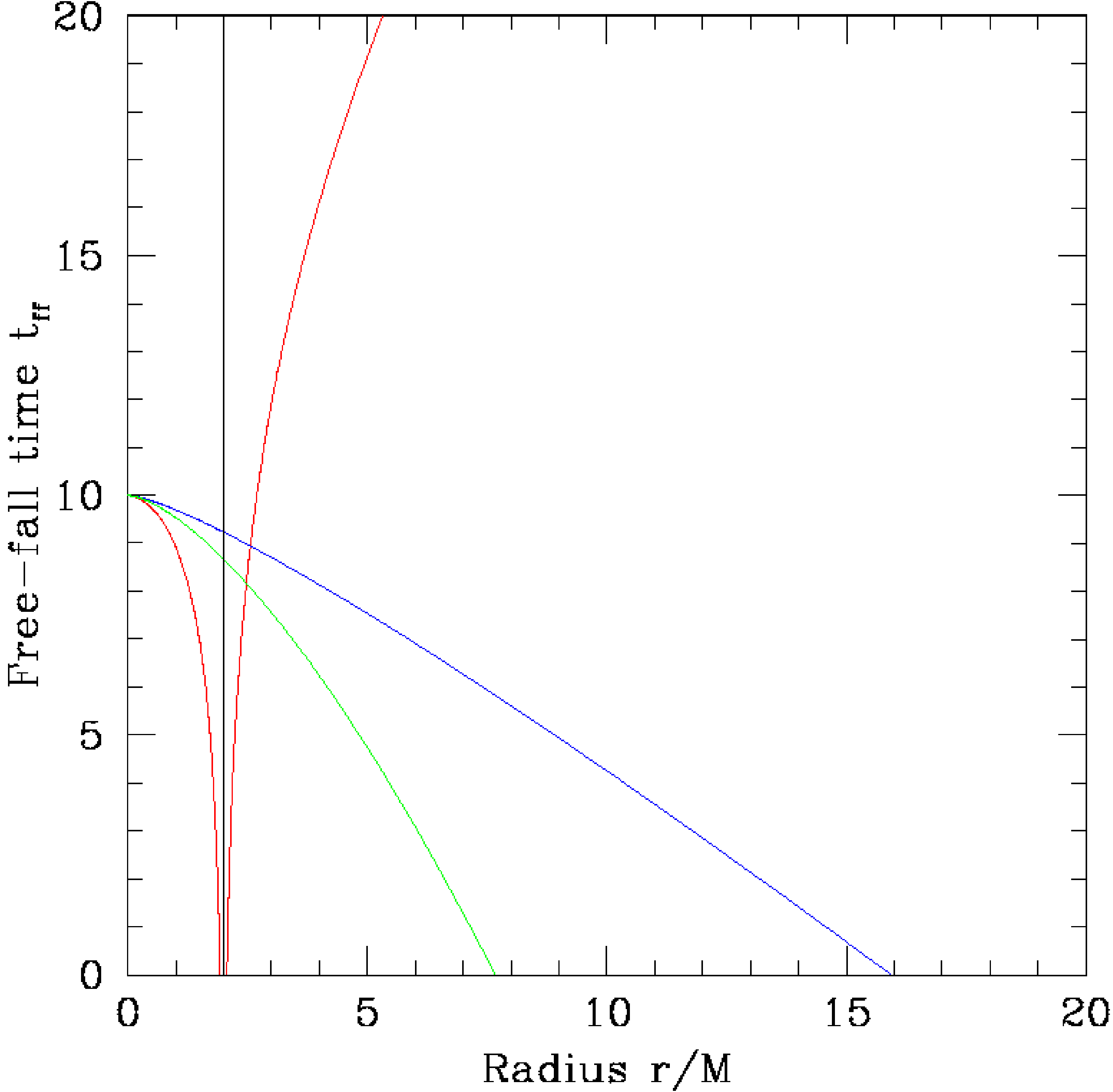
Book Keeper Radius r



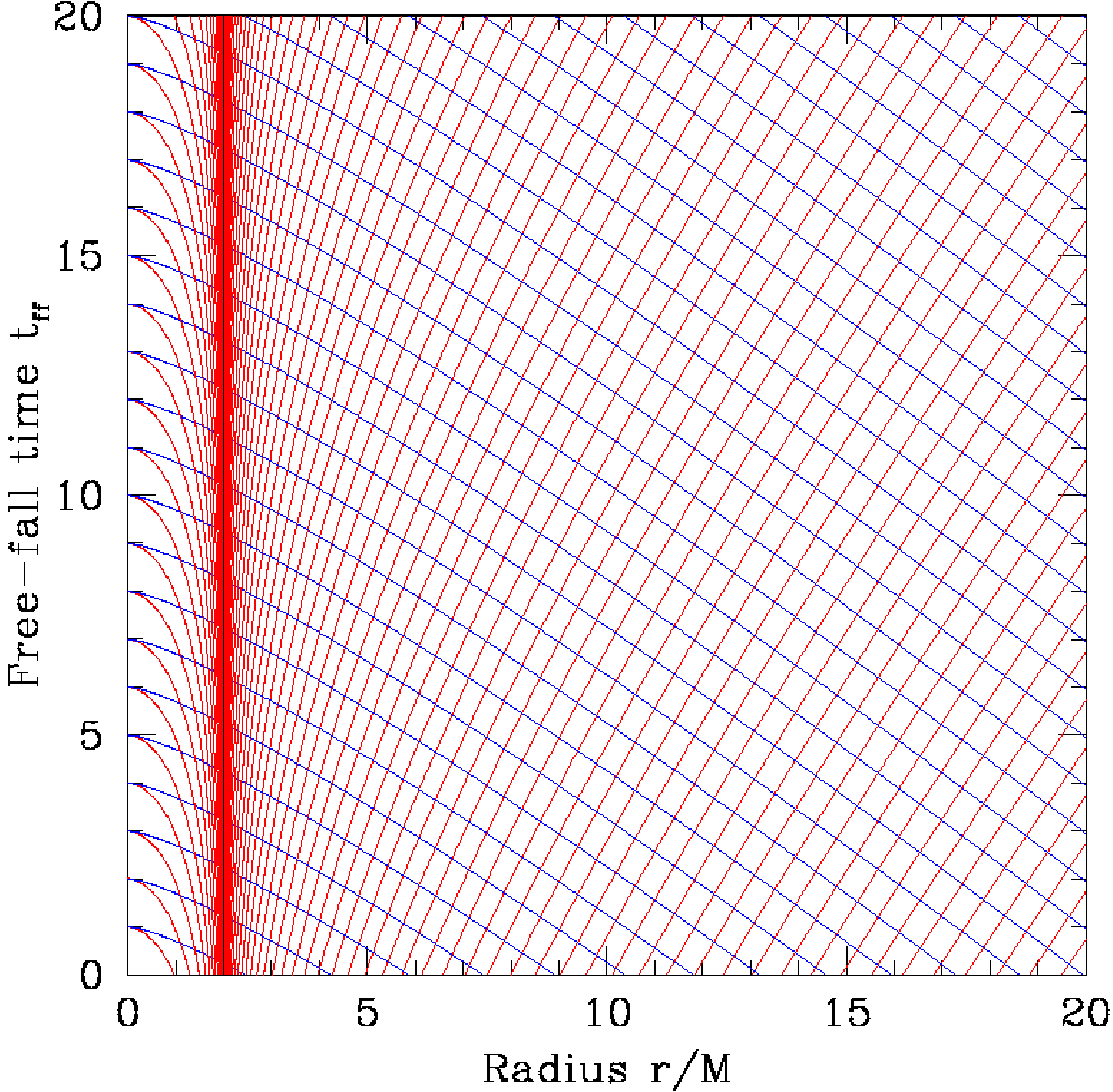
Free-fall time t_{ff}
Radius r/M
0
5
10
15
20
0
5
10
15
20



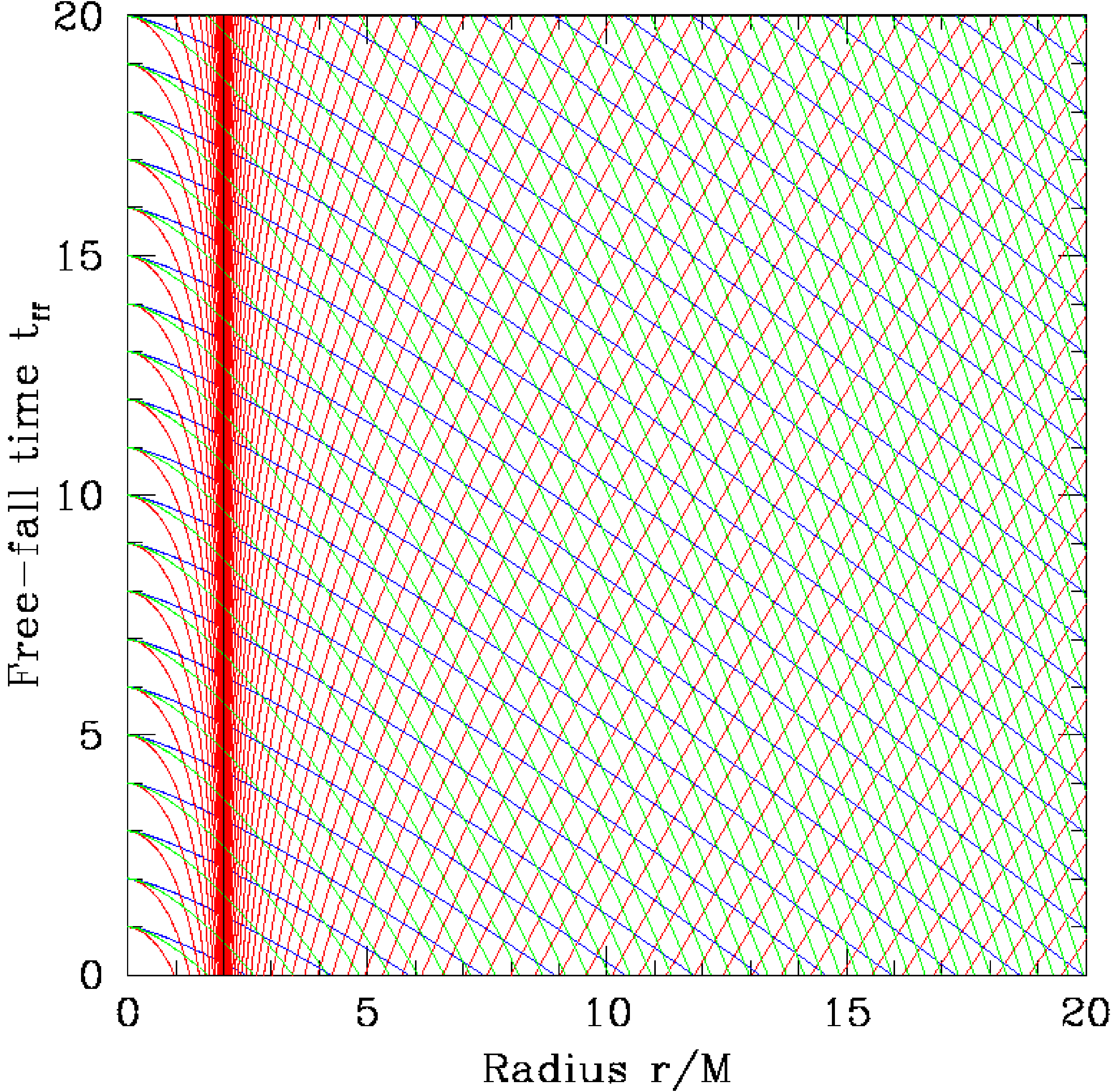
Free−fall time t_{ff}
Radius r/M
0
5
10
15
20
0
5
10
15
20



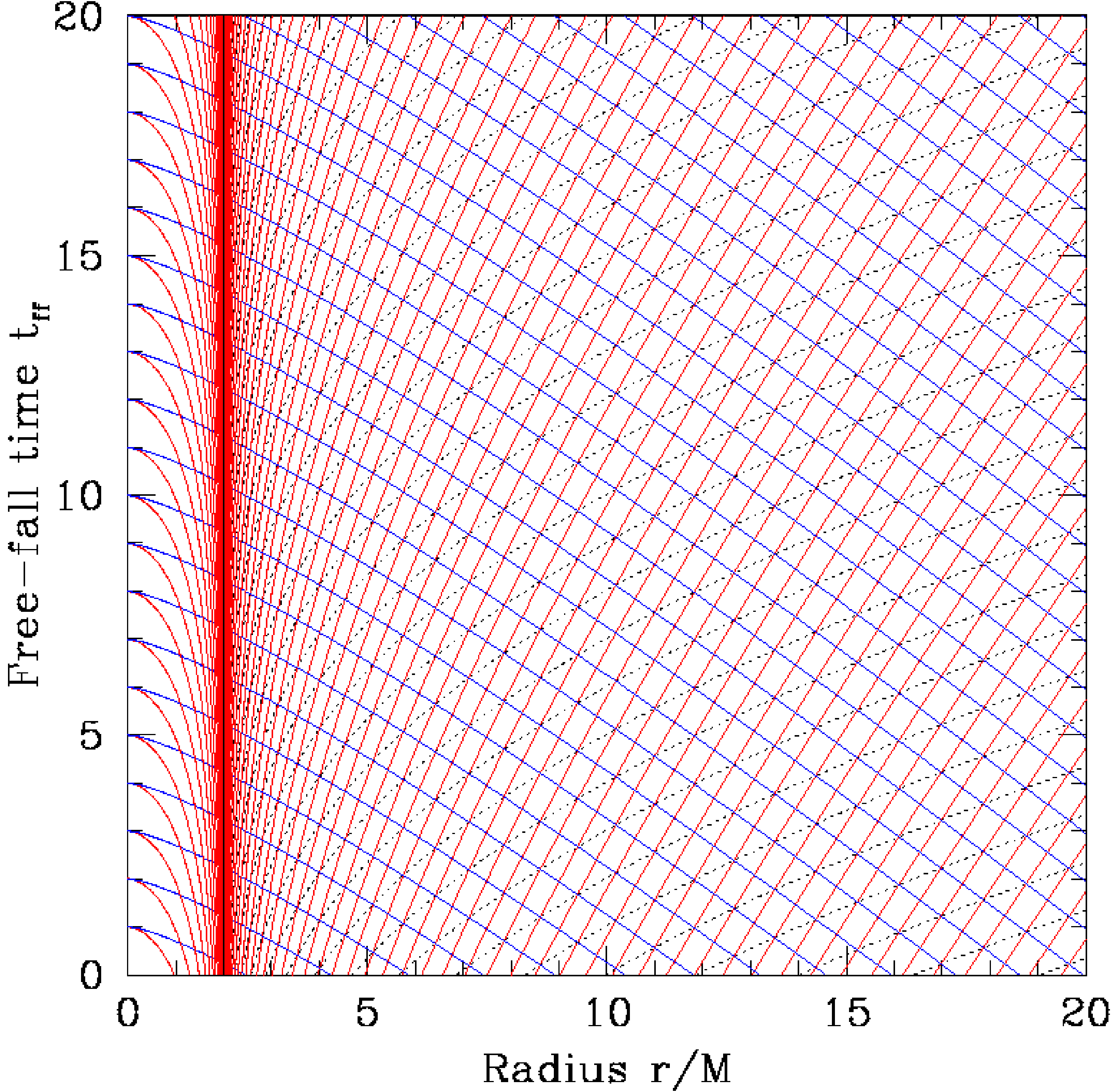
Free-fall time t_{ff}
Radius r/M
0
5
10
15
20



Free−fall time t_{ff}
Radius r/M
0
5
10
15
20
0
5
10
15
20



Free−fall time t_ff
Radius r/M
0
5
10
15
20
0
5
10
15
20



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Radius r/M
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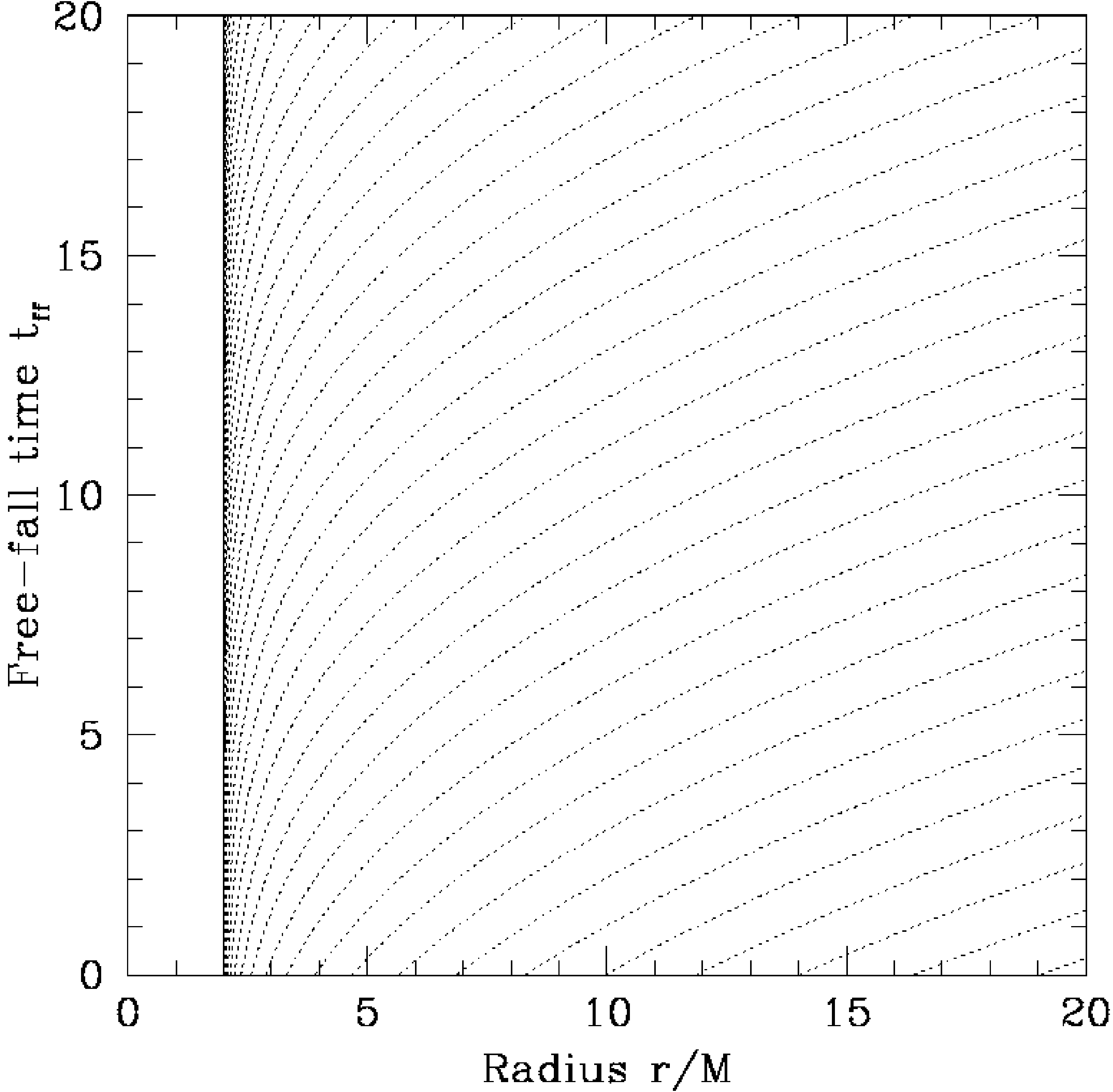
15
20



Free-fall time t_{ff}
Radius r/M
0
5
10
15
20
0
5
10
15
20